

# Determination of lightning currents from far electromagnetic fields: Effect of a strike object

J.L. Bermudez<sup>a,1</sup>, F. Rachidi<sup>b,\*</sup>, W. Janischewskyj<sup>c</sup>, V. Shostak<sup>d</sup>, M. Rubinstein<sup>a</sup>,  
D. Pavanello<sup>b</sup>, A.M. Hussein<sup>e</sup>, J.S. Chang<sup>f</sup>, M. Paolone<sup>g</sup>

<sup>a</sup>University of Applied Sciences of Western Switzerland, Yverdon, Switzerland

<sup>b</sup>Swiss Federal Institute of Technology, Power Systems Laboratory, EPFL-LRE, 1015 Lausanne, Switzerland

<sup>c</sup>University of Toronto, Department of Electrical and Computer Engineering, Toronto, Canada

<sup>d</sup>Kyiv Polytechnic Institute, Kiev, Ukraine

<sup>e</sup>Ryerson University, Electrical and Computer Engineering Department, Toronto, Canada

<sup>f</sup>McMaster University, Hamilton, Canada

<sup>g</sup>University of Bologna, Bologna, Italy

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## Abstract

We discuss in this paper the influence of the presence of an elevated strike object on the peak of the lightning return stroke current determined from remote field measurements. We develop analytical expressions relating the lightning return stroke channel-base current and the far electromagnetic field for different specific cases, namely, (1) ground-initiated return strokes (classical transmission line (TL) model), (2) ground-initiated return strokes including possible reflections at ground level, (3) tall strike objects for which the current's zero-to-peak time is smaller than the travel time along the object, and (4) electrically short strike objects. It is shown that for tall structures, the field enhancement relative to a return stroke initiated at ground level is expressed through a factor equal to  $k_{\text{tall}} = [1 + c/v(1 - 2\rho_t)]/(1 - \rho_t)$ , where  $v$  and  $c$  are the return stroke front speed and the speed of light in vacuum, respectively, and  $\rho_t$  is the top reflection coefficient. For very short towers and/or very slow return stroke current wavefronts, when the condition  $t_f \gg h/c$  applies, expressions relating the far electromagnetic field and the return stroke current were also derived. For case (2), return strokes initiated at ground level, the field enhancement relative to a return stroke initiated at ground level, case (1), is expressed through a factor equal to  $k_{\text{short}} = (1 + (c/v)\rho_{\text{ch-g}})/(1 + \rho_{\text{ch-g}})$ , where  $\rho_{\text{ch-g}}$  represents the reflection coefficient between the lightning channel and the grounding impedance.

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## 1. Introduction

The determination of the peak return stroke current from remotely measured electric or magnetic fields considerably facilitates the collection of lightning return stroke current data without having to instrument towers or trigger the lightning artificially, and without the inherent relative inefficiency associated with those methods. This is especially true nowadays because of the widespread use of

lightning location systems. Indeed, such systems are already used also to provide estimates of lightning current parameters (e.g. [1–3]).

The theoretical estimation of return stroke currents from remote electromagnetic fields depends on the adopted return stroke model. Based on the transmission line (TL) model, for an observation point at ground level, the radiated (far) electric and magnetic field peaks produced by a vertical lightning channel terminated directly at ground are simply proportional to the channel base current peaks [4]

$$E_Z^{\text{far}}(\mathbf{r}, t + r/c) = -\frac{v}{2\pi\epsilon_0 c^2 r} i(0, t), \quad (1)$$

\*Corresponding author.

E-mail address: [farhad.rachidi@epfl.ch](mailto:farhad.rachidi@epfl.ch) (F. Rachidi).

<sup>1</sup>Currently with ABB-Sécheron, Geneva, Switzerland.

$$H_{\varphi}^{\text{far}}(\mathbf{r}, t + r/c) = \frac{v}{2\pi cr} \mathbf{i}(0, t), \quad (2)$$

where  $\mathbf{i}(0, t)$  is the current at the channel-base,  $v$  is the return stroke speed, and  $r$  is the distance from the channel-base to the observation point. Eqs. (1) and (2) are derived assuming that the return stroke is initiated at ground.

Recently, experimental data on instrumented towers and theoretical investigations have shown that the above expressions are not applicable to lightning strikes to elevated objects [5–8]. The aim of this paper is to analyze the relation between lightning currents and distant electromagnetic fields for different specific cases, namely:

- ground-initiated return strokes: reference case corresponding to the classical return stroke models;
- strokes terminating on tall structures, where the current risetime,  $t_f$ , is smaller than  $2h/c$ ,  $h$  being the height of the structure;
- strokes terminating on electrically short structures, where  $t_f \gg h/c$ , and, finally
- the special case of electrically short structures when  $h = 0$ , in other words ground-initiated strokes but taking into account possible reflections at ground level (channel-grounding impedance discontinuity).

For each case, we will derive expressions relating the peak value of the far field to the peak value of the channel base current. The analysis is based on the TL model. We will discuss, however, the generalization of the results to other engineering models.

## 2. TL model for lightning return strokes extended to include an elevated strike object

Several engineering return stroke models have been extended to account for the presence of a tall strike object in the expressions of the lightning return stroke current along the channel (e.g. [5,9–15]). In this paper, we will use the version of the TL return stroke model recently extended in [16] to include the effect of the elevated strike object.

Representing the elevated strike object of height  $h$  as an ideal (lossless) transmission line of length  $h$  with constant current reflection coefficients  $\rho_t$  and  $\rho_g$  at its extremities, the expression for the spatial-temporal distribution of the current along the strike object and along the channel are given by [16,17]

$$\mathbf{i}(z, t) = (1 - \rho_t) \sum_{n=0}^{\infty} \left[ \rho_t^n \rho_g^n \mathbf{i}_o \left( \mathbf{h}, t - \frac{h-z}{c} - \frac{2nh}{c} \right) + \rho_t^n \rho_g^{n+1} \mathbf{i}_o \left( \mathbf{h}, t - \frac{h+z}{c} - \frac{2nh}{c} \right) \right], \quad 0 \leq z \leq h, \quad (3a)$$

$$\mathbf{i}(z, t) = \mathbf{i}_o \left( \mathbf{h}, t - \frac{z-h}{v} \right) - \rho_t \mathbf{i}_o \left( \mathbf{h}, t - \frac{z-h}{c} \right) + (1 - \rho_t)(1 + \rho_t) \sum_{n=0}^{\infty} \rho_g^{n+1} \rho_t^n \times \mathbf{i}_o \left( \mathbf{h}, t - \frac{h+z}{c} - \frac{2nh}{c} \right), \quad h < z < H_T, \quad (3b)$$

where the ‘undisturbed’ current  $\mathbf{i}_o(\mathbf{h}, t)$  is the idealized current that would be measured at the tower top if the current reflection coefficients at both its extremities were equal to zero. Note that Eqs. (1) and (2) can be equivalently expressed in terms of the short-circuit current  $\mathbf{i}_{\text{sc}}(t) = 2\mathbf{i}_o(\mathbf{h}, t)$  [20].

$H_T$  represents the total height of both the lightning channel and the elevated strike object.

We will disregard any upward connecting leader and any reflections at the return stroke wavefront, even though it has been shown in [15] that some features of the radiated field could be attributed to these phenomena.

The bottom reflection coefficient for the current in the tower can be expressed in terms of the characteristic impedance of the tower  $Z_t$  and the grounding system impedance  $Z_g$ ,

$$\rho_g = \frac{Z_t - Z_g}{Z_t + Z_g}. \quad (4)$$

The top reflection coefficient for the current in the tower can be expressed in terms of the characteristic impedance of the tower  $Z_t$  and the equivalent impedance of the lightning return-stroke channel  $Z_{\text{ch}}$ ,

$$\rho_t = \frac{Z_t - Z_{\text{ch}}}{Z_t + Z_{\text{ch}}}. \quad (5)$$

This top reflection coefficient can be defined in a similar manner for currents observed in the channel and, in that case,  $Z_t$  and  $Z_{\text{ch}}$  should be interchanged in (5).

## 3. Radiated electromagnetic field

The general expressions for the vertical electric field and the azimuthal magnetic field from a vertical antenna above a perfectly conducting ground, for an observation point at ground level (see Fig. 2), are given by [18]

$$E_z(\mathbf{r}, t) = \frac{1}{2\pi\epsilon_0} \left[ \int_0^H \frac{2z^2 - r^2}{R^5} \int_{R/c}^t \mathbf{i}(z, \tau - R/c) d\tau dz + \int_0^H \frac{2z^2 - r^2}{cR^4} \mathbf{i}(z, t - R/c) dz - \int_0^H \frac{r^2}{c^2 R^3} \frac{\partial \mathbf{i}(z, t - R/c)}{\partial t} dz \right], \quad (6)$$

$$H_{\varphi}(\mathbf{r}, t) = \frac{1}{2\pi} \left[ \int_0^H \frac{r}{R^3} \mathbf{i}(z, t - R/c) dz + \int_0^H \frac{r}{cR^2} \frac{\partial \mathbf{i}(z, t - R/c)}{\partial t} dz \right], \quad (7)$$

where  $H$  is the height of the return stroke wavefront as seen by the observer,  $r$  is the horizontal distance between the channel and the observation point, and  $R$  is the distance between a single dipole located at a height  $z$  above ground and the observation point ( $R = \sqrt{r^2 + z^2}$ ).

Let us consider here only the radiated electric field. For distant observation points, neglecting the static and induction components of the electric field, and considering  $R \cong r$  and  $r \gg H$ , the general expression for the electric field [18] for an observation point located at ground level reduces to

$$E_z(\mathbf{r}, t) \cong -\frac{1}{2\pi\epsilon_0 c^2 r} \int_0^H \frac{\partial i(z', t - r/c)}{\partial t} dz'. \quad (8)$$

Introducing the expressions for the spatial–temporal distribution of the current in (3a) and (3b) into (8), and after appropriate mathematical manipulations, we obtain expression (9) for the radiated (far) electric field [17,19]

$$\begin{aligned} E_z(\mathbf{r}, t + r/c) = & -\frac{v}{2\pi\epsilon_0 c^2 r} \left[ 1 + \frac{c}{v} (1 - 2\rho_t) \right] i_o(\mathbf{h}, t) \\ & - \frac{(1 - \rho_t)}{2\pi\epsilon_0 c r} \sum_{n=0}^{\infty} \left\{ (\rho_g - 1) \rho_g^n \rho_t^n \right. \\ & \times i_o \left( \mathbf{h}, t - \frac{(2n+1)h}{c} \right) + 2\rho_g^{n+1} \rho_t^{n+1} \\ & \left. \times i_o \left( \mathbf{h}, t - \frac{2(n+1)h}{c} \right) \right\}. \end{aligned} \quad (9)$$

Similarly, for the magnetic field,

$$\begin{aligned} H_\phi(\mathbf{r}, t + r/c) = & \frac{v}{2\pi c r} \left[ 1 + \frac{c}{v} (1 - 2\rho_t) \right] i_o(\mathbf{h}, t) \\ & + \frac{(1 - \rho_t)}{2\pi r} \sum_{n=0}^{\infty} \left\{ (\rho_g - 1) \rho_g^n \rho_t^n \right. \\ & \times i_o \left( \mathbf{h}, t - \frac{(2n+1)h}{c} \right) + 2\rho_g^{n+1} \rho_t^{n+1} \\ & \left. \times i_o \left( \mathbf{h}, t - \frac{2(n+1)h}{c} \right) \right\}. \end{aligned} \quad (10)$$

Eqs. (9) and (10) represent the general expressions for the far radiated electric and magnetic fields in the presence of an elevated strike object, under the assumptions introduced in [17,19].

## 4. Relations between return stroke far fields and currents

### 4.1. Ground-initiated return strokes

This case corresponds to the classical TL model. Eqs. (1) and (2) can be rewritten in terms of current and field peak values:

$$E_{z \text{ peak}}^{\text{far}} = -\frac{v}{2\pi\epsilon_0 c^2 r} I_{\text{peak}}, \quad (11)$$

$$H_{\phi \text{ peak}}^{\text{far}} = \frac{v}{2\pi c r} I_{\text{peak}}. \quad (12)$$

### 4.2. Tower-initiated return stroke

Let us call  $t_f$  the zero-to-peak risetime of the lightning return stroke current waveform. We will consider two cases: (1) ‘tall’ structures, for which the round-trip propagation time from top to bottom within the tower ( $2h/c$ ) is greater than the time  $t_f$ , in which case the current transmitted into the tower reaches its peak before the arrival of any ground reflections (none of the reflections overlap with it); and, (2) electrically short structures, for which the round-trip propagation time is much shorter than the lightning return stroke current wavefront  $t_f$ , in which case we can neglect propagation delays along the tower.

#### 4.2.1. ‘Tall’ strike object ( $t_f < 2h/c$ )

Noting that all terms on the right-hand side of Eqs. (9) and (10) except for the first are zero for times satisfying the inequality ( $t_f < 2h/c$ ), we can write for  $t = t_f$ ,

$$E_z^{\text{far}}(\mathbf{r}, t_f + r/c) = -\frac{v}{2\pi\epsilon_0 c^2 r} \left[ 1 + \frac{c}{v} (1 - 2\rho_t) \right] i_o(\mathbf{h}, t_f), \quad (13)$$

$$H_\phi^{\text{far}}(\mathbf{r}, t_f + r/c) = \frac{v}{2\pi c r} \left[ 1 + \frac{c}{v} (1 - 2\rho_t) \right] i_o(\mathbf{h}, t_f). \quad (14)$$

Since the current on the right-hand side of Eqs. (13) and (14) is the first peak of the undisturbed current, and the fields on the left-hand side are the first peak in the far electric and magnetic fields, we can write

$$E_{z \text{ peak}} = -\frac{v}{2\pi\epsilon_0 c^2 r} \left[ 1 + \frac{c}{v} (1 - 2\rho_t) \right] I_{o \text{ peak}}, \quad (15)$$

$$H_{\phi \text{ peak}} = \frac{v}{2\pi c r} \left[ 1 + \frac{c}{v} (1 - 2\rho_t) \right] I_{o \text{ peak}}, \quad (16)$$

where  $I_{o \text{ peak}}$  is the first peak of the undisturbed current  $i_o(\mathbf{h}, t)$ .

It is important to note that the undisturbed current  $i_o(\mathbf{h}, t)$  is different from the actual current pulse injected from the channel into the tower top. It would be therefore, more appropriate to express the electromagnetic field peaks as a function of the current transmitted into the tower, for which experimental data are usually available. To do that, one needs to express the undisturbed current peak  $I_{o \text{ peak}}$  as a function of the peak of the current transmitted into the tower,  $I_{\text{peak}}$ . Under the current conditions of tall towers ( $t_f < h/c$ ), these two quantities are simply related by

$$I_{\text{peak}} = (1 - \rho_t) I_{o \text{ peak}}, \quad (17)$$

where  $\rho_t$  is given by (5).

Introducing (17) into (15) and (16), we obtain

$$E_{z \text{ peak}} = -\frac{v}{2\pi\epsilon_0 c^2 r} k_{\text{tall}} I_{\text{peak}}, \quad (18)$$

$$H_{\phi \text{ peak}} = \frac{v}{2\pi c r} k_{\text{tall}} I_{\text{peak}}, \quad (19)$$

where  $k_{\text{tall}}$  is given by

$$k_{\text{tall}} = \frac{1 + (1 - 2\rho_t)c/v}{1 - \rho_t}. \quad (20)$$

Comparing Eqs. (18) and (19) with Eqs. (11) and (12), we can see that the enhancement effect of the tower can be quantified through the factor  $k_{\text{tall}}$ .

Because of the condition  $t_r < 2h/c$  imposed on the current, Eqs. (18) and (19) are independent of the structure's height  $h$  and of the ground reflection coefficient  $\rho_g$ .

It is also important to note that, as the grounding impedance of the tower is generally noticeably smaller than its own characteristic impedance and since the characteristic impedance is, in turn, appreciably lower than the equivalent impedance of the lightning channel (e.g. [20,21]), the current reflection coefficient at the ground  $\rho_g$  is positive and the top reflection coefficient  $\rho_t$  is negative. Thus, the factor  $k_{\text{tall}}$  in Eqs. (19) and (20) is greater than 1, implying that the presence of the strike object enhances the electric and magnetic field peaks in comparison to return strokes initiated at ground level (Section 4.1).

Fig. 1 presents the variation of the enhancement factor  $k_{\text{tall}}$  as a function of the top reflection coefficient and the return stroke speed.

#### 4.2.2. Electrically short strike object ( $t_r \gg 2h/c$ )

Let us now consider the special case in which the strike object is electrically short. This would be the case, for instance, when lightning is initiated artificially from short platforms, or for very long-front pulses. This condition can be expressed mathematically by,  $t_r \gg 2h/c$ .

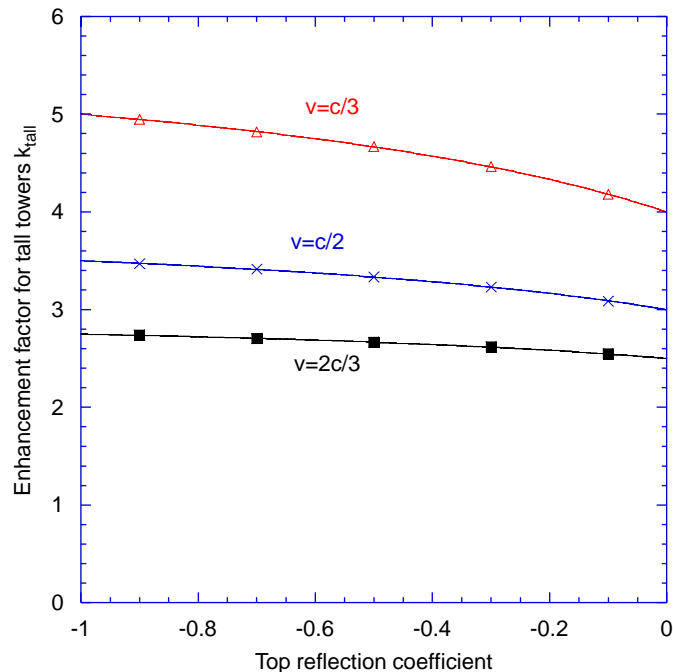


Fig. 1. Variation of the enhancement factor for a tall tower as a function of the top reflection coefficient  $\rho_t$  and the return stroke speed  $v$ .

In this case, propagation along the tower can be neglected and closed-form expressions can be derived for the spatial-temporal distribution of the current along the strike object and along the channel.

Neglecting propagation phenomena along the tower (note that propagation effects in the channel still need to be taken into account), after straightforward mathematical manipulations [19], the expressions for the far electric and magnetic fields are given by

$$E_z^{\text{far}}\left(\mathbf{r}, t + \frac{r}{c}\right) \cong -\frac{v}{2\pi\epsilon_0 c^2 r} \left(1 + \frac{c}{v}\rho_{\text{ch-g}}\right) i_o(\mathbf{h}, t) - \frac{(1 + \rho_{\text{ch-g}})h}{2\pi\epsilon_0 c^2 r} \frac{\partial i_o(\mathbf{h}, t)}{\partial t}, \quad (21)$$

$$H_\varphi^{\text{far}}\left(\mathbf{r}, t + \frac{r}{c}\right) \cong \frac{v}{2\pi c r} \left(1 + \frac{c}{v}\rho_{\text{ch-g}}\right) i_o(\mathbf{h}, t) + \frac{(1 + \rho_{\text{ch-g}})h}{2\pi c r} \frac{\partial i_o(\mathbf{h}, t)}{\partial t}, \quad (22)$$

where  $\rho_{\text{ch-g}} = (\rho_g - \rho_t)/(1 - \rho_g\rho_t)$ .

It is interesting to note that after insertion of the expressions for  $\rho_g$  and  $\rho_t$  in terms of impedances, ( $\rho_g = (Z_t - Z_g)/(Z_t + Z_g)$  and  $\rho_t = (Z_t - Z_{\text{ch}})/(Z_t + Z_{\text{ch}})$ ),  $\rho_{\text{ch-g}}$  reduces to  $\rho_{\text{ch-g}} = (Z_{\text{ch}} - Z_g)/(Z_{\text{ch}} + Z_g)$ , which is the reflection coefficient at ground level, and it is independent of the impedance of the electrically short tower.

As in the case of tall towers, we can express the undisturbed current  $i_o(\mathbf{h}, t)$  in Eqs. (21) and (22) in terms of the current that would be measured at the top of the tower,  $i(\mathbf{h}, t)$ , for which experimental data are usually available. These two currents can be related through [19]

$$i(\mathbf{h}, t) = (1 + \rho_{\text{ch-g}})i_o(\mathbf{h}, t). \quad (23)$$

Inserting Eq. (23) into Eqs. (21) and (22) yields

$$E_z^{\text{far}}\left(\mathbf{r}, t + \frac{r}{c}\right) \cong -\frac{v}{2\pi\epsilon_0 c^2 r} \frac{1 + (c/v)\rho_{\text{ch-g}}}{1 + \rho_{\text{ch-g}}} i(\mathbf{h}, t) - \frac{h}{2\pi\epsilon_0 c^2 r} \frac{\partial i(\mathbf{h}, t)}{\partial t}, \quad (24)$$

$$H_\varphi^{\text{far}}\left(\mathbf{r}, t + \frac{r}{c}\right) \cong \frac{v}{2\pi c r} \frac{1 + (c/v)\rho_{\text{ch-g}}}{1 + \rho_{\text{ch-g}}} i(\mathbf{h}, t) + \frac{h}{2\pi c r} \frac{\partial i(\mathbf{h}, t)}{\partial t}. \quad (25)$$

The far electric and magnetic fields associated with lightning strikes to electrically short structures include two terms. The first one, which is proportional to the undisturbed current, represents the contribution of the channel, while, the second one, which depends on the current derivative, represents the contribution of the strike object.

### 4.3. Ground-initiated return strokes including reflections at ground level

Note that setting  $h = 0$  in (24) and (25) we can obtain a generalized form of Eqs. (1) and (2) for return strokes initiated at ground level, in which the reflections at ground level are taken into account,

$$E_z^{\text{far}}\left(r, t + \frac{r}{c}\right) \cong -\frac{v}{2\pi\epsilon_0 c^2 r} \frac{1 + (c/v)\rho_{\text{ch-g}}}{1 + \rho_{\text{ch-g}}} i(0, t), \quad (26)$$

$$H_\phi^{\text{far}}\left(r, t + \frac{r}{c}\right) \cong \frac{v}{2\pi cr} \frac{1 + (c/v)\rho_{\text{ch-g}}}{1 + \rho_{\text{ch-g}}} i(0, t). \quad (27)$$

In terms of the peak amplitudes of the current and the fields (for  $t = t_f$ ), (26) and (27) read

$$E_{z \text{ peak}} = -\frac{v}{2\pi\epsilon_0 c^2 r} k I_{\text{peak}}, \quad (28)$$

$$H_{\phi \text{ peak}} = \frac{v}{2\pi cr} k I_{\text{peak}}, \quad (29)$$

where  $k$  is given by

$$k = \frac{1 + (c/v)\rho_{\text{ch-g}}}{1 + \rho_{\text{ch-g}}}. \quad (30)$$

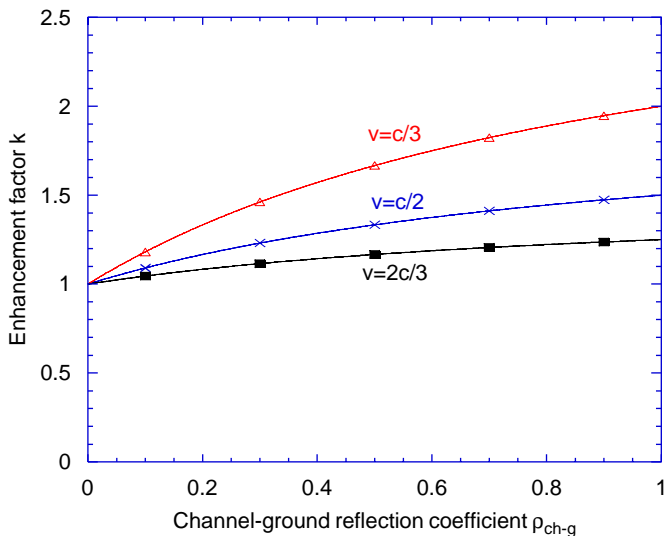


Fig. 2. Variation of the enhancement factor  $k$  as a function of the reflection coefficient  $\rho_{\text{ch-g}}$  and the return stroke speed  $v$ .

In terms of impedances, (30) can be written as

$$k = \frac{Z_{\text{ch}}(1 + (c/v)) + Z_g(1 - (c/v))}{2Z_{\text{ch}}}. \quad (31)$$

Eqs. (28) and (29) will reduce to (11) and (12), respectively, when the reflection coefficient  $\rho_{\text{ch-g}}$  equals 0 or, equivalently, when  $Z_{\text{ch}} = Z_g$ . This consideration, that  $\rho_{\text{ch-g}}$  equals 0, implies that no current reflections will occur between the lightning channel and the ground, as assumed implicitly in the classical TL model. Thus, (28) and (29) can be considered as extended forms of (11) and (12) for return strokes initiated at ground level, in which the reflections at ground are incorporated in the TL model.

Eqs. (28) and (29) can be employed to find the return stroke electric and magnetic field peak amplitudes associated with lightning impacting the ground. These equations take into account the impedance discontinuity between the lightning channel and the grounding impedance. Comparing Eqs. (28) and (29) with Eqs. (11) and (12), we can see that including the grounding condition will result in an enhancement of far fields, which is quantified through the factor  $k$ . This enhancement factor  $k$  is generally much smaller than the factor associated with tall towers  $k_{\text{tall}}$ . Indeed, considering the ideal case when the grounding impedance is 0 and the return stroke speed  $v = c/2$ , the enhancement factor  $k$  is equal to 1.5 (for the same speed  $k_{\text{tall}} > 3$ , see Fig. 1).

Fig. 2 presents the variation of the enhancement factor  $k$  as a function of  $\rho_{\text{ch-g}}$  and the return stroke speed.

Table 1 summarizes the expressions relating the far field and the current for the different considered cases.

## 5. Comparison

Fig. 3 presents the results of comparison between the various expressions in Table 1. Note that, for the case of an electrically short structure, the peak value of the far field depends not only on the peak value of the current, but also on its derivative. However, in a first approach for the calculation of the field peak, the term involving the current derivative can be neglected and the case of electrically short towers becomes similar to the case of a ground-initiated return stroke, considering the reflection at ground level. The calculations in Fig. 3 are obtained considering  $\rho_t = -0.37$ ,  $\rho_{\text{ch-g}} = 0.66$ , and  $v = 1.2 \times 10^8$  m/s. A similar figure can be obtained for the magnetic field using the far-

Table 1  
Summary of far electric field-current relationships for TL model for different considered cases

| Case   | Far field-current relation  |
|--|---|
| Ground initiated (classical TL model)                                  | $E_z^{\text{far}}(r, t + r/c) = -\frac{v}{2\pi\epsilon_0 c^2 r} i(0, t)$  |
| Tall towers, $t_f < 2h/c$  | $E_{z^{\prime}}^{\text{far}}(r, t + r/c) = -\frac{v}{2\pi\epsilon_0 c^2 r} \frac{1 + (1 - 2\rho_t)c/v}{1 - \rho_t} i(h, t)$   |
| Electrically short structures, $t_f \gg h/c$                           | $E_z^{\text{far}}\left(r, t + \frac{r}{c}\right) \cong -\frac{v}{2\pi\epsilon_0 c^2 r} \frac{1 + (c/v)\rho_{\text{ch-g}}}{1 + \rho_{\text{ch-g}}} i(0, t) - \frac{h}{2\pi\epsilon_0 c^2 r} \frac{\partial i(0, t)}{\partial t}$ |
| Ground initiated, taking into account reflection coefficient at ground | $E_z^{\text{far}}\left(Y, t + \frac{Y}{c}\right) \cong -\frac{v}{2\pi\epsilon_0 c^2 r} \frac{1 + (c/v)\rho_{\text{ch-g}}}{1 + \rho_{\text{ch-g}}} i(0, t)$  |



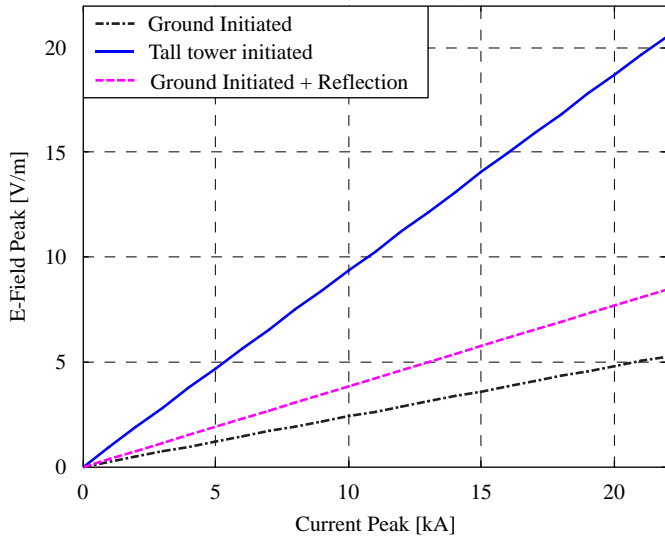


Fig. 3. Electric field peaks as a function of return stroke current peak, calculated at 100 km. Other parameters,  $\rho_t = -0.37$ ,  $\rho_{ch-g} = 0.66$ ,  $v = 1.2 \times 10^8$  m/s.

field relationship for an electromagnetic wave propagating in free space ( $E/H = Z_0$ ).

It can be seen that the presence of a tall tower enhances considerably the radiated fields. Also, possible reflections at ground level would increase, but to a lesser degree, the radiated fields.

## 6. Conclusions

The derived expressions show that, for tall structures satisfying the condition  $t_f < 2h/c$ , the field enhancement relative to a return stroke initiated at ground level according to the classical TL model is expressed through a factor equal to  $k_{tall} = [1 + (c/v)(1 - 2\rho_t)] / (1 - \rho_t)$ , where  $v$  and  $c$  are the return stroke front speed and the speed of light in vacuum, respectively, and  $\rho_t$  is the top reflection coefficient. Since the top reflection coefficient  $\rho_t$  is typically negative, lightning strikes to tall towers can result in a significant enhancement of the radiated electromagnetic field, which can be a few times larger than the field radiated by a similar return stroke but initiated at ground level. For very short towers and/or very slow return stroke current wavefronts, when condition  $t_f \gg h/c$  applies, expressions relating the far electromagnetic field and the return stroke current were also derived. For the case of return strokes initiated at ground level ( $h = 0$ ), the derived expressions represent a generalization of Eqs. (1) and (2), in which grounding conditions are taken into account. In this case, the field enhancement relative to a return stroke initiated at ground level (according to the classical TL model) is expressed through a factor equal to  $k_{short} = (1 + (c/v)\rho_{ch-g}) / (1 + \rho_{ch-g})$ , where  $\rho_{ch-g}$  represents the reflection coefficient between the lightning channel and the grounding impedance.

The presented results are based on the TL model for the return stroke. However, in a recent study [22], Pavanello

et al. have shown that the early-time response of the electromagnetic fields radiated by lightning strikes to elevated strike objects is nearly independent of the return stroke model. Therefore, the derived expressions for tower-initiated strikes can also be considered as applicable for other engineering models.

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