A One Dimensional Model for Blood Flow: Application to Vascular Prosthesis

Luca Formaggia[1], Fabio Nobile[1], Alfio Quarteroni[1,2]

[1] Mathematics Department, EPFL, Lausanne, CH-1015, Switzerland
[2] Mathematics Department, Politecnico di Milano, I-20133 Milano, Italy

Abstract. We investigate a one dimensional model of blood flow in human arteries. In particular we consider the case when an abrupt variation of the mechanical characteristic of an artery is caused by the presence of a vascular prosthesis (e.g. a stent). The derivation of the model and the numerical scheme adopted for its solution are detailed. Numerical experiments show the effectiveness of the model for the problem at hand.

1 Introduction

The growing interest in the use of mathematical modelling and numerical simulations for the investigation of biomedical issues and, in particular, the human cardiovascular system, is testified by the numerous works which have appeared on the subject in recent years, among which we mention [1,6,10] and the references therein. In this context, often simple models are already able to provide good indications for the practitioners, at a reasonable computational cost.

Here, we will focus on the application of a one-dimensional model of blood flow in a compliant vessel to study the effect on the flow pattern caused by the local stiffening of an artery. This can be due to a stent implantation, or to the presence of a vascular prosthesis. A common pathology in the human circulatory system is the on-rise of atherosclerotic plaques which cause a restriction of the arterial lumen called a stenosis; in the most severe cases this may hinder, or even stop, the flow of blood. One of the techniques nowadays used for curing this problem is the implantation of a *stent* (an expandable metal mesh) into the affected region which has the purpose of returning the artery lumen to approximately its original shape. Whenever possible, this procedure is preferred to more invasive ones, such as surgical by-pass.

However, besides other effects, the presence of a stent causes an abrupt variation in the elastic properties of the vessel wall, since the stent is usually far more rigid than the rather soft arterial tissue. This fact may cause a disturbance in the blood flow pattern (and in particular in the pressure) with the appearance of reflected waves. Indeed, the so–called pressure pulse is generated by the interaction between the blood flow and the compliance of the circulatory system and is intrinsically related to the elastic properties of the arteries. The alteration in the pressure pattern is even more important

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