CONSIDERATION OF THE DYNAMIC EFFECT OF INCREASED TRAIN LOADS FOR THE FATIGUE EXAMINATION OF CONCRETE BRIDGES

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Abstract

The influence of increased train loads on the dynamic amplification for bridge action effects are investigated using simple models such as two-mass oscillators.

The results show, the bridge dynamic amplification factor decreases when carriage weights are increased. This procedure facilitates more accurate calculation of fatigue stresses.

Keywords

stress calculation, dynamic train-bridge interaction, fatigue, dynamic amplification factor.

1 Introduction

In view of higher future rail traffic loads on the European Rail Network the accurate determination of the stress range in reinforced concrete bridges is an important input data for the evaluation of the residual fatigue life as the stress influences considerably the result, particularly in the relevant domain of high cycle fatigue.

The parameters velocity, fundamental frequency and length of the bridge are currently used to define the dynamic amplification factor for bridge action effects (UIC 1994). The effects of un sprung wheel set mass due to track irregularities is given by a separate formula. However, the height of the axle load is not considered.

Simple models are suggested to investigate the influence of the height of the train mass on dynamic response of the bridge. The model has to represent three major components: the vehicle, the bridge and the excitation. The advantages of simple models are that they are easy to build and render possible the analysis of the actual problem (Ludescher 2004).

This paper highlights the influence of increased train loads on the dynamic amplification factor. The objective is to investigate dynamic amplification factors for determination of fatigue stress in bridge elements. First the influence of carriage weight on wheel force amplifications than the amplification for bridge action effects due to moving trains is mapped.

2 Wheel force amplifications due to track irregularities

Wheel force amplifications are the most important cause for the initiation of dynamic interaction between bridge main girder and fatigue effective freight trains. Resonance is unlikely to occur as the speeds generally are much lower than the critical speed for resonance (Yang 2004) of the fundamental mode of the bridge which is most relevant for bridge force amplifications.

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2.1 Modelling of track irregularities

The most important cause for wheel force amplifications are track irregularities which excite vibrations of car body and “un sprung” wheel set masses.

Irregularities in vertical direction with equal shape for both rails are assumed as it can be encountered mostly anywhere on track. High differences in elevation (degradation of ballast, settlements) are related to high wavelengths whilst low differences in elevation such as rail corrugation and wheel flats have short wave lengths. The wavelength of the track irregularities together with the running speed determine the excitation frequencies which accordingly to the transfer behaviour leads to wheel force amplifications of the carriage. Figure 1 shows the result of a vertical track bed measurement. The available scale gives information on track irregularity of wave length in the upper domain. The dominant wave length is about 35 m. With a velocity of 35 m/s (130 km/h) the excitation frequency would be \( f_{exc} = 1 \text{Hz} \) with a difference in elevation of about 7 mm.

![Figure 1 Typical measured track irregularities](image)

Track irregularities are not always of periodic nature; there are also isolated track irregularities such as differences in elevation at the abutments of bridges, locally unsupported sleepers or solitary supported sleepers.

2.2 Modelling of carriages

Most goods wagons have no secondary suspension (secondary suspensions are arranged between bogie and car body and used in passenger cars to ensure adequate riding comfort). Subsequently, a two-mass oscillator with excitation at its base is used, reproducing only vertical movements of the carriage system. Figure 2 shows the model which is used to describe the behaviour of two axle goods wagons, which are still in use in European network and are in general dynamically more aggressive than goods wagons with two axle bogies. The wheel force behaviour of one wheel can be represented by the equivalent car body mass (½ car body mass). The model, subsequently called “carriage” covers 4 components: the car body mass, its suspension, the “un sprung” wheel set mass and the ballast layer. The track superstructure (rail, sleepers and ballast) is considered as a spring moving with the axle where the mass is neglected (the mass of track superstructure will be added to the bridge mass). The track superstructure has a non linear behaviour; for higher forces a higher stiffness has to be considered.

The excitation of vibrations due to track irregularities is placed below the spring.

![Figure 2: Simplified model for the acquisition of dynamic wheel forces](image)
2.3 Car weight influence on the wheel force amplification

In order to investigate the behaviour of the carriage excited by track irregularities the transfer function of the two-mass oscillator in figure 1 is established. The transfer function describes the response of the system at harmonic excitation in the stationary state.

![Graph showing variations in wheel force amplification with car body mass and excitation frequency.](image)

**Figure 3:** Influence of the car body mass over the whole frequency spectra: left: carriage with linear suspension, \( f_{\text{car body}} = 1.5 \text{ Hz}, \ \zeta_{\text{susp}} = 15\% \), right with leaf spring; common parameters: excitation frequency \( a_{\text{exc}} = 5 \text{ mm}, m_{\text{car body}} = 10 \text{ to}, m_{\text{wheel set}} = 1 \text{ to}, k_{\text{tyres}} = 2\cdot2 \text{ MN/m}, \ \zeta_{\text{tyres}} = 3\% \)

Figure 3 left shows the result of a linear suspended and at the right side a leaf suspended “carriage”. Clearly visible in both cases are the marked amplifications in the domain of the car body natural frequency (left hump) and in the domain of the axle natural oscillation (right hump). The curve for the higher car body mass in both cases is situated clearly below the curve of the lower car body mass. It should be mentioned that these curves are obtained with a model representing the dynamic properties of vehicles, where the stiffness of the wheel set is given through a rather soft suspension-and-tyre system. In the case of a railway vehicle, the “tyre” includes also the track superstructure, which has the function of absorbing wheel set vibrations. Due to the high stiffness of the track superstructure, the natural frequency of wheel set vibrations is thus expected to be rather high which means that the curves in the domain of wheel set oscillations (right hump of the curve of figure 3) should be displaced to the right relative to the mapped one. However the curves are useful to highlight the influence of car body mass. Attention should be paid to the fact that in addition the track superstructure has non linear force-displacement behaviour.

![Graph showing wheel force amplification as a function of the carriage weight and stiffness.](image)

**Figure 4:** Wheel force amplification factor as a function of the carriage weight and stiffness

Figure 3 can also be seen as an information of the devolution of the amplification factor as a function of the excitation frequency caused by the track irregularities \( f_{\text{exc}} \) (running speed divided by the dominant wave length of track irregularity). In the case of linear as well of leaf springs the frequency ranges with maximum excitation are limited on the domains 1-2 and 10-15 Hz. Assuming in every case the biggest amplification would thus lead to very conservative amplification factors. On the other hand, the track profile at a specific bridge can exhibit the biggest unevenness at exactly the same wave length, which together with the
running speed of a heavily loaded train gives the most adverse excitation frequency. The peak values therefore can’t be ignored completely.

Particularly demonstrative, the influence of the car body mass can be identified by using a one-mass oscillator as carriage model which can be seen as a car body and its suspension (figure 4). As action, an excitation by a located track irregularity is simulated by introducing an impulse in the system by the release of an imposed deflection. In figure 4 left the amplification factor as a function of the carriage weight and the suspension stiffness is mapped. The decrease of the amplification factor with increased car body mass is clearly recognizable for 3 different values of the suspension stiffness. This decrease can be explained by the decreasing significance of the dynamic part of the maximum force. An invariable amplification factor with increased car body weight would necessitate also an increased stiffness of the suspension.

3 Bridge action effect amplifications

3.1 Dynamic carriage behaviour

In the above sections it could be demonstrated that wheel force amplifications due to track irregularities of different type decrease with increased carriage load. To be able to create a model for the study of the influence of increased carriage load on bridge action effects, first a new model for the carriage is established.

Figure 5: Modelling of a carriage as composition of one-mass oscillators under forced vibration (left) and transfer function of the linear one-mass oscillator for a damping rate of 10 % (right)

Figure 5 schematically shows how the acquisition of equivalent one-mass oscillators is made whose properties correspond to the different natural modes which can be obtained through a modal analysis of a carriage as a multi body oscillator. Thereby the carriage can be traced back to the base of the one-mass oscillator with excitation at its base. (The carriage model of above sections, consisting of two masses can be reduced to two one-mass oscillators, each consisting of a modal mass and the corresponding natural frequency). At right a typical transfer function for a one-mass oscillator with forced vibration is mapped. This transfer function is valuable for every kind of linear one-mass oscillator with a damping rate of 10%. Parameter a is the amplitude of the harmonic displacement which excites the oscillator and $\Delta z$ is the amplitude of the oscillation in stationary state. As medium and short span bridges always carry two carriages (the two halves of adjacent carriages) when maximum action effect occurs, the ends of two carriages are joined in the model and is hereafter called "carriage". Within the present studies it is assumed that the two halves always move in phase which is a conservative assumption.

3.2 Dynamic bridge behaviour

Different natural modes of bridge vibrations can be covered by equivalent one-mass oscillators just like it is possible with the carriages. Figure 6 schematically shows how to get through modal analysis to the different natural modes with corresponding modal masses.
Each natural mode can be modelled as a one-mass oscillator of equal dynamic properties. At right, a typical transfer function of a linear one-mass oscillator with 1% damping is mapped. The difference with the transfer function of the carriage is evident; the amplification factor is 10 times higher but in a very narrow domain. However, damping normally is higher with high velocities which considerably reduces the peak value.

![Modal analysis](image)

**Figure 6:** Modal analysis of a simple beam (left) and transfer function of a simple oscillator for a damping rate of 1 % (right)

As in the rather theoretic appearing curve of figure 6 narrow domains of maximal amplification can be observed also in response spectra of real bridges (Frýba 1998) which confirms the assumptions made.

### 3.3 Modelling of the system carriage + bridge

Two components for the model used in the parametric study for the carriage-bridge interaction have been erected; a one-mass oscillator for the carriage and the one-mass oscillator for the bridge, each one representing the behaviour of its (uncoupled) natural modes.

![Model of the two-mass oscillator](image)

**Figure 7:** Model of the two-mass oscillator for the simulation of the carriage-bridge interaction

On the real bridge, the carriage changes its position which has an influence on the intensity of the excitation and also higher modes are excited. Furthermore, in particular heavy carriages influence the fundamental frequency of the system during their passage (Ludescher 2004). But the position of the carriage is a parameter which falls out when comparisons between mass ratios are made. Hence, a two-mass oscillator model can be used.

The scenario where the dynamic behaviour of the system is dominated by the 1st natural mode of the carriage and the fundamental mode of the bridge is the base of the model presented in figure 7. The excitation function \( z_{\text{exc}}(t) \) of the model consists of a simplified track bed in form of a sinusoidal curve.

The scenario, where the 2nd natural mode of the vehicle is dominant is treated in 3.6.
3.4 Analysis in the time domain, influence of carriage mass

Figures 8 and 9 show the response of the model under sinusoidal excitation with an assumed amplitude of 1 mm. The velocity is chosen such that the excitation frequency corresponds exactly to the fundamental frequency $f_1$ of the system which leads to the strongest reaction of the bridge. On the left, the calculated displacements of “carriage” and “bridge” and on the right the corresponding forces are mapped. For the bridge, an effective mass of 100 to (total mass 200 to) and a fundamental frequency of 3 Hz are chosen. The damping rate of the “bridge” and the “carriage” is 1% and 10% respectively. The carriage is modelled linear elastically.

Figure 8 shows on the left the result for a mass ratio $m_{\text{eff}}/m_{\text{car}} = 10$ and a frequency ratio $f_{\text{br}}/f_{\text{car}} = 1$. Due to the relatively high mass ratio only small deflections occur in the “bridge”. However, the amplification factor (which is related to the static deflection due to carriage weight) is rather important and amounts to 1.8. From the force deviation it can be read off that the dynamic amplification of the static action due to dead loads is small both for the “carriage” and for the “bridge”.

Figure 9 shows the behaviour of the system for a mass ratio $m_{\text{eff}}/m_{\text{car}} = 2$ and a frequency ratio $f_{\text{br}}/f_{\text{car}} = 1$, i.e., a five times heavier “carriage”. In this case the reaction of the “bridge” is more pronounced, but the amplification factor with a value of 1.45 is nearly halved. The main reason for this is the higher “carriage” weight, whose action effect serves as a reference for the indication of the amplification factor. The higher damping of the system in addition is significant due to the larger portion of the carriage mass in the total mass of the system.

On a real bridge, the excitation is not implicitly of periodic nature; the duration of the excitation is shorter or the bridge can be vibrating due to the excitation of a previous axle. A simple model in form of the two-mass oscillator allows showing that dynamic amplification factor for bridge action effect decreases with increased carriage mass.
3.5 Excitation by an impulse

Besides periodic track irregularities also located track irregularities, such as settlements near the abutments of bridges are of concern. Figure 10 shows as an example the response of the system to an initial displacement of the “carriage” mass from the rest position which can be interpreted as a simple modelling of a pitch in the track.

![Figure 10: Model for the analyse of the reaction of the system „carriage“+ „bridge“ due to an impulse like excitation](image)

Figure 11 left shows the deviation of the wheel force amplification and right the amplification factor for the bridge action effect. The upper diagrams were arranged for a “bridge” mass $m_{br} = 100\text{t}$, the lower for $m_{br} = 20\text{t}$. The result for the wheel force amplification factor (left) shows a clear decrease with increasing “carriage” weight. This confirms the results of section 2. In the case of the amplification factor of bridge action effect (right), the results are coherent, whereby the resonance effect becomes slightly apparent with very light vehicles.

![Figure 11: Amplification factor of wheel forces (left) and bridge displacements (right) as a function of carriage mass and bridge stiffness](image)

The decrease of dynamic amplification factor with increased traffic load can be confirmed by the interpretation of measurements (Hirt 1976), (ORE 1990).

If the behaviour of the system regarding the influence of the damping is analyzed, the cognition is made that not the influence of the mass ratio itself is responsible but the increased damping in the system. The carriages have to a certain extent the effect of energy dissipaters. But in reality the damping rate is not the same for heavy and light vehicles. Heavily loaded carriages have a lower damping rate. But the effect of lower natural
frequency of the carriage due to heavier loading is dominant and it can be assumed that in all the amplification factor decreases (Ludescher 2004).

3.6 The 2nd natural mode of the carriage is dominant

The scenario where the dynamic behaviour of the “carriage” is dominated by its 2nd natural mode (when the un sprung wheel set is moving against the car body but also when running instabilities of the wheel set occur) leads to high wheel force amplifications as it can be seen in figure 3. In the domain of excitation where important axle vibrations occur, the car body nearly doesn't move at all due to its inertia (Ludescher 2004). If linear behaviour is assumed, increasing car body weight (heavier loading) doesn’t change the dynamic properties of the wheel set. The significance of wheel force amplification decreases with increased car body mass as the dynamic amplification factor has the static effect as reference.

4 Conclusions

Amplification factors for high traffic loads are distinctly lower than for trains with light carriages:
1. The wheel force amplification factor due to track irregularities decrease with increased weight of carriage.
2. In the bridge element the dynamic amplification factor decreases with increased train load.
3. Where the dynamic behaviour of the carriage is dominated by its 2nd natural mode or running instabilities of the wheel set occur, a decrease of dynamic amplification factor with increased carriage weight can be expected.

These results confirm observations made by measurements. For proof with respect to the fatigue limit, a reduction may be made with respect to load model UIC71* representing maximum traffic action effect considering the same dynamic amplification factors for all axle loads.

For the calculation of fatigue stresses using characteristic trains, lower dynamic amplification factors for high traffic load may lead to a considerable increase in the calculated remaining fatigue live.

References

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