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Quasi-Three-Dimensional Characteristics Method for a Supersonic Compressor Rotor

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ABSTRACT

A new quasi-3D characteristics method has been developed to calculate the supersonic flow in a compressor rotor on an axisymmetric stream surface with variable radius and stream tube thickness. The method is valid for supersonic inlet flow with an attached shock wave at the blade leading edge where the inlet Mach number and flow angle are related by the unique incidence condition. On the basis of the quasi-3D characteristics method, a simple quasi-3D method has also been derived that calculates the unique incidence condition and the Mach number distribution on the suction side of the profile for a given stream surface and stream tube thickness. The reliability of the simple method is established by comparing results with those obtained by Denton's time-marching method as well as those from the quasi-3D characteristics method. Systematic calculations performed using the simple quasi-3D method show the effects of varying the geometrical parameters of the cascade for different stream tube thickness and stream surface radius evolutions through the rotor.

NOMENCLATURE

a	speed of sound	m/s
B	stream tube thickness	m
d	blade thickness	m
h	enthalpy	J/kg-K
h_u	rothalpy	J/kg-K
i	unique incidence with respect to the camberline at the leading edge	deg
L	chord length	m
L_d	abscissa of maximum blade thickness	m
M	Mach number	-
$\dot{m}$	mass flow rate	kg/s
p	pressure	N/m^2
R	gas constant (for air R = 287.06)	J/kg-K
r	stream tube mean radius	m
s	entropy	J/kg-K
T	temperature	K
t	pitch	m
u	circumferential velocity	m/s
v	component of velocity	m/s
w	relative flow velocity	m/s
x	axial coordinate on the stream surface	m
z	axial coordinate	m
β	relative flow angle with respect to the axial coordinate z	deg
β_g	stagger angle with respect to the axial coordinate z	deg
γ	shock angle	deg
η	right-running characteristic	-
Θ_*	Prandtl-Meyer angle	deg
κ	ratio of specific heats (for air $\kappa = 1.4$)	-
λ	constant value on ξ characteristic	deg
μ	constant value on η characteristic	deg
ϑ	shock turning angle	deg
ξ	left-running characteristic	-
ρ	density	kg/m^3
σ	Mach angle ($\sin\sigma = 1/M$)	deg
ϕ	circumferential angular coordinate	deg
ω	angular rotor velocity	rad/s

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Subscripts

1	inlet values atupstream infinity
2	outlet values at downstream infinity
n	axial component of the Mach number
o	quantities of the homogeneous inlet flow
u	circumferential values
w	relative stagnation values
x	axial direction on the stream surface
ϕ	circumferential direction on the stream surface

Superscripts

*	referring to critical flow
^	referring to the flow state after a shock wave

INTRODUCTION

The most common method for blade design in industrial applications is based on a quasi-3D through-flow method coupled with a blade-to-blade calculation (Wu, 1952). For a given cascade geometry in the blade-to-blade calculation, at subsonic or low supersonic inlet Mach number, the inlet flow angle can be varied within certain limits. However, for a supersonic inlet flow with an attached shock wave at the blade leading edge and a specific inlet Mach number, there is only one possible inlet flow angle, called the "unique incidence" angle (Lichtfuß and Starken, 1984). For a 2D case, the supersonic flow field can be calculated by the characteristics method in a simple way. From this method the unique relation between the inlet Mach number and the flow angle can be established.

For a variable stream tube thickness (and stream surface radius constant) a quasi-3D characteristics method was developed by Bölcs (1981). Systematic calculations have shown that the unique incidence and the Mach number distribution on the suction side are strongly influenced by changes in the stream tube thickness and cascade geometry. Measurements performed in a quasi-2D linear cascade confirm the importance of stream tube thickness variation, or axial velocity density ratio (AVDR) (Tweedt et al., 1988). An additional consideration in such calculation is the effects of the leading edge radius on the creation of a detached shock. This parameter influences the position and shape of the shock wave and therefore the energy losses across the normal part of the shock. As a consequence the unique incidence angle is modified. These effects were included in the 2D characteristics method of York and Woodard (1975).

However, in a transonic compressor rotor the axisymmetric stream surface radius varies as well as the stream tube thickness. In the following, the governing equations of the quasi-3D characteristic method are derived. The method used in the full quasi-3D calculation of the compressor rotor cascade is also presented. A simplified method which permits the calculation of the unique incidence angle and the Mach number distribution on the suction side for a given stream surface radius and stream tube thickness variation, has also been derived.

To validate the two characteristics methods, a comparison is performed with Denton's (1983) "time-marching" method (DTMM), for different stream surface radius and stream tube thickness distributions. 2D and quasi-3D flow calculations having increasing or decreasing stream surface radius and stream tube thickness through the rotor have been performed. Finally, the geometry and the boundary conditions were varied to determine their influence on the entrance region flow field.

THE QUASI-3D CHARACTERISTIC EQUATIONS

A typical transonic compressor stage (formed by the fan and a stator) is illustrated in Fig. 1. A stream surface, defined by the radius r(z) and the variable stream tube thickness B(z), is shown.

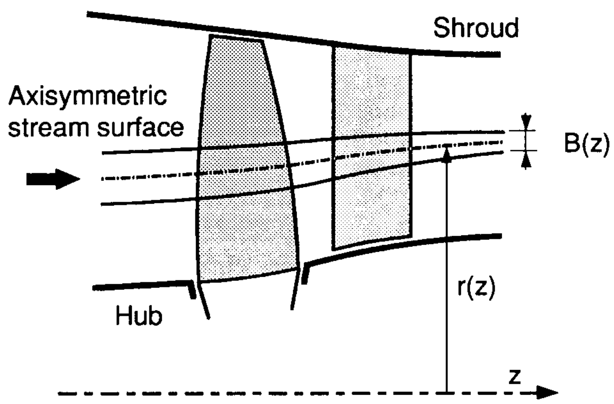


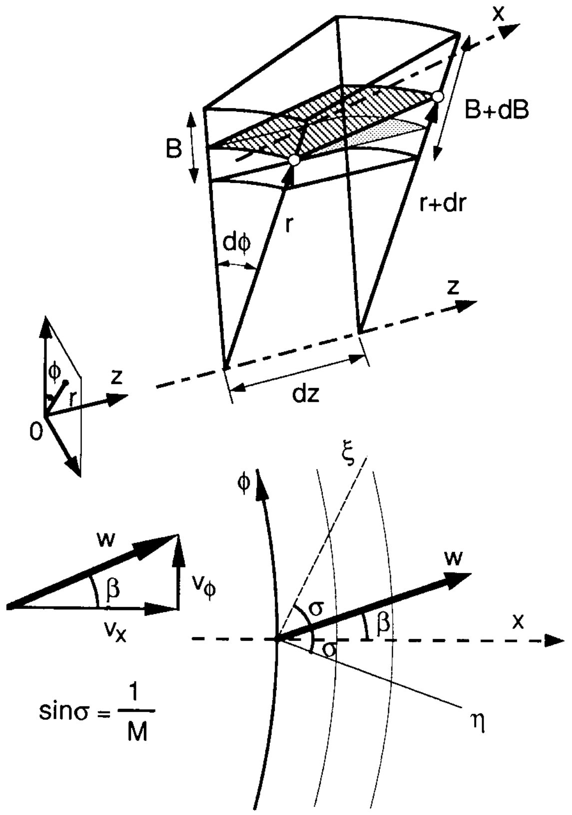
Fig. 1: Stream tube and stream surface geometry.

The flow through the rotating blade row is calculated in the relative coordinate system using the constancy of the rothalpy on the stream surfaces. For the quasi-3D calculation the following assumptions are made:

- the upper and lower stream tube boundaries are chosen to coincide with two stream surfaces which are assumed axisymmetric,
- the flow quantities are constant over the stream tube thickness and are taken equal to the mean value over the stream tube thickness,
- the axisymmetric stream surface radius varies with the axial direction r = f(z),

- the stream tube thickness depends only on the axial direction B = f(z),
- the axial velocity component is subsonic,
- radial forces are neglected, and
- the relative flow is supersonic and inviscid.

The equations for the conservation of mass and momentum are applied to the control volume illustrated in Fig. 2, in the x and ϕ coordinate system, which represent the circumferential and the axial direction on the axisymmetric stream surface, respectively.



ξ = left-running characteristic

η = right-running characteristic

***Fig. 2:** Control volume and coordinate system.*

Because the derivation of the equations is rather long, only the important intermediate steps will be presented. In the Appendix at the end of the paper a more detailed development of the quasi-3D characteristic equations is given.

<u>Continuity:</u>

$$\frac{\partial(v_x \rho)}{\partial x} + \frac{1}{r}\frac{\partial(v_\phi \rho)}{\partial \phi} = - v_x \rho \left(\frac{1}{r}\frac{dr}{dx} + \frac{1}{B}\frac{dB}{dx}\right) \qquad (1)$$

<u>Euler equation in the x direction:</u>

$$\frac{\partial(v_x^2 \rho)}{\partial x} + \frac{1}{r}\frac{\partial(v_x v_\phi \rho)}{\partial \phi} = -\frac{\partial p}{\partial x} - v_x^2 \rho \left(\frac{1}{r}\frac{dr}{dx} + \frac{1}{B}\frac{dB}{dx}\right) \qquad (2)$$

<u>Euler equation in the ϕ direction:</u>

$$\frac{\partial(v_x v_\phi \rho)}{\partial x} + \frac{1}{r}\frac{\partial(v_\phi^2 \rho)}{\partial \phi} =$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \phi} - v_\phi v_x \rho \left(\frac{1}{r}\frac{dr}{dx} + \frac{1}{B}\frac{dB}{dx}\right) \qquad (3)$$

The right-hand side of Eqs. (1) to (3) include quantities of mass and momentum that depend on the variations of r(x) and B(x). Combining Eq. (1) with Eqs. (2) and (3) and employing the definition of the speed of sound

$$a^2 = \frac{\partial p}{\partial \rho} \qquad (4)$$

the fundamental equation of gas dynamics for the quasi-3D flow is found:

$$(v_x^2 - a^2)\frac{\partial v_x}{\partial x} + v_x v_\phi \left(\frac{1}{r}\frac{\partial v_x}{\partial \phi} + \frac{\partial v_\phi}{\partial x}\right) + \frac{1}{r}(v_\phi^2 - a^2)\frac{\partial v_\phi}{\partial \phi} =$$

$$= a^2 v_x \left(\frac{1}{r}\frac{dr}{dx} + \frac{1}{B}\frac{dB}{dx}\right) \qquad (5)$$

The left-hand side of this equation is identical to that for axisymmetric quasi-2D flow. The terms on the right-hand side include the additional influence of variable r and B.

The energy equation in the relative coordinate system is defined by the conservation of rothalpy on the stream surfaces, which is expressed by

$$h_u = h + \frac{w^2}{2} - \frac{u^2}{2} = \text{constant} \qquad (6)$$

and may be rewritten, with the use of Eqs. (1) to (3), as

$$\frac{1}{r}\frac{\partial v_x}{\partial \phi} - \frac{\partial v_\phi}{\partial x} = - u \frac{\partial u}{\partial x}\frac{1}{(v_\phi - v_x)} \qquad (7)$$

A new set of dependent variables, w (flow velocity) and β (flow angle), are introduced by the following definitions (see Fig. 2):

$$v_x = w \cos\beta \qquad (8)$$

$$v_\phi = w \sin\beta \qquad (9)$$

Eqs. (5) and (7) can be expressed in terms of w and β in the coordinate system defined by the left-running characteristic ξ and the right-running characteristic η (see Fig. 2).

$$\frac{\partial\beta}{\partial\xi}+\frac{\partial\Theta_*}{\partial\xi} = \sin\sigma\cos\beta\left(\frac{1}{r}\frac{dr}{dx}+\frac{1}{B}\frac{dB}{dx}\right) + \frac{1}{r}\frac{dr}{dx}\left(\frac{M_u}{M}\right)^2\frac{\cos\sigma}{\sin\beta-\cos\beta} \quad (10)$$

and

$$\frac{\partial\beta}{\partial\eta}-\frac{\partial\Theta_*}{\partial\eta} = -\sin\sigma\cos\beta\left(\frac{1}{r}\frac{dr}{dx}+\frac{1}{B}\frac{dB}{dx}\right) + \frac{1}{r}\frac{dr}{dx}\left(\frac{M_u}{M}\right)^2\frac{\cos\sigma}{\sin\beta-\cos\beta} \quad (11)$$

Eq. (10) is valid along the right-running characteristic η and Eq. (11) is valid along the left-running characteristic ξ.

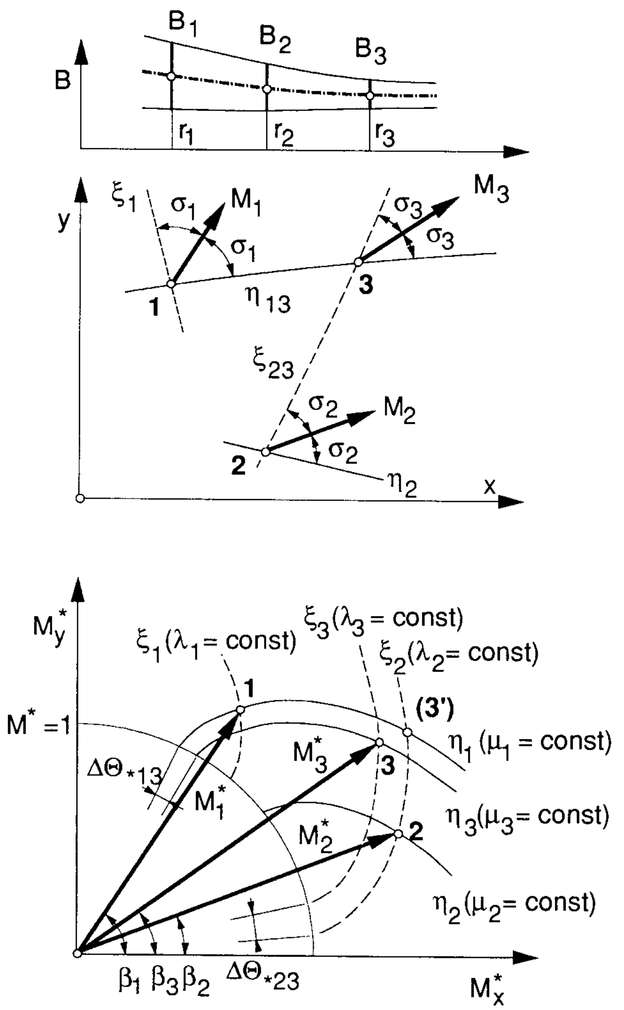


Fig. 3: *Graphical construction in the physical and hodograph plane.*

The left-hand sides of Eqs. (10) and (11) are identical to the expressions for 2D supersonic flow which have the solution

$$\Theta_* = \sqrt{\frac{\kappa+1}{\kappa-1}}\,\mathrm{arctg}\sqrt{\frac{\kappa-1}{\kappa+1}(M^2-1)} - \arctan\sqrt{M^2-1} \quad (12)$$

The Θ_* is known as the "Prandtl-Meyer angle". Eq. (12) represents two families of hodograph characteristics $\pm\Theta_*$ (see Fig. 3).

The characteristics in the physical and the hodograph plane can be considered as a natural curvilinear coordinate system and may also be used in place of M and β that specify the velocity vector. Each of the hodograph characteristic curves is identified by a particular value (Bölcs and Suter, 1986), which is

$$\mu = \frac{1}{2}(1000 - \Theta_* - \beta) \quad \text{for the } \eta \text{ characteristic} \quad (13)$$

$$\lambda = \frac{1}{2}(1000 - \Theta_* + \beta) \quad \text{for the } \xi \text{ characteristic} \quad (14)$$

with β the flow angle and Θ_* the Prandtl-Meyer angle defined by Eq. (12). The physical characteristics are identical to the Mach lines. The symbol σ is used to represent the Mach angle between the characteristics and the velocity vector as shown on Fig. 3.

With a given geometry and known flow properties at two points (points 1 and 2 in Fig. 3) the numerical values of λ_1, μ_1, λ_2, and μ_2 can be calculated. The values of β and Θ_* at point 3 may then be deduced by the simultaneous solution of

$$\beta_3 = \lambda_3 - \mu_3 \quad (15)$$

$$\Theta_{*3} = 1000 - \lambda_3 - \mu_3 \quad (16)$$

The Mach number M_3 is then found from the Prandtl-Meyer relation, Eq. (12).

While for an isentropic 2D flow $\lambda_3 = \lambda_2$ and $\mu_3 = \mu_1$ (point 3 coincides with point 3' on Fig. 3), for the quasi-3D flow additional terms $\Delta\lambda$ and $\Delta\mu$ need be introduced, so that

$$\mu_3 = \mu_1 + \Delta\mu_{13} \quad (17)$$

$$\lambda_3 = \lambda_2 + \Delta\lambda_{23} \quad (18)$$

These additional terms (corresponding to a change of the characteristic lines as shown in the hodograph plane on Fig. 3) have been found on the basis of the Eqs. (10) and (11) and are given by

$$\Delta\mu = -\frac{1}{2\cos(\beta-\sigma)}\left\{\sin\sigma\cos\beta\left(\frac{\Delta r}{r}+\frac{\Delta B}{B}\right) + \frac{\Delta r}{r}\left(\frac{M_u}{M}\right)^2\frac{\cos\sigma}{\sin\beta-\cos\beta}\right\} \quad (19)$$

$$\Delta\lambda = -\frac{1}{2\cos(\beta+\sigma)}\left\{\sin\sigma\cos\beta\left(\frac{\Delta r}{r}+\frac{\Delta B}{B}\right) - \frac{\Delta r}{r}\left(\frac{M_u}{M}\right)^2\frac{\cos\sigma}{\sin\beta-\cos\beta}\right\} \quad (20)$$

where $M_u = u/a = \omega r/a$ and $M = w/a$ are the Mach numbers related to the circumferential and relative velocities respectively.

Three different terms in the expression for the corrections $\Delta\mu$ and $\Delta\lambda$ on the right-hand side of Eqs. (19) and (20) can be distinguished. The first two of these take into account the effects of the variable stream tube mean radius and thickness, and the last one the combined effects of the rotor angular velocity and the stream tube mean radius variation.

For a 2D flow with constant stream surface radius and stream tube thickness ($\Delta B = 0$, $\Delta r = 0$), the corrections $\Delta\mu$ and $\Delta\lambda$ are zero. For a quasi-3D flow the variables used in the corrections $\Delta\lambda_{23}$ and $\Delta\mu_{13}$ are taken to be the average value between the two connecting points, and differences such as ΔB and Δr are calculated with the opposite order included in the subscripts (i.e. $\Delta B_{23} = B_3 - B_2$).

FULL CHARACTERISTICS METHOD

For the calculation of the supersonic cascade flows with shock waves, the following quantities are required:

- the profile coordinates with respect to the reference system (x,ϕ) and the stagger angle β_g measured from the axial direction x,
- the number of blades and the rotor angular velocity (ω), and
- the stream tube thickness B(z) and the stream surface radius r(z).

The flow field calculation begins from the first profile of the semi-infinite cascade (profile P1 on Fig. 4) assuming a homogeneous supersonic flow at infinity upstream of the oblique shock wave formed at the profile leading edge. For the determination of this inlet supersonic flow the Mach number M_o and the flow angle β_o are specified together with the stagnation temperature T_{wo} and pressure p_{wo} in the relative coordinate system at the radius r_o and the stream tube thickness B_o.

The flow quantities just upstream of the oblique shock wave on the first profile are calculated by the conservation of mass which takes into account the radius of the stream surface r(z) and the stream tube thickness B(z) in the axial direction z.

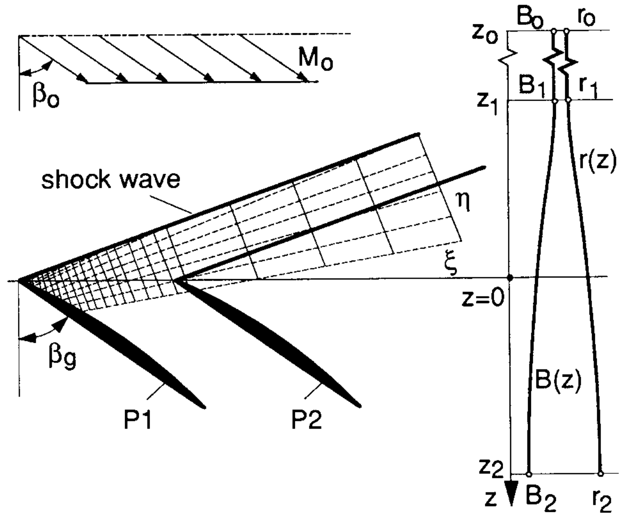


Fig. 4: Calculation of the first profile (P1).

The oblique shock wave attached to the leading edge of the profile is calculated in accordance with the turning angle associated with the tangent at the profile contour at the nose. The shock pattern is found by enforcing the oblique shock relations at the "shock-points" defined as the intersection between the shock wave and the left-running characteristic lines ξ (points 1 and 3 on the shock wave on Fig. 5).

For better accuracy a number of intermediate points between any two shock wave points on the oblique shock wave are introduced, from which right-running characteristics η depart (points a, b, c, d and e on Fig. 5). The density of the characteristics network depends on the position of the point 1 as well as on the choice of the number of the intermediate points. It has been found effective to choose the position of point 1 at about 1% of the blade chord and then vary, according to experience, the number of intermediate points as a function of the inlet Mach number and the camber of the profile suction side near the leading edge. This produces a sufficiently dense characteristics network.

The characteristics network is determined point by point starting from the leading edge (LE) and the first shock-point (1), in accordance with the quasi-3D characteristics method described above. The state of the flow at a point in the flow field (e.g. point k) is obtained from the known quantities at points i and j using the equations derived previously. The state of the flow at a point on the profile contour (e.g. point 2) is calculated from the know data at the point 1 and the boundary condition of the velocity tangent to the contour.

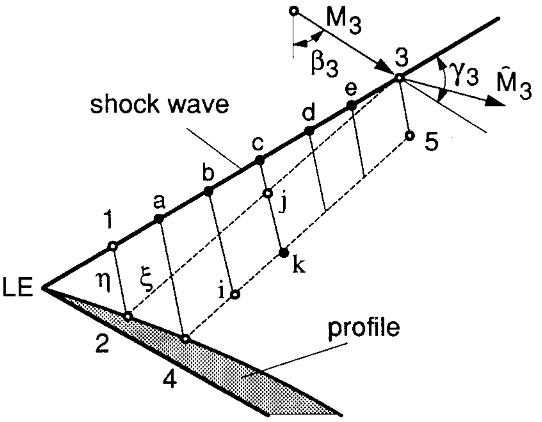


Fig. 5: *Calculation of the oblique shock wave.*

For the cascade, the flow is built up as in a linear cascade test facility. The flow upstream of each blade is calculated from the flow conditions of the previous blade (Fig. 6a). Flow periodicity is determined by comparing the Mach number and the flow angle at the leading edge of consecutive profiles. Even though the initial flow is not periodic, the periodic state is usually achieved after three or four blade passages.

The characteristic ξ intersecting the leading edge of the profile is calculated from point S to P (Fig. 6b). On this characteristic, the interblade mass flow rate is determined by adding the mass flow rates between all pairs of adjacent grid points. Some additional calculations are also performed. The characteristics network downstream of the characteristic ξ_{PS} in the interblade area is constructed. The oblique shock in the interblade flow area which lies in the characteristic network constructed previously, is also calculated (Fig. 6c). Finally, the region behind that oblique shock is determined (Fig. 7).

The uniform inlet Mach number $M_1 = w_1/a$ and the corresponding flow angle β_1 are found by solving a system of two equations. The first equation is obtained by setting the interblade mass flow rate ($\dot{m}_{PS}$) equal to the mass flow rate upstream which passes through a gap equal to the pitch t_1 ($\dot{m}_1$) with

$$\dot{m}_1 = \rho_1 w_1 t_1 \cos\beta_1 B_1 = \dot{m}_{PS}. \qquad (21)$$

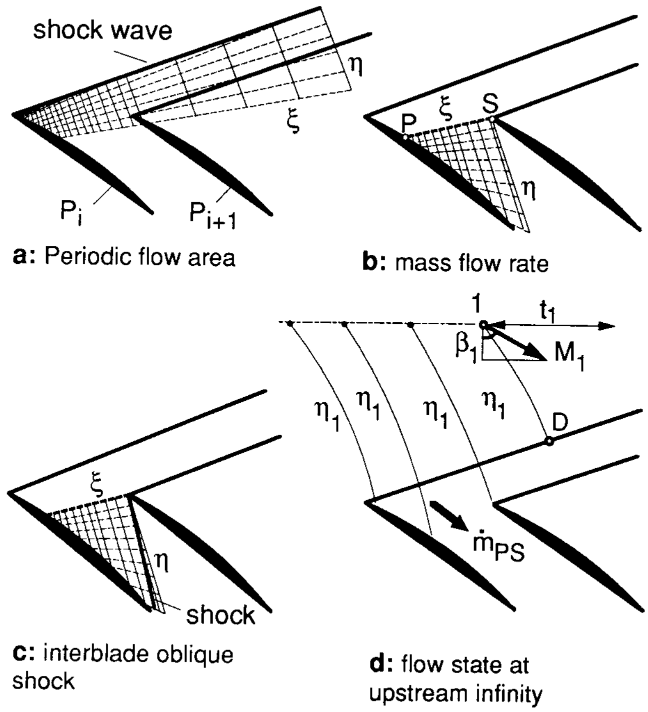


Fig. 6: *Elements of the full characteristics method.*

The second equation is found by the following characteristic relation between a known point D (taken on the shock wave) and the inlet conditions (Fig. 6d)

$$\mu_1 = \mu_D + \Delta\mu_{1D} \quad \Rightarrow$$

$$\Theta_*(M_D) + \beta_D = \Theta_*(M_1) + \beta_1 - 2\Delta\mu_{1D} \qquad (22)$$

where $\Delta\mu_{1D}$ is defined by the stream surface radius and the stream tube thickness variation between the points 1 and D (see Eq. (19)). Any other point in the flow can be used in place of point D, since characteristics η issuing from the homogeneous inlet flow reach all points in the field (see Fig. 6d). The point D is chosen as far as possible in the direction of the oblique shock, so that the characteristic η_{1D} crosses the weak part of the previous oblique shock waves and losses can be neglected. Losses along the streamlines can also be taken into account by introducing the following corrections to μ and λ (Bölcs,1986):

$$\Delta\mu = \Delta\lambda = -\frac{1}{2\kappa} \frac{\sqrt{\bar{M}^2-1}}{\bar{M}^2} \ln \frac{p_w}{p_{wref}} \qquad (23)$$

where $\bar{M}$ is the mean Mach number between the reference and the calculation points which must be on the same streamline. For the flow field investigations presented in this paper, the oblique shock waves were sufficiently weak and therefore these corrections are negligible. Shown in Fig. 7 is an example of the entire characteristics network together with the shock configuration in a typical supersonic compressor rotor.

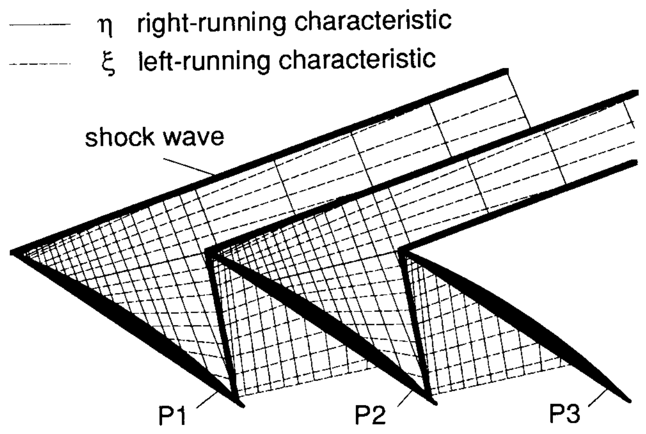


***Fig. 7:** Characteristic lines network.*

In a 2D flow, the left-running characteristics (ξ) are almost straight lines and are identical with the lines of constant Mach numbers (iso-Mach lines). In the quasi-3D flow the characteristics are curved and do not coincide with the curved iso-Mach lines.

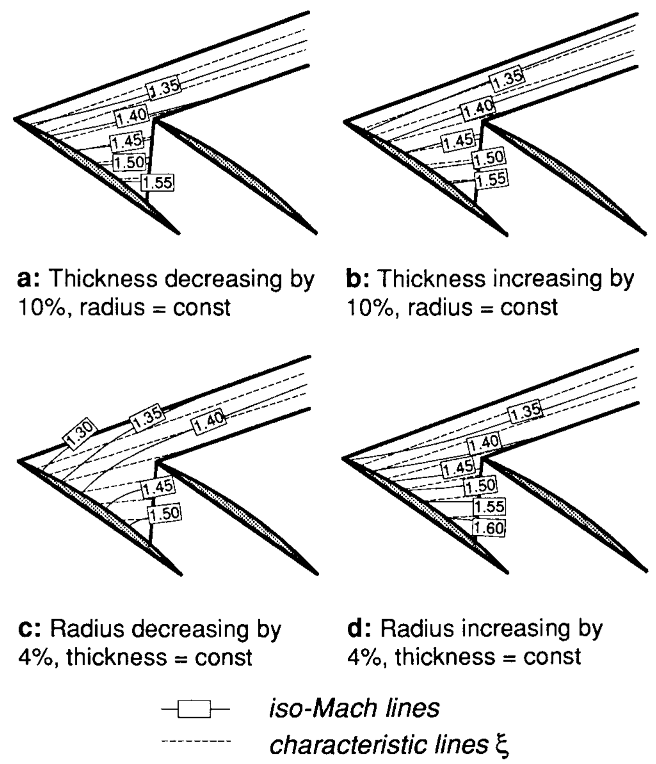


***Fig. 8:** Different flow configurations for quasi-3D geometries.*

In Fig. 8 the influence of the variation through the cascade of the stream tube thickness B(z) and the stream surface radius r(z) on the evolution of the left-running characteristics (ξ) and the iso-Mach lines is shown.

For constant mean radius r(z), and decreasing stream tube thickness through the cascade, the supersonic flow decelerates and consequently the iso-Mach lines curve upstream from the characteristic lines. When the stream tube thickness increases, the flow accelerates and the iso-Mach lines curve downstream (see Fig. 8a and 8b). The variation of the stream surface radius when B(z) = const produces a strong variation in Mach number. When the radius decreases, the additional deceleration of the flow makes the iso-Mach lines curve forward and when it increases, the additional acceleration makes them curve backwards.

A SIMPLE QUASI-3D CHARACTERISTICS METHOD

For the calculation of the supersonic flow field through the cascade by the simplified quasi-3D method, the same geometrical parameters as in the full method are required.

The inlet state of the supersonic flow is specified by the stagnation temperature T_{w1} and stagnation pressure p_{w1} in the relative coordinate system at the stream surface radius r_1 and the thickness B_1. The flow angle β_1 (corresponding to the unique incidence i_1) is one of the parameters determined by the present method, given the Mach number M_1.

The simple method is based on the calculation of the state of the flow at some chosen points in the flow field. These points, which are shown on Fig. 9, are: point 1 at infinity in the upstream direction, point S at the leading edge of the blade just upstream of the oblique shock wave, point E also at the leading edge just after the oblique shock wave, and point P at the intersection of the blade surface and the characteristic ξ issuing from the point S. Finally, a simple estimate of the normal shock wave limit position in the interblade flow area is made by considering the straight line SF perpendicular to the blade chord. The state of the flow at the point F just upstream of the normal shock wave is also calculated in order to estimate the intensity of the shock wave.

In the characteristics network between two consecutive oblique shock waves, there is a unique characteristic ξ_{1N}, which starts from a point N on the profile surface and reaches the inlet homogeneous supersonic flow without crossing any shock waves (Fig. 9). This characteristic will be called the "neutral characteristic" and the intersection point with the blade surface the "neutral point". The state of the flow at the neutral point N is required in order to calculate the rest of the flow field. For the given Mach number M_1 the corresponding unique incidence angle i_1, defined by the periodicity of the flow, is calculated below.

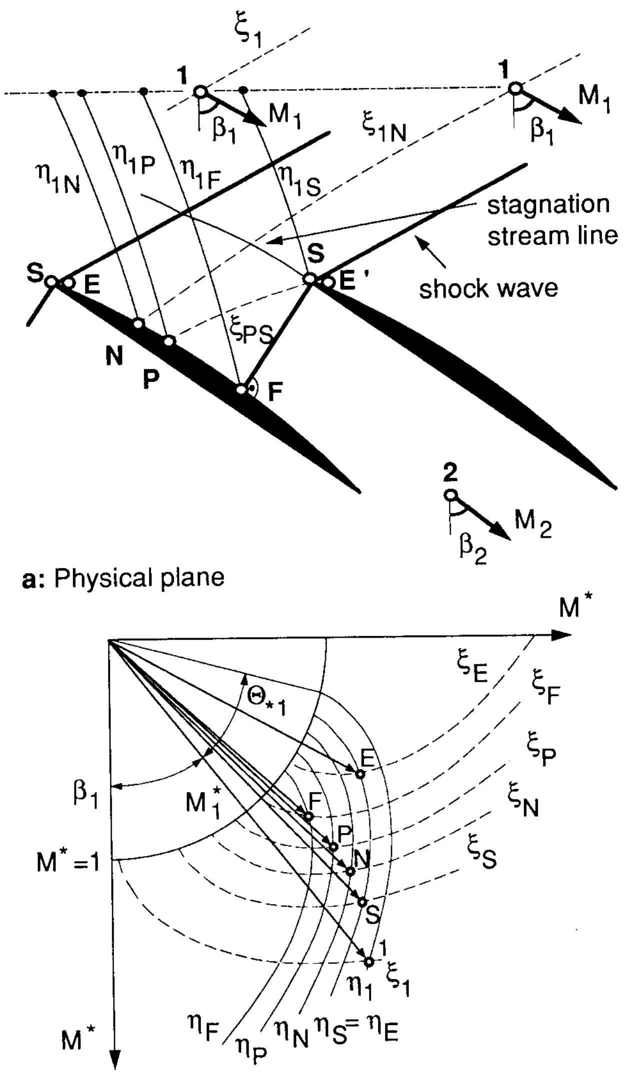


***Fig. 9:** Simplified quasi-3D method.*

A first approximation for the inlet flow angle β_1 is made by choosing a value close to the stagger angle of the profile β_g. As shown on Fig. 9, point N is on the intersection of the characteristics ξ_{1N} and η_{1N}, so that the following relations are valid:

$$\lambda_N = \lambda_1 + \Delta\lambda_{1N} \quad \text{along the } \xi_{1N} \text{ characteristic} \qquad (24)$$

$$\mu_N = \mu_1 + \Delta\mu_{1N} \quad \text{along the } \eta_{1N} \text{ characteristic} \qquad (25)$$

where the losses on characteristic η_{1N} across the oblique shock waves are neglected (as in the full characteristics method). These two relations permit the calculation of the Mach number M_N as well as the other flow quantities at point N. The flow angle β_N determines the position of the point N on the profile surface in accordance to the flow tangency condition.

The next step is to find the characteristic ξ_{PS} shown on Fig. 9. In a quasi-3D flow, the real left-running characteristics ξ are curved, but for the simple method, the characteristic ξ_{PS} is replaced by a straight line and its mean Mach angle is given by $\sigma_{PS} = (\sigma_P + \sigma_S)/2$. The position of point P and the state of the flow there is calculated from the expansion along the profile surface, in accordance with the following relations

$$\lambda_P = \beta_P + \mu_P \qquad (26)$$

$$\mu_P = \mu_N + \Delta\mu_{NP} \qquad (27)$$

The final position of the point P is determined in a way such that the characteristic ξ_{PS} touches the leading edge of the neighboring profile (point S). The flow quantities at point S are subsequently found by the following two relations. The first

$$\lambda_S = \lambda_P + \Delta\lambda_{PS} \qquad (28)$$

is valid along the characteristic ξ_{PS}, and the second

$$\mu_S = \mu_P + \Delta\mu_{PS} \qquad (29)$$

can be derived by considering the expansion of the fluid along the blade surface. The state of the flow at point E is determined in two different ways to enforce periodicity.

The flow variables at point E which lies on the profile P1 are determined by the boundary condition

$$\lambda_E = \beta_E + \mu_E \qquad (30)$$

and the relation analogous to Eq. (29)

$$\mu_E = \mu_N + \Delta\mu_{NE} \qquad (31)$$

The flow variables at E', lying on profile P2, are obtained by the application of the standard oblique shock wave relations, which requires knowledge of the state of the flow at point S and the slope of the profile surface on the suction side of the leading edge. If the flow variables at points E and E' are identical, the calculation is considered complete. Otherwise the differences in the flow at these points are used to derive a new flow angle β_1, and the entire calculation is repeated.

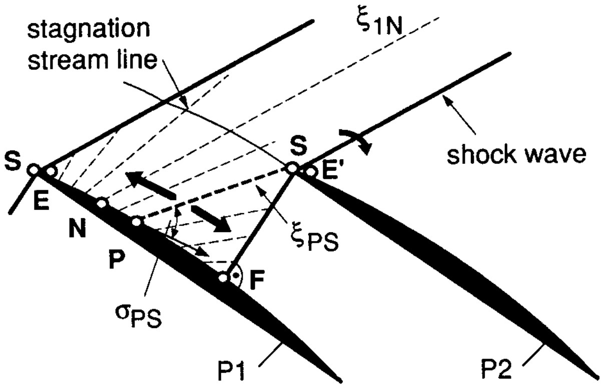


Fig. 10: *Calculation of points E and F, control of the flow periodicity.*

Finally, using the known position of point F, the flow is calculated in a similar way to that used for points P and E employing the following two relations:

$$\lambda_F = \beta_F + \mu_F \tag{32}$$

$$\mu_F = \mu_N + \Delta\mu_{NF} \tag{33}$$

Generally, after three or four iterations convergence is achieved and periodicity of the flow is attained.

ACCURACY OF THE SIMPLE METHOD

Using Denton's (1983) "time-marching" method (DTMM) and the full quasi-3D characteristics method, the accuracy of the simple method will be assessed. In DTMM the Euler equations are solved for the inviscid compressible flow on a blade-to-blade stream surface with allowance for changes in stream tube thickness and radius. The rotation of the rotor is also taken into account (Denton, 1983) in this method. The characteristics method calculates supersonic flow only, whereas DTMM is able to calculate both supersonic and subsonic flows. For this reason a very low back pressure was imposed in DTMM so that the normal shock in the interblade area moves back to the trailing edge of the blade creating a large supersonic region which can be solved using the simple method.

Six different flow solutions are compared. For all of the solutions the geometry is kept the same (profile DCA-(3%), stagger angle $\beta_g = 60°$, and number of blades N = 25). The inlet Mach number and stagnation values at infinity upstream are identical ($M_1 = 1.374$, $p_{w1} = 1$ bar, $T_{w1} = 300$ K). The angular velocity of the compressor rotor is a constant n = 11000 rpm.

Only the stream tube geometry is varied. Case 1 is 2D with constant radius and stream tube thickness.For case 2 the thickness decreases by 10% while the mean radius remains constant, and for case 3 the thickness increases by 10%. For case 4 the radius decreases by 4% while the thickness remains constant, and for case 5 the radius increases by 4%. For the final case 6 the thickness decreases by 10% and the mean radius increases by 4%.

The variation of the stream tube thickness or the surface radius through the cascade has the form of a cosine with extremes at the axial positions $z_1 = -0.25z_L$ and $z_2 = 1.25z_L$, where z_L (= $L\cos\beta_g$) is the axial length of the profile chord (Fig. 12). For either $z < z_1$ or $z > z_2$ the flow is 2D with $B_1 = 0.01$m and $r_1 = 0.32$m.

With the chosen set of parameters, the unique incidence i_1, flow angle β_1, mass flow rate through the interblade area $\dot{m}$, Mach number M_S and flow angle β_S at the blade leading edge were obtained by the three different methods. The results are shown in Table 1.

Time marching method (DTMM)

	M_1	i_1(°)	β_1(°)	$\dot{m}$(kg/s)	M_S	β_S (°)
1	1.375	1.038	64.471	0.0730	1.446	62.976
2	1.372	1.898	65.331	0.0708	1.448	63.332
3	1.373	0.170	63.603	0.0754	1.438	62.392
4	1.373	0.990	64.423	0.0732	1.428	62.770
5	1.378	0.381	63.814	0.0746	1.469	61.958
6	1.373	1.178	64.611	0.0727	1.457	62.658

Full characteristics method

	M_1	i_1(°)	β_1(°)	$\dot{m}$(kg/s)	M_S	β_S (°)
1	1.374	1.038	64.471	0.0730	1.449	62.315
2	1.374	1.899	65.333	0.0707	1.443	63.108
3	1.374	0.165	63.598	0.0753	1.455	61.498
4	1.374	1.145	64.578	0.0727	1.452	61.927
5	1.374	0.826	64.259	0.0736	1.435	62.756
6	1.374	1.665	65.098	0.0713	1.435	63.474

Simple method

	M_1	i_1(°)	β_1(°)	$\dot{m}$(kg/s)	M_S	β_S (°)
1	1.374	0.948	64.381	0.0733	1.446	62.291
2	1.374	1.788	65.222	0.0710	1.439	63.059
3	1.374	0.081	63.515	0.0756	1.452	61.536
4	1.374	0.953	64.386	0.0733	1.444	61.931
5	1.374	0. 886	64.320	0.0734	1.447	62.647
6	1.374	1.667	65.101	0.0713	1.438	63.390

Table 1: *Comparison of numerical results for cases 1-6*

The calculated values of M_S and β_S from DTMM are not as reliable owing to grid related inaccuracies near the blade leading edge. Because the value of M_1 in the time marching method is a derived and not an input quantity, some effort was made to achieve a M_1 as close as possible to the reference value of 1.374.

For cases 1 to 3 (2D flow or variable thickness of the stream tube), results are very close. For cases 4 to 6 (variable surface radius), the results of the two characteristic methods are also very close and in good agreement with those from DTMM. Similar trends are observed for all cases.

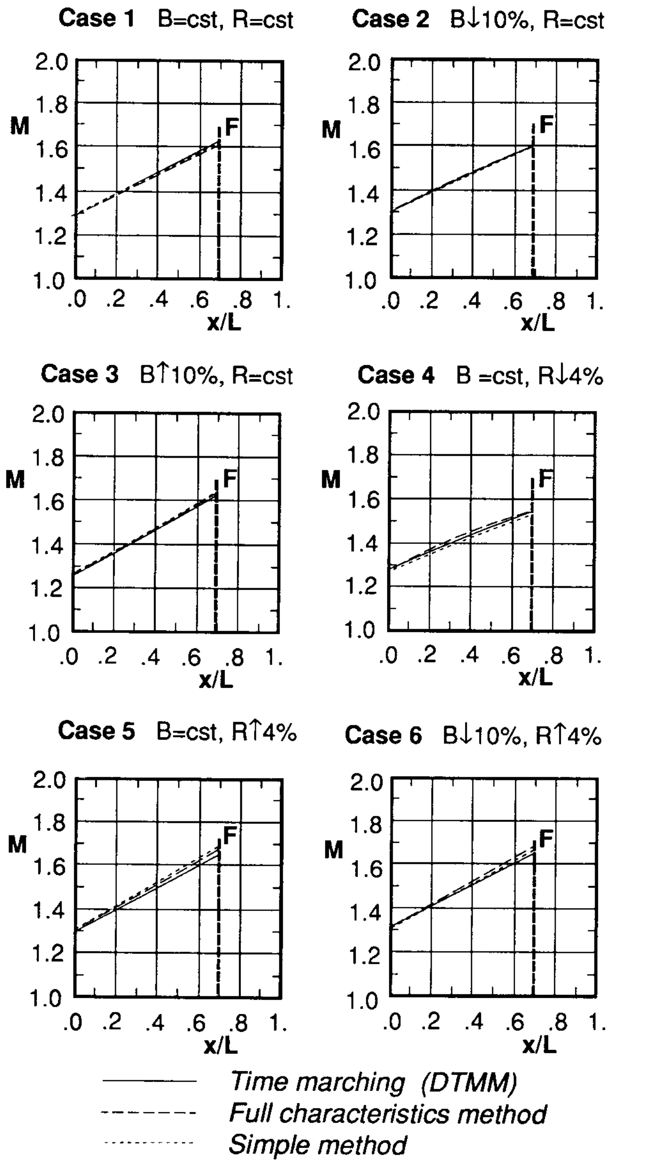


Fig. 11: *Comparison of Mach distribution on the blade suction side of cases 1-6.*

The distribution of the Mach number on the blade suction side, starting at the leading edge until point F (Fig. 9), is shown on Fig. 11.

As already noted, the Mach number at the point F just upstream of the limit position (S-F) of the normal shock gives an estimate of the intensity of the shock and therefore the losses across it. Fig. 11 shows that there is excellent agreement in the distribution of the Mach number between DTMM and the full characteristics method.

Small differences are observed when the results from the simple method are also considered. This could be attributed to the differing abilities of the various methods to follow the curvature of the characteristics. The full characteristics method employs a greater number of grid points and is therefore able to capture the curvature of the characteristics themselves more accurately than the simple method.

It is interesting to note that the Mach number distribution on the profile suction surface is less sensitive to the stream tube thickness variation as compared with the effects of the stream surface radius variation. The deceleration of the flow with 4% radius decrease is greater than the deceleration caused by a 10% thickness decrease.

The value of the Mach number at the point F, M_F, is found to be sensitive to changes in geometry. For instance, the results of cases 4 and 5 suggest that a stream surface radius decrease of 4% though the cascade leads to a M_F = 1.55 and an increase of 4% to M_F = 1.7. It could therefore be concluded that a reduction in the value of M_F can be achieved by affecting changes to the stream surface radius as an alternative to using blades of complicated geometries.

The results of the computations presented above show that the simple method can be used for the blade-to-blade calculation for the supersonic region of the flow field in a through flow calculation. With this method it is possible to determine overall averages for every variable through the cascade. The calculation of the mean flow rate by this method leads to the determination of the stagnation stream line (Fig. 10). This can be used to determine mean flow variables in the field between consecutive leading edges.

A FLOW FIELD INVESTIGATION USING THE SIMPLE METHOD

A set of calculations has been performed using the simple method on different cascade geometries to evaluate the influence of the geometrical parameters on the simulated flow.

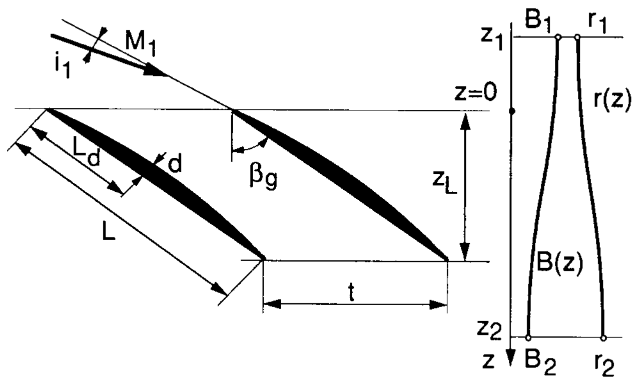


Fig. 12: Definition of the cascade, the stream tube thickness and the surface radius.

The reference cascade for these calculations is defined by (see Fig. 12)

- MCA profile
- chord length $L = 0.1$ m
- stagger angle $\beta_g = 60°$
- relative maximum blade thickness $d/L = 0.03$
- abscissa at maximum relative thickness $L_d/L = 0.5$
- relative pitch $t/L = 0.8$

The stream tube thickness and the surface radius of the reference case are constant (i. e. 2D flow) and equal to

- $B_1 = 0.01$ m, and
- $r_1 = 0.32$ m.

The definition of the variation of the stream tube thickness and the surface radius through the cascade is given on Fig. 12. It has the form of a cosine with extremes at the axial positions $z_1 = -0.25z_L$ and $z_2 = 1.25z_L$, where z_L ($= L\cos\beta_g$) is the axial chord. For either $z < z_1$ or $z > z_2$ the flow is 2D with $B_1 = 0.01$m and $r_1 = 0.32$m. Stagnation values at the inlet are kept the same throughout at

- $T_{w1} = 300$ K,
- $p_{w1} = 1$ bar,

and the angular velocity of the compressor rotor is n = 11000 rpm.

On Fig. 13 the variation of the unique incidence angle i_1 as a function of the inlet Mach number M_1 is presented for the reference cascade for six different stream tube evolutions (identical to those used for the comparison between the methods on Fig. 11). All curves start from the minimum Mach number M_1 for which an attached shock on the leading edge is possible and they end at the Mach number for which its axial component becomes equal to unity.

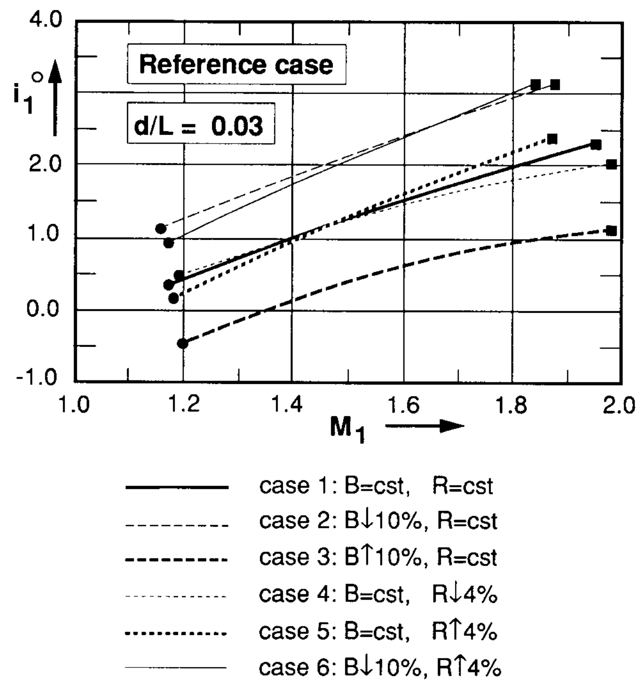


Fig. 13: Variation of unique incidence with Mach number for reference cascade, d/L=0.03.

For the reference cascade it can be seen that the decrease in the stream tube thickness causes an increase in the unique incidence angle (as compared with the 2D case) and, as a consequence, the mass flow rate through the cascade decreases. On the contrary, an increase in the stream tube thickness results in the decrease in the unique incidence angle relative to the 2D case.

Any variation in the surface radius will influence the slope of the unique incidence distribution. For case 4, a decrease in the surface radius reduces the rate of increase with respect to M_1 when compared with the 2D case. The difference in i_1 between the two cases at the extremes of the Mach number range are of opposite sign, with i_1 at the lowest M_1 of case 4 being larger than the result of the 2D case. Any increase in surface radius results in an increase of the slope of the unique incidence distribution relative to the 2D case. Finally, the combined effects of the stream tube thickness decrease and the surface radius increase (case 6) appear to be the sum of those of cases 2 and 5.

In the following, the influence of the cascade parameters (d/L, β_g, t/L and L_d/L) on the unique incidence i_1 as a function of the inlet Mach number M_1 is presented. The line style convention, used on Fig. 13, will be retained for the following figures.

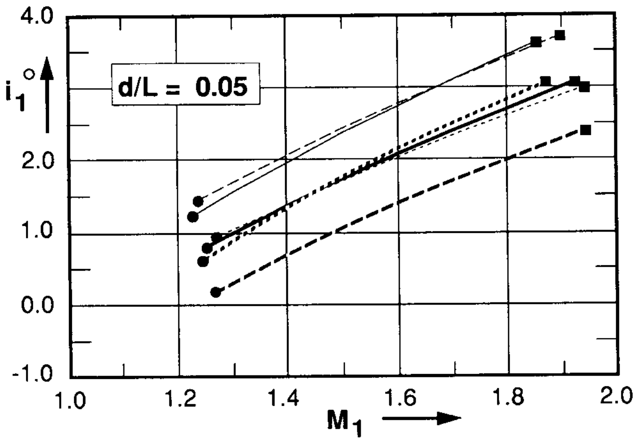


Fig. 14: Variation of unique incidence with Mach number for d/L=0.05.

It can be seen from Figs. 13 and 14 that an increase in the relative blade thickness increases the camber of the profile and shifts the unique incidence angle for given M_1 to higher values but the overall trend of the curves remains similar to that of the reference cascade.

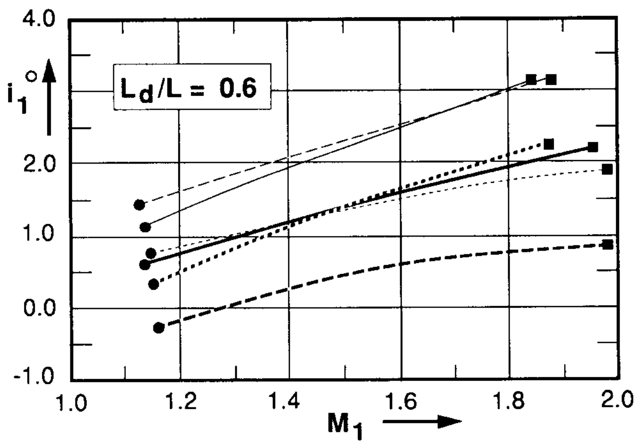


Fig. 15: Variation of unique incidence with Mach number for $L_d/L=0.6$.

When the maximum thickness of the profile is positioned closer to the trailing edge both the camber of the profile suction side and the leading edge angle decrease. Consequently, the shock wave attaches at the leading edge at lower inlet Mach numbers than in the reference cascade (Fig. 15). The curves are shifted slightly to lower incidence angles.

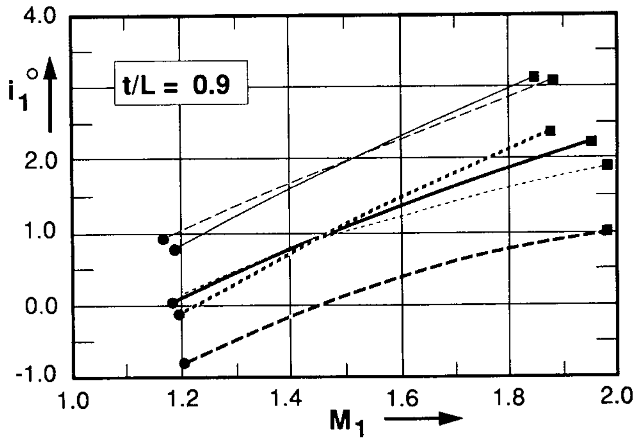


Fig. 16: Variation of unique incidence with Mach number for t/L=0.9.

Fig. 16 shows the influence of the ratio of blade spacing to the chord. At larger values the incidence angle for the 2D case is smaller compared with to that of the reference cascade.

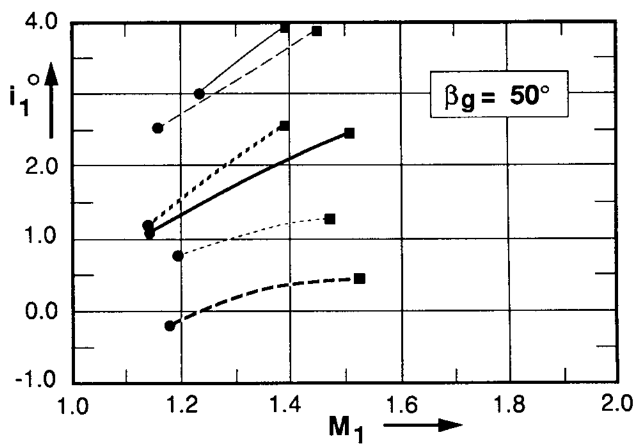


Fig. 17: Variation of unique incidence with Mach number for $\beta_g=50°$.

The influence of reduced stagger angle on the unique incidence angle is shown on Fig. 17. The axial component of the inlet Mach number approaches unity faster than in the other curves, and therefore the range of the possible values of M_1 at the inlet of the cascade is smaller than in the reference cascade.

CONCLUSIONS

A new quasi-3D characteristics method for supersonic cascade flows has been presented. The results obtained by this method are in good agreement with Denton's (1983) time marching approach. From the full characteristics method, a simple method has also been developed. The accuracy of this simple method has been checked by comparison with the time marching and the full characteristics methods. The simple method allows one to calculate the unique incidence and the Mach number distribution on the suction side, given the inlet Mach number in a quasi-3D stream tube in the rotor. The leading edge of the blade is chosen to be sharp so that the oblique shock is attached to it. The introduction of a blunt leading edge can easily be perfomed by the use of, for example, the Moeckel mass method. Finally, the flow is considered irrotationnal but the method may be extended to rotational flows by the use of Eq. (23).

Systematic calculations for various cascade parameters demonstrate the influence of the stream tube thickness and mean radius evolution through the cascade on the unique incidence. The influence of the stream tube thickness on the unique incidence is stronger than that of the radius variation. Consequently the stream tube thickness is more important for optimizing the mass flow rate through the cascade than the radius evolution. On the other hand, the variation of the stream surface radius has a stronger influence on the Mach number distribution on the suction side of the blade than the variation of the stream tube thickness.

The present simple method can be used as a tool for predicting the unique incidence in a quasi-3D supersonic flow with relatively very small computer time. In fact, the simple method is more economical than the full characteristics method by a factor of between 150 and 200. The simple method will take about 1 sec on the VAX 3100 station (for calculating the supersonic region of the flow only) when the DTMM needs 5 to 10 min on the CRAY-2S. The former method can therefore be used routinely for parametric investigations on different cascade and stream tube geometries.

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