Physics Informed Neural Networks for Surrogate Modelling and Inverse Problems in Geotechnics

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Finite elements methods have benefited from decades of development to solve partial differential equations and to simulate physical systems. In the recent years, machine learning and artificial neural networks have shown great potential to approximate such systems. Combining sparse or even no data, physics informed neural networks can be trained simultaneously on available data and the governing differential equations to fit a specific model or to compute the solution of an ordinary and partial differential equations. This master project focuses on the implementation of ANN models in order to predict the evolution of a hyperbolic PDE that is the acoustic wave equation in geotechnic problems. The U-Net model trained in a semi-supervised approach with a physical loss yields promising results, the lower average RMSE and the higher SSIM. This model is able to extrapolate the acoustic propagation on data not shown during its training but lacks robustness when predicting wavefields at larger time than the ones provided for its training.

Physics Informed Neural Networks

We aim at solving the wave equation in 2D. When the initial conditions are silent (i.e., no source term) the equation reads with p the pressure and c the wave velocity :

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - c^2 \frac{\partial^2 p}{\partial t^2} = 0 \qquad (1)$$

To create a surrogate model, convolutional neural networks are used. Such architecture has already been employed to predict the wave propagation in equation [1,2] or Saint-Venant equation by [3,4] using supervised learning. In this problem formulation, each wavefield is considered as an image, the only available data is the pressure and automatic differentiation cannot be used to compute the partial derivatives required to compute the PDE residuals. Using CNN provide multiple advantages. It requires less parameters and allows a faster training of the models. Moreover, the convolutional filters consider the structure of the images which yield a more robust extrapolation.

Currently, there is no paper dealing with PINN and the wave equation. To train the CNN, the proposition from [5] is used. They state that the L2 minimization between the model's prediction and a consistent numerical time integrator method is analogous to the L2 minimization of the discretized PDE residual. In this work the finite time difference method is as the time integrator. For a heterogeneous domain, the models are trained by computing the next wavefield using Equation (2) with $q_{i,j} = c_{i,j}^2$.

$$u_{i,j}^{n+1} = -u_{i,j}^{n-1} + 2u_{i,j}^n$$
$$+ \frac{\Delta t^2}{\Delta x^2} \left(\frac{1}{2}(q_{i,j} + q_{i+1,j})(u_{i+1,j}^n - u_{i,j}^n) - \frac{1}{2}(q_{i-1,j} + q_{i+1,j})(u_{i,j}^n - u_{i-1,j}^n) \right) \quad (2)$$
$$+ \frac{\Delta t^2}{\Delta y^2} \left(\frac{1}{2}(q_{i,j} + q_{i,j+1})(u_{i,j+1}^n - u_{i,j}^n) - \frac{1}{2}(q_{i,j-1} + q_{i,j+1})(u_{i,j}^n - u_{i,j-1}^n) \right)$$

Experiments

To select the network architecture for the surrogate model, the Salvus python package is used to generate the training data for the networks. A layered heterogeneous domain of size 1500x1500 [m] with a wavefield propagating is modelled. The metrics are computed for each network and training scenarios. There are 6 training scenarios : 3 data driven and 3 physics informed combining different losses (MSE, GDL and MAE). The RMSE and SSIM metrics are computed to compare on three experiments (Training, Space and Time extrapolation) how the model can reconstruct the next wavefield.

Figure 1. Model selection domains

Figure 2. Surrogate models experiments setup, the small crack is used to train the surrogate and the big crack for testing
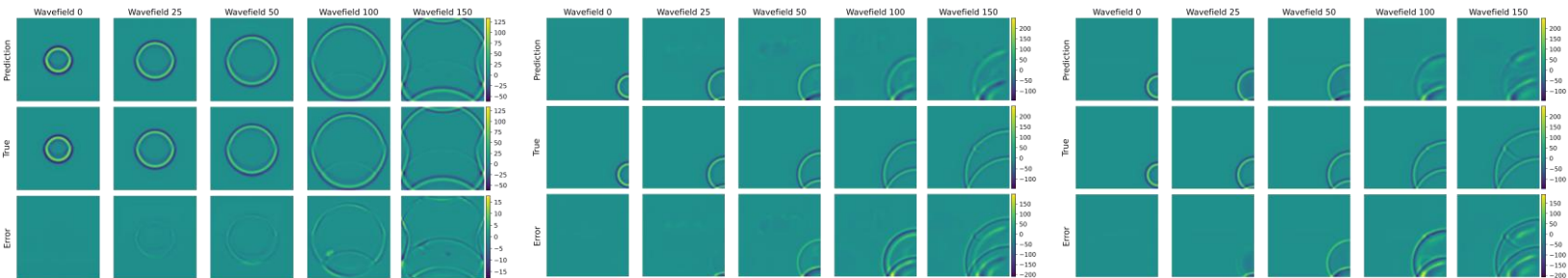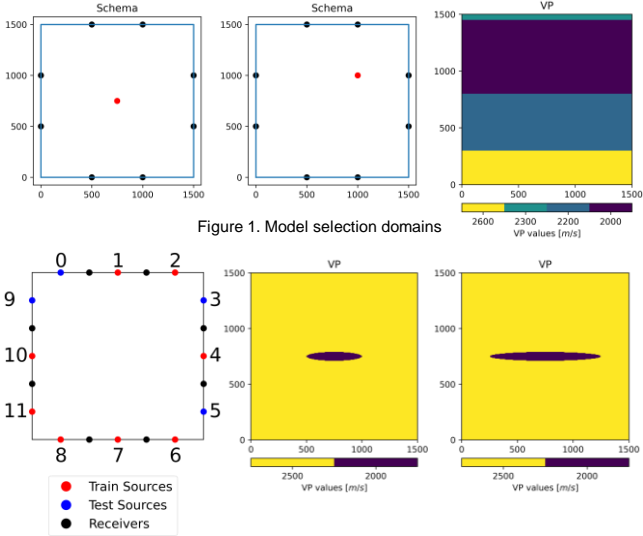
Results

Figure 3 (a). Wavefield propagation for the model selection

Figure 3 (b). Wavefield propagation on the small crack

Figure 3 (c). Wavefield propagation on the large crack

Table 1. Metrics computed over the 150 recursively predicted timesteps for the model selection

	RMSE [Pa]	SSIM [-]
(a) Model Selection	6.20e+00	8.14e-01
(b) Small Crack - Test	1.64e+01	7.63e-01
(c) Large Crack - Test	1.71e+01	8.22e-01

Key findings and further works

The best model found during the model selection is the U-Net architecture when it is trained by minimizing the residuals of the wave equation. It shows great predictions capabilities for heterogeneous domain was chosen to implement the surrogate model. It yields promising performance for the surrogate model to simulate the waves when a crack driven by fluid injection is propagating. The predictions with the neural networks are faster than the traditional FEM (15 [s] compared to 4-5 [min]). To improve the existing model, one could use more training scenarios with different initial conditions.

The main contributions of the master project are that multiple artificial neural networks are trained with different training scenarios involving non-physical and physical constrained loss functions. The performances of the surrogate model are assessed with clear and reproducible metrics to create state of the arts results currently missing for that specific problem. A dataset of acoustic wave simulations using Finite Element Method in Salvus is also created.

Further works include assessing the performances for different model sizes, geometries, different time intervals, different initial conditions. New models can also be tested to improve the reliability. Useful for problem inversion, record pressure at receivers and compare it with real data. test on real experimental setup

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[4] Sorteberg, W. E., Garasto, S., Pouplin, A. S., Cantwell, C. D., & Bharath, A. A. (2018). Approxi-mating the solution to wave propagation using deep neural networks.
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