Multiplexing of heterogeneous VBR connections over VBR trunk

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Abstract

We analyze the performance of the multiplexing of heterogeneous VBR connections over a VBR trunk, to provide guaranteed lossless or statistical service. The method extends the results of Lo Presti et al [1] and uses recent results of network calculus [2]. For the lossless case, we consider the multiplexed and the segregated systems, and we show that the resources required are the same for both systems if an optimal allocation is performed. For statistical service, we develop a new method for deriving an upper bound on the loss probability.

Keywords : lossless multiplexing, statistical multiplexing, VBR trunk.

Technical area : ATM

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1 Introduction

We consider the multiplexing of several variable bit rate (VBR) over one variable bit rate connection (called "the VBR trunk"). This occurs for example with ATM when a number of VBR virtual channel connections (VCCs) are multiplexed over one virtual path connection (VPC) [9] which is also of the VBR type. Another example is the multiplexing of several IP flows with reservations (using a protocol such as RSVP [10] or ST.II [8]) over one ATM VCC. Discussion on the use of VBR trunks is out of the scope of the paper, for more detailed information see [5, 3].

In the paper we analyze the performance of the multiplexing of heterogeneous VBR connections over a VBR trunk. For the lossless service we show that the multiplexed and the segregated systems require the same amount of buffer resources if an optimal allocation is performed. For the statistical service we calculate an upper bound on global congestion probability (called "loss probability") that we believe is accurate. The upper bound is calculated by splitting a VBR trunk into two independent CBR trunks. This enables us to apply results of Lo Presti et al [1] for each CBR case. Thus, the upper bound for the VBR case is given by sum of two independent CBR cases.

Now, we will introduce some notation and assumptions that we will use throughout the paper.

We assume a buffered network node with buffer X_0 (buffer size, in bits). The output of the network node is a virtual VBR trunk described by leaky bucket parameters P_0 (peak rate), M_0 (sustainable rate), B_0 (burst tolerance, in bits). It includes, in particular , the CBR trunk considered by Lo Presti et al, and characterized by the bandwidth C_0 and buffer size X_0 if we take $B_0 = 0$ or $P_0 = M_0$. Input to the network node are leaky bucket regulated heterogeneous connections. We assume that there are I classes of traffic with leaky bucket regulator parameters p_i (peak rate), m_i (sustainable rate) and b_i (burst tolerance, in bits) for each each class i, $1 \leq i \leq I$, that contains K_i such streams. The maximum burst duration of each connection is given by

$$T_{on_i} = \frac{b_i}{p_i - m_i}. \tag{1}$$

The classes are labeled in decreasing burst duration:

$$T_{on_1} \geq T_{on_2} \geq T_{on_3} \geq \ldots \geq T_{on_I}.$$

For stability, we will assume that

$$\sum_{i=1}^{I} K_i m_i < M_0.$$

Throughout the paper we use fluid model as we consider large buffers and we neglect cell delay variation tolerance.

2 Guaranteed lossless service

In this section, we assume that the parameters $\{P_0, M_0, B_0, X_0\}$ are given, and we first study the deterministic case, where the resource allocation guarantees a lossless service. As in Lo Presti et al [1], we consider two different scenarios: the first one, called the *lossless multiplexed system*, in which all connections share the same resources, and the second one, called the *lossless segregated system*, in which each connection is allocated a fixed portion of the resources, without any sharing between them. We can show that the following theorem holds:

Theorem 1 *Let the descriptors $\{p_i, m_i, b_i\}$ of the input connections be given and let $\{P_0, M_0, B_0, X_0\}$ be any set of resources guaranteeing a lossless operation for the multiplexed system. Then there exists an algorithm that allocates a fixed portion of these resources to each connection, and still guarantees that the resulting segregated system is lossless.*

The remainder of this section will be devoted to the description of this allocation algorithm.

The worst case is when the sources are greedy and synchronized, i.e. start emitting at peak rate p_i at the same time each (that, with no loss of generality, will be taken as $t = 0$) and then continue at the sustainable rate m_i after the burst duration T_{on_i}. The aggregated input traffic (the total number of bits) that has arrived in the time interval $[0, t]$ is then given by

$$R_{in}(t) = \sum_{i=1}^{I} K_i \cdot \min\{p_i t, b_i + m_i t\} = \min_{0 \le i \le I} \{\sigma_i + \rho_i t\} \tag{2}$$

with, for $0 \le i \le I$,

$$\sigma_i = \sum_{j=i+1}^{I} K_j b_j \tag{3}$$

$$\rho_i = \sum_{j=1}^{i} K_j p_j + \sum_{j=i+1}^{I} K_j m_j. \tag{4}$$

2.1 Lossless multiplexed system

We will use network calculus [2, 11] to compute the resources needed by the multiplexed system in order to guarantee a lossless service. The service curve offered by the trunk to the aggregated traffic is given by

$$\beta(t) = \min\{P_0 t, B_0 + M_0 t\} \tag{5}$$

The minimal buffer space X_0 which is required in order to prevent any loss is then given by

$$X_0 \ge \sup_{t \ge 0, u \le t} [R_{in}(t) - R_{in}(t - u) - \beta(u)] \tag{6}$$

where $[x]^+ = x$ if $x > 0$ and $[x]^+ = 0$ otherwise. Let k and l be indices such that

$$\rho_{k-1} \leq \quad M_0 \quad < \rho_k \tag{7}$$

$$\rho_{q-1} \leq \quad P_0 \quad < \rho_q. \tag{8}$$

Then, using (2) and (5), (6) becomes

$$X_0 \geq \max\{[\sigma_q + (\rho_q - P_0)T_{on_q})]^+, [\sigma_k - B_0 + (\rho_k - M_0)T_{on_k}]^+\}. \tag{9}$$

2.2 Lossless segregated system

Now each source in class i is allocated a fraction of the resources, namely a sustainable rate m_{0i}, a peak rate p_{0i}, a burst tolerance b_{0i} and buffer space x_{0i}. The total resources must therefore satisfy the four constraints

$$\sum_{i=1}^{I} K_i m_{0i} \leq M_0; \quad \sum_{i=1}^{I} K_i p_{0i} \leq P_0; \quad \sum_{i=1}^{I} K_i b_{0i} \leq B_0; \quad \sum_{i=1}^{I} K_i x_{0i} \leq X_0; \tag{10}$$

where $\{P_0, M_0, B_0, X_0\}$ have the same values (given by (5) and (9)) as above.

The first resources which are allocated are the sustainable rates m_{0i}. We must at least allocate $m_{0i} = m_i$ for all classes i in order to fulfill the stability constraint. We next assign what is left from the total sustainable rate M_0 to the classes with the largest burst duration. Therefore the sustainable rates allocation is, with the index k being given by (7),

$$m_{0i} = \begin{cases} p_i & \text{for} \quad 1 \leq i \leq k-1 \\ m_k + (M_0 - \rho_{k-1})/K_k & \text{for} \quad i = k \\ m_i & \text{for} \quad k+1 \leq i \leq I, \end{cases} \tag{11}$$

As a consequence, the $k-1$ first classes do not need any burst tolerance nor buffer space resources. For the other classes one considers simultaneously the total peak rate P_0 and the total burst tolerance B_0 by defining A_0 as

$$A_0 = \min\left\{B_0, \sum_{i=1}^{q-1} K_i(p_i - m_{0i})T_{on_i} + (P_0 - \rho_{q-1})T_{on_q}\right\} \tag{12}$$

and index l as the index such that

$$\sum_{i=1}^{l-1} K_i(p_i - m_{0i})T_{on_i} \leq A_0 < \sum_{i=1}^{l} K_i(p_i - m_{0i})T_{on_i}. \tag{13}$$

One can show that if $A_0 = B_0$ then $k \leq l \leq q$ whereas if $A_0 < B_0$ then $l = q$. If $A_0 = B_0$ then the limitation is due to the burst tolerance B_0 rather than the peak rate P_0, whereas if $A_0 < B_0$, it is the peak rate P_0 which is limiting and not the burst tolerance B_0. The allocation is then as follows:

$$p_{0i} = \begin{cases} p_i & \text{for} \quad 1 \le i \le l-1 \\ m_l + \left(A_0 - \sum_{i=1}^{l-1} K_i(p_i - m_{0i})T_{on_i}\right) / K_l T_{on_l} & \text{for} \quad i = l \\ m_i & \text{for} \quad l+1 \le i \le I \end{cases} \tag{14}$$

$$b_{0i} = (p_{0i} - m_{0i})T_{on_i} \tag{15}$$

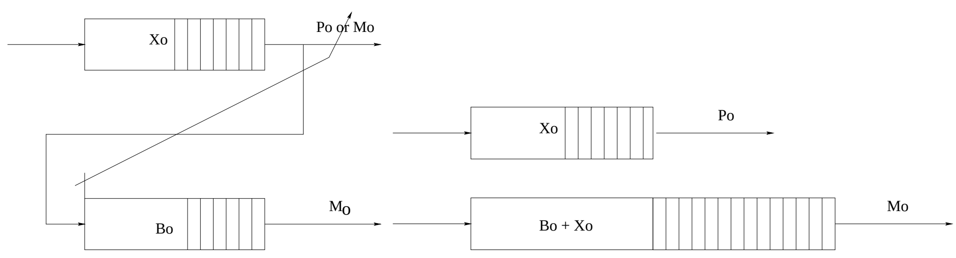
$$x_{0i} = (p_i - p_{0i})T_{on_i}. \tag{16}$$

With this allocation, only class l needs both buffer and burst resources, whereas all classes $k, \ldots, l-1$ need no buffer and all classes $l+1, \ldots, I$ no burst resource. This allocation strategy (11), (14), (15) and (16) can be proven to always verify the four constraints (10).

3 Model of the VBR trunk

In this section we propose a model of the VBR trunk that provides an upper bound on loss probability for an exact model. We analyze the exact model (Model A) shown in Figure (1)(a) and the simplified model (Model B) shown in Figure (1)(b) for both guaranteed lossless service and statistical service. For the lossless service we prove the following proposition:

Proposition 1 *The original VBR trunk (Model A) is lossless if and only if the combination of the two CBR trunk systems (Model B) is lossless.*



(a) Exact model: Model A (b) Model with 2 CBR servers: Model B

Figure 1: Models of the VBR trunk

The VBR trunk system can be considered as a shaper, with shaping curve $\beta(s) = \min(P_0 s, B_0 + M_0 s)$. From network calculus [2], we know that, for a buffer size X_0, there is no loss if and only if for all $s \ge 0$

$$\alpha(s) - \beta(s) \le X_0 \tag{17}$$

where $\alpha(s)$ is the maximal arrival curve of the aggregated input. This equation can be re-written as

$$\begin{cases} \alpha(s) \leq X_0 + P_0 s \\ \alpha(s) \leq X_0 + B_0 + M_0 s \end{cases} \tag{18}$$

The two lines in Equation (18) are for the system made of two CBR trunks in parallel, see Model B, where the two CBR trunks receive the same input. We have thus proven that the original VBR trunk in Model A is lossless if and only if the combination of the two CBR trunk systems in Model B is lossless. □

We conjecture that we can extend this result to the statistical case:

Conjecture 1 *The congestion probability for Model B is an upper bound of the congestion probability for the original model Model B.*

We do not have a formal proof but we believe that the following intuitive proof should work. For the case of statistical service, Equation (17) may not hold leading to the possibility of cell loss. An important fact following from Proposition (1) is that this loss first time will happen at the same time instant for both models, but if we look at the number of cells buffered in each system after the end of the first loss period we can see that the number of cells stored in Model B is larger than or equal to the number of cells stored in the exact model. Therefore, Model B will have less or equal buffer capacity to absorb the next burst that will potentially lead to larger cell loss. □

4 Statistical service with small loss probabilities

In this section we use Model B of the VBR trunk for upper bounding loss probability in a buffered VBR network node. The model of VBR trunk reduces the need for the estimation of loss probability for VBR case to problem of estimation of loss probability in two independent CBR cases that is already solved by Lo Presti et al [1]. The reminder of this section discuses application of results of [1] to our case. We will discuss in detail only one of the two CBR servers as for both the same formalism is applicable.

For estimating the loss probability, we will assume on-off sources with random phases ϕ_{ij} as traffic model maximizing loss probability estimate by Chernoff bound [13, 5].

According to [1] we can separate the virtual output rate $u_{ij}(t)$ and the virtual buffer occupancy $v_{ij}(t)$. The total output rate will be $U_X(t) = \sum_{i=1}^{I}\sum_{j=1}^{K_i} u_{ij}(t)$ and total buffer occupancy $V_X(t) = \sum_{i=1}^{I}\sum_{j=1}^{K_i} v_{ij}(t)$. Separation of virtual resources allows us to apply the Chernoff bound for estimating probabilities $\boldsymbol{P}_{P_0^v}(V_X(t) > X)$ and $\boldsymbol{P}_{P_0^v}(U_X(t) > P)$. A single source $u_{ij}(t)$ is on-off process with period $d_i = T_{on_i} + \frac{x_{0i}T_{on_i}}{p_{0i}}$ and with random phase ϕ_{ij}. For a given t, $u_{ij}(t)$ has Bernoulli distribution. It takes value x_{0i} with probability $w_i = d_i/e_i$, where $e_i = T_{on_i} + \frac{b_i}{m_i}$, and value 0 with probability $1 - w_i$

$$\boldsymbol{P}_{P_0^v}(U_X(t) > P_0) \leq \exp\left(-\Lambda^*_{P_0^v}(P_0)\right)$$

where $\Lambda^*_{P_0^v}(P_0)$ is Legendre transform of $\Lambda^a_{P_0^v}(\theta)$

$$\Lambda^a_{P_0^v}(\theta) = \sum_{i=1}^{I} K_i w_i \exp(\theta p_{0i}) + 1 - w_i$$

For fixed t, $v_{ij}(t)$ is distributed uniformly in $[0, x_{0i}]$ with probability w_i and is 0 with probability $1 - w_i$.

$$\Lambda^b_{P_0^v}(\theta) = \sum_{i=1}^{I} Kiw_i \frac{\exp(\theta x_{0i}) - 1}{\theta x_{0i}} + 1 - w_i$$

$$\boldsymbol{P}_{P_0^v}(V_X(t) > X_0) \leq \exp(-\Lambda^*_{P_0^v}(X_0))$$

where $\Lambda^*_{P_0^v}(X_0)$ is Legendre transform of $\Lambda^b_{P_0^v}(\theta)$ Similarly, we have

$$\boldsymbol{P}_{M_0^v}(U_B(t) > M_0) \leq \exp(-\Lambda^*_{M_0^v}(M_0))$$

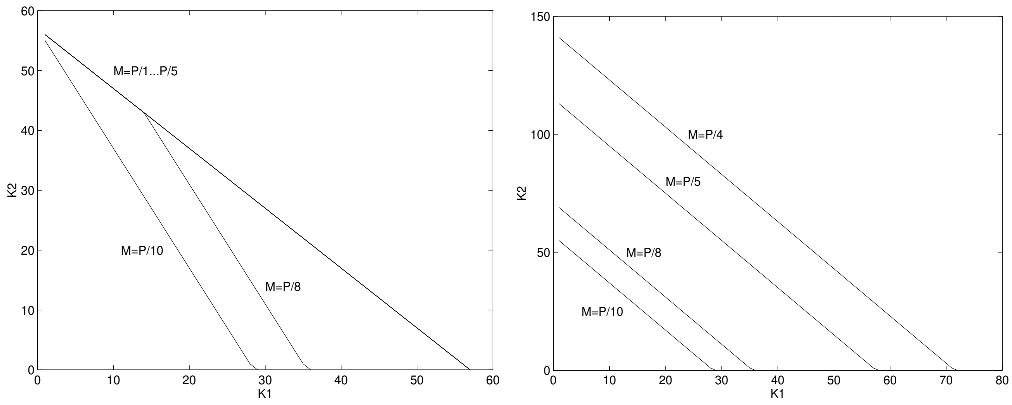
$$\boldsymbol{P}_{M_0^v}(V_B(t) > B_0 + X_0) \leq \exp(-\Lambda^*_{M_0^v}(B_0 + X_0))$$

where $U_B(t)$ is the total utilized mean rate and $V_B(t)$ is the sum of the total utilized buffer and total utilized mean rate. Now, we have loss probability estimates due to exceeding each separate trunk resource that still depend on the chosen virtual segregated system (see [1]). The final loss probability estimate is the infimum of the sum of all individual estimates taken over all possible virtual segregated systems. For our case, because of the independence of the two CBR trunks, the final loss probability estimate is given as the sum of the two following infimum.

$$\boldsymbol{P}_{loss} = \inf_{P_0^v}(\boldsymbol{P}_{P_0^v}(U_X > P_0) + \boldsymbol{P}_{P_0^v}(V_X > X_0)) + \inf_{M_0^v}(\boldsymbol{P}_{M_0^v}(U_B > M_0) + \boldsymbol{P}_{M_0^v}(V_B > B_0 + X_0)) \tag{19}$$

5 Numerical simulations

We give some numerical examples illustrating performance of the suggested multiplexing method in this section. We consider both lossless service and statistical service with loss probability. The output trunk in both cases is VBR type with parameters $P_0 = 115Mb/s$ and $B_0 = 5Mb$. Buffer size is $X_0 = 5Mb$. The value of the sustainable rate is indicated for each curve separately. The input to the network node is aggregation of two heterogeneous source classes. The first is characterized by the parameters $p_1 = 2Mb/s$, $m_1 = 0.4Mb/s$ and $b_1 = 0.05Mb$. The second class has the same peak rate $p_2 = 2Mb/s$ and burst tolerance $b_2 = 0.05Mb$, but smaller value of sustainable rate $m_2 = 0.2Mb/s$. The second class is thus more bursty. Lossless multiplexing is shown in Figure (2)(a). Curves, shown in Figure 2(a) illustrate some advantages of the use of VBR output trunk instead of CBR. The upper curve represents the bound of the admissible region when the value of the sustainable rate of output trunk is variated from $M_0 = P_0$(CBR output trunk) to $M_0 = P_0/5$. Note that the bound remains constant. Thus, we are able to serve the same traffic aggregation with less bandwidth by choosing an appropriate VBR trunk. If we further decrease the sustainable rate then the admissible region starts to shrink. From curve $M_0 = P_0/8$, we can see that this effects first the relatively less bursty class I and only then class II.



(a) Lossless multiplexing of two heterogeneous classes

(b) Statistical multiplexing of two heterogeneous classes

Figure 2: VBR over VBR network node

Statistical service is illustrated in Figure (2)(b). We still have the same source classes, but instead of guaranteed lossless service we provide service with target loss probability $L = 10^{-9}$.

Note that, contrary to the lossless case, a decrease in the sustainable rate yields decrease of the admissible region (compare curves $M_0 = P_0/4$ and $M_0 = P_0/5$). This is due to the multiplexing gain of a relatively large number of bursty input sources.

We can conclude that VBR output trunk has advantages when multiplexing a relatively small or medium number of VBR input sources and it allows yield multiplexing gain in the following network nodes. The latest is particularly important for efficient network utilization as it enables traffic aggregation [5].

Conclusions

We have presented an analysis of the multiplexing of heterogeneous VBR connections over a VBR trunk. If the trunk and buffer parameters are sufficient to provide lossless service for the multiplexed system, we have developed a resource allocation algorithm guaranteeing that the segregated system also operates without any losses. Thus, we have shown that, in the lossless case, there is no penalty in segregating the resources, provided an optimal allocation is performed. For statistical service, we have proposed a reduction of the model of the VBR node into two independent CBR systems, that enabled us to provide an upper bound on the loss probability in the VBR system. We believe that these results can be extended to more general settings where the output is constrained by more than two leaky buckets. Finally, another extension of this approach can be used for bounding the delay (instead of the loss probability) in a network node.

References

[1] F. Lo Presti, Z.-L. Zhang, D. Towsley, J. Kurose, *Source time scale and optimal buffer/bandwidth trade-off for regulated traffic in an ATM node*, To appear in Proc. IEEE INFOCOM'97, Kobe, Japan, 1997.

[2] J.-Y. Le Boudec, *Network calculus made easy*, EPFL Technical Report DI 96/218, http://lrcwww.epfl.ch/PS_files/d4paper.ps

[3] E. Gauthier, S. Giordano, J.-Y. Le Boudec, *Reduce connection awareness* EPFL Technical Report DI 95/145, http://lrcwww.epfl.ch/scone/scone_paper2.ps.

[4] J.-Y. Le Boudec, *Traffic control part I: virtual partition method for statistical multiplexing*, lecture notes.

[5] J.-Y. Le Boudec, A. Ziedins, *A CAC algorithm for VBR Connections over a VBR Trunk*, To appear in Proc. ITC 15, Washington, D.C., USA, 1997.

[6] E. Wallmeier, C.M. Hauber, *Blocking probabilities in ATM pipes controlled by a connection acceptance algorithm based on mean and peak bit rates*, Queuing, performance and control in ATM (ITC-13), Elsevier science publishers, IAC,1991.

[7] ATM Forum, User Network Interface 4.0

[8] C. Topolcic, *Experimental Internet Stream Protocol, Version 2 (ST-II)*, IEEE Network, September 1993

[9] ITU Telecommunication Standardization Sector - Study group 13, *ITU Recommendation I.371, Traffic Control and Congestion Control in B-ISDN*, 1995.

[10] L. Zhang, S. Deering, D. Estrin, S. Shenker and D. Zapalla, *RSVP: A New Resource ReSerVation Protocol*, IEEE Network, September 1993.

[11] R.L. Cruz, *A Calculus for Network Delay, Part I : Network Elements in Isolation* , IEEE Trans. Inform. Theory, vol 37-1,pages 114–131, January 1991.

[12] A. Elwalid, D. Mitra, R. H. Wentworth, *A new approach for allocating buffers and bandwidth to heterogeneous, regulated traffic in an ATM node*, IEEE Journal on Selected Areas in Communications, vol.13, N0.6., August 1995.

[13] D. Mitra, J. Morrison, *Multiple time scale regulation and worst case processes for ATM network control*, CD-ROM Proc. of 34th Conference on Decision and Control, 1995.