Second Order Phase Transition Describes Maximally Informative Encoding in the Retina

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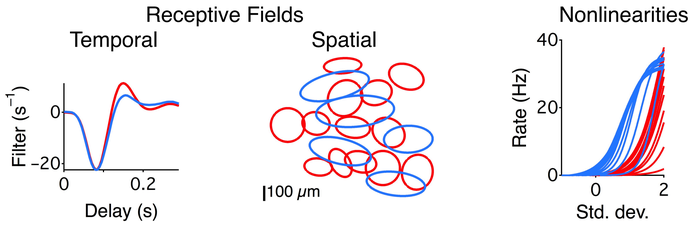
Neurons have limited dynamic ranges. Theoretical studies have derived how a single neuron should place its dynamic range to optimize the information it transmits about the input. However, neural circuits encode with populations of neurons.

Two ganglion cell populations encode similar visual features, but have distinct sensitivity and plasticity

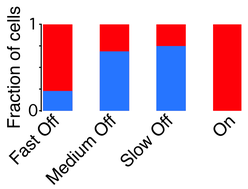


Adaptation

Sensitization

Steady state nonlinearities for the two populations measured across multiple contrasts

Only off cell types, in salamanders , split the encoding between distinct cell types. On cells do not split into multiple cell types.

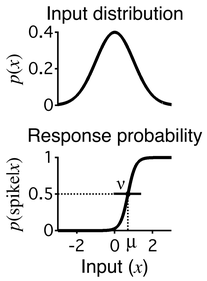


Kastner & Baccus 2011

Population Encoding Model

Neurons modeled as having binary outputs

$$p(\text{spike}|x) = \frac{1}{1+\exp\left(\frac{\mu - x}{\nu}\right)}$$



$$P(\text{spike}) = \int p(\text{spike}|x)\,p(x)\,dx$$

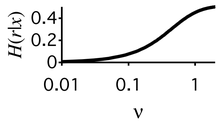
$$P(1) \equiv P(\text{spike})$$

$$P(0) = 1 - P(1)$$

Four responses patterns for two neurons

$$P(11), P(10), P(01), P(00)$$

Response slope introduces noise



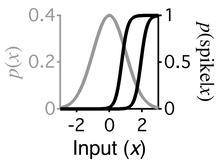
Mutual information

$$I(x,r) = H(r) - H(r|x)$$

$$H(r) = -\sum_i p(r_i)\log_2 p(r_i)$$

$$H(r|x) = -\int p(x)\sum_i p(r_i|x)\log_2 p(r_i|x)\,dx$$

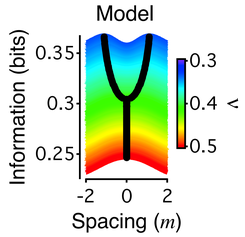
Rate limitation enforces sparse responses



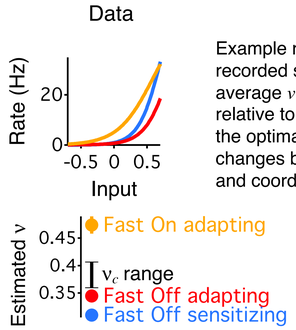
Placement of response probabilities that maximizes information given noise and rate.

$$\hat{r} = P_1(1) + P_2(1)$$

Second order phase transition occurs with increasing response function noise

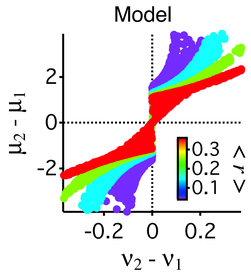


The information provided by two response functions with the same slope (ν) as the spacing ($\mu_2 - \mu_1$) between the functions vary. Black line indicate the maximal information at a given ν.

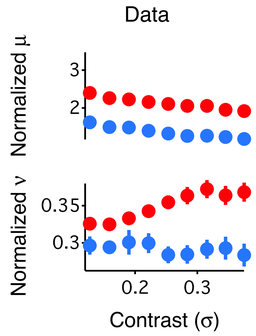


Example response functions, recorded simultaneously, and average ν for many cells plotted relative to ν_c, the point where the optimal coding strategy changes between redundant and coordinated encoding.

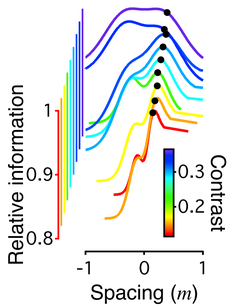
The response function with less noise should have the lower threshold



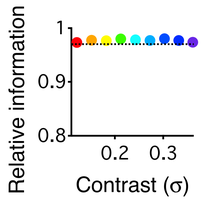
The spacing that provides the maximal information for two response functions with the different slopes (ν) across many different slopes and average rates.



Optimal dynamic range placement in the retina

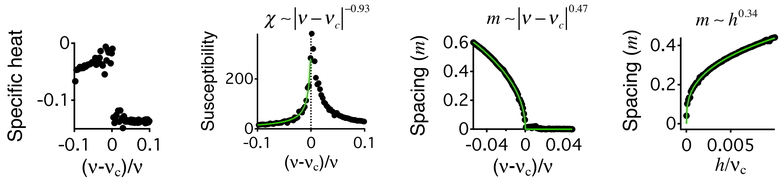


Information as a function of the spacing between two response functions that have slopes and a rate constraint taken from two cells recorded simultaneously. Black dots indicate the spacing of the data. Curves for the different contrasts are offset for clarity.

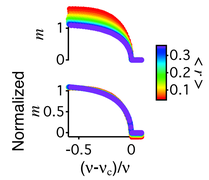


Average fraction of the maximal information captured by 7 pairs of simultaneously recorded adapting and sensitizing cells across different contrasts

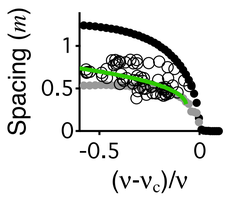
All divergences and power-law scaling within the model are consistent with a second order phase transition



Retinal populations remain poised at a critical point



The differences in the model due to different average rates (as occurs across contrasts), can be normalized, allowing for a comparison across different experimental conditions



Data, placed onto the normalized model, and fit by equation $m = A|\nu - \nu_c|^{\alpha} + B(h)^{\gamma}$. Grey points indicate the spinodal line.

Mapping between maximally informative solutions in neural circuits and the Ising model of phase transitions in physics

	Magnetic systems (Ising model)	Maximally informative coding
Optimal states defined by:	minima of free energy	maxima of information
Transition occurs with respect to:	temperature	input noise (average slopes of input response functions)
Symmetry broken below critical temperature	magnetization direction	exchange symmetry between neurons
Order parameter	magnetization	deviation of thresholds from the mean (difference for n = 2) across a neural population
Conjugate field	applied magnetic field	deviation of slopes from the mean (difference for n = 2) across a neural population
Exponent with respect to temperature for h = 0	mean-field value: ½ experiment: 0.316 – 0.327	mean-field value: ½ experiment: 0.39 ± 0.12
Critical isotherm exponent	mean-field value: ⅓ experiment: 0.2 – 0.21	mean-field value: ⅓ experiment: 0.15 ± 0.08

Conclusions

- Coordinated fast-Off populations provide the optimal amount of information about their inputs given noise and an energy constraint
- The absence of multiple types of On cells in salamanders is predicted by optimal information transmission
- Maximal information transmission with populations of neurons has a direct parallel with second order phase transitions from physics
- fast-Off ganglion cells reside near a critical point

References:

Kastner, D.B. Baccus, S.A. Coordinated dynamic encoding in the retina using opposing forms of plasticity. (2011). Nature Neuroscience.

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