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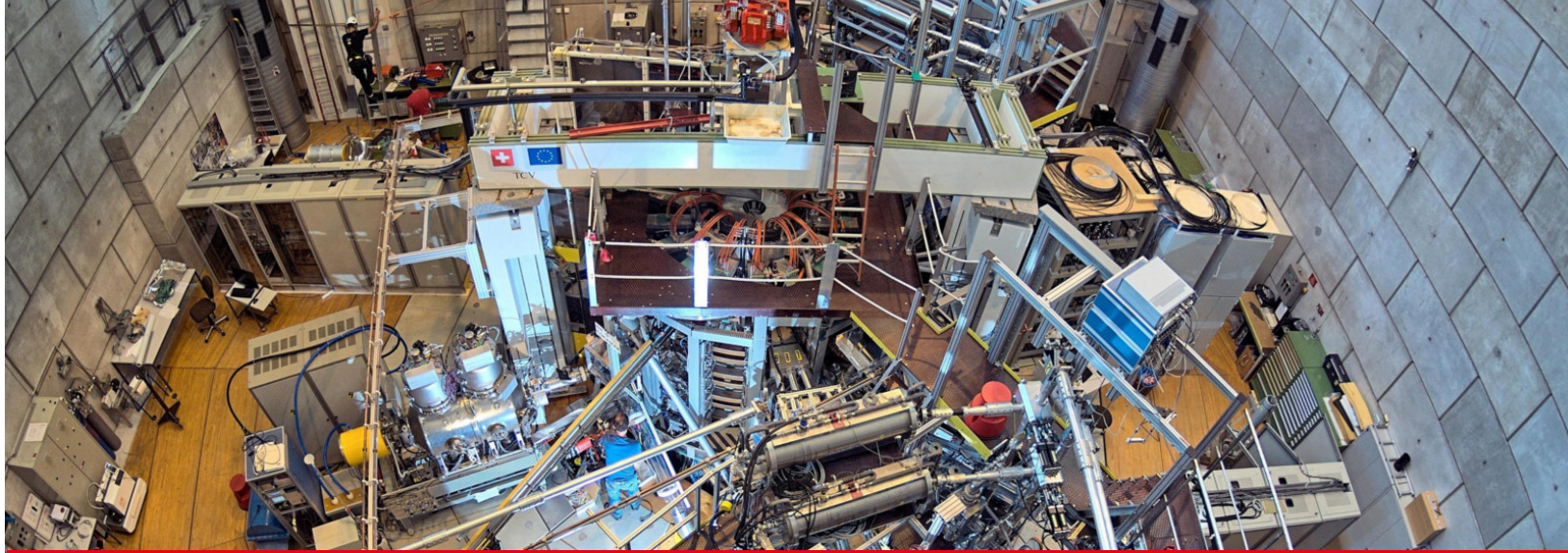

A Gyrokinetic Moment-Based Approach for the Simulation of the Boundary Plasmas in Fusion Devices

B. J. Frei[1], P. Ricci[1], J. Ball[1], A. C. D. Hoffmann[1], R. Jorge[2], and L. Stenger[1]

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[1] Ecole Polytechnique Fédérale de Lausanne (EPFL), Swiss Plasma Center, CH-1015 Lausanne, Switzerland
[2] Institute for Research in Electronics and Applied Physics, University of Maryland, College Park MD 20742, United States of America

Swiss Plasma Center

Challenges of Simulating the Turbulent Boundary Plasmas

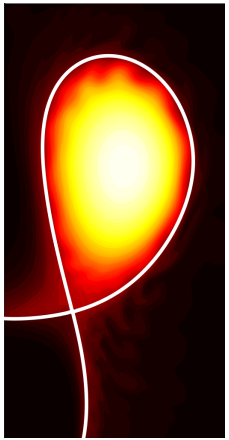
- **Boundary region** (edge and scrape-off-layer) controls the performance of fusion devices (H-L transition, pedestal, ELMs, …)
- **Boundary region** is characterised by **different plasma collisionality regimes**:

ITER: $T_e \sim 10 - 10^4$ eV and $n \sim 10^{18} - 10^{20}$ m^{-3} $\Rightarrow \lambda_{mpf}/R_0 \sim \dfrac{T_e^2}{n_e R_0} \sim 10^{-1} - 10^3$

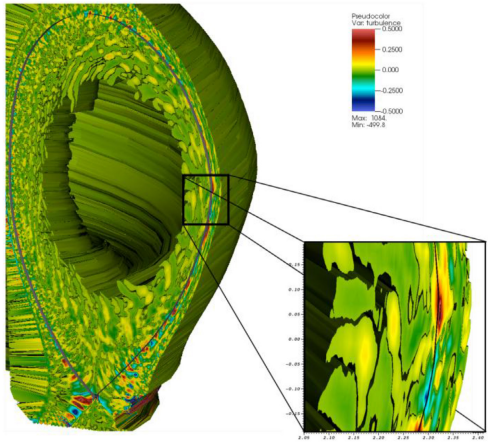
- Two currently-used turbulent modelling approaches:
  - **Drift-Kinetic (DK) fluid modelling** (lowest-order moments, less expensive, $k_\perp \rho_i \ll 1$ and $\lambda_{mfp} k_\parallel \ll 1$, no kinetic effects, Full-F)
  - **GyroKinetic (GK) modelling** (expensive, $k_\perp \rho_i \sim 1$, kinetic effects, also suitable for $\lambda_{mfp} k_\parallel \gtrsim 1$)

GK Moment Approach

$$k_\parallel \lambda_{mfp} \ll 1 \quad \& \quad k_\parallel \lambda_{mfp} \gtrsim 1$$



Giacomin M. *et al.* JPP, **2020**



Chang C. S. *et al.* Nucl. Fusion, **2017**

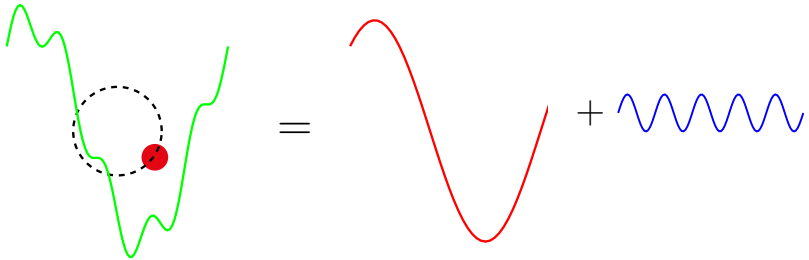
Tokamak Boundary Assumptions

- Small parameters **in the boundary region:**

$$\epsilon = \frac{\rho_s}{L_P} \sim \frac{\rho_s}{L_\phi} \ll 1, \quad \epsilon_B = \frac{\rho_s}{L_B} \ll \epsilon$$

- **<u>Large</u>** fluctuations on **<u>large scales</u>** and **<u>small</u>** fluctuations on **small scales:**

$$k_\perp \rho \frac{e\phi}{T_e} \sim \frac{k_\parallel}{k_\perp} \sim \epsilon$$



- Low frequency ordering:

$$\frac{\omega}{\Omega_i} \sim k_\perp \rho \epsilon \qquad \epsilon_\nu = \frac{\nu_{ii}}{\Omega_i} \sim \epsilon^2$$

- From $\dot{R}$ and $\dot{v}_\parallel$ to collective dynamics, $F_a = F_a(\mathbf{R}, v_\parallel, \mu, t)$ (Full-F):

$$\frac{\partial}{\partial t}\left(B_\parallel^* F_a\right) + \nabla \cdot \left(B_\parallel^* \dot{\boldsymbol{R}} F_a\right) + \frac{\partial}{\partial v_\parallel}\left(B_\parallel^* \dot{v}_\parallel F_a\right) = B_\parallel^* \sum_b \langle C_{ab} \rangle$$

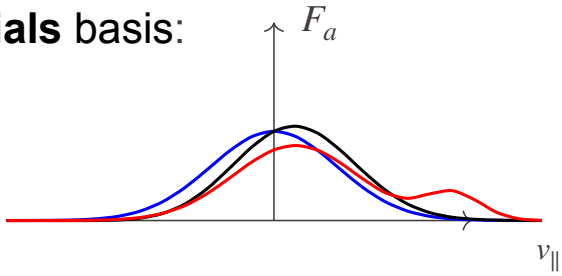
Gyroaverged collision operator

The new GK Moment Approach to the GK Boundary Model

- Expansion of F_a (Full-F) on a **Hermite-Laguerre polynomials** basis:

Gyro-Moment of F_a

$$F_a = \sum_p \sum_j \boxed{N_a^{pj}(\boldsymbol{R}, t)} \frac{H_p(v_\parallel) L_j(\mu)}{\sqrt{2^p p!}} F_{Ma}$$



- **Equation for N_a^{pj}** from the projection **onto the Hermite-Laguerre basis** of GK Equ.

$$||\ldots||^{pj} = \int d\theta \int d\mu dv_\parallel H_p(v_\parallel) L_j(\mu) \ldots \qquad ||(\text{GK Equ.})||^{pj}$$

- **Gyro-Moment Hierarchy Equation** and **GK Field Equations**

Projection of GK collision operator

$$\frac{\partial}{\partial t} N_a^{pj} + \nabla \cdot ||\dot{\boldsymbol{R}}||^{pj} - \frac{\sqrt{2l}}{v_{th\parallel a}} ||\dot{v}_\parallel||^{p-1j} + \mathcal{F}_a^{pj} = \sum_b C_{ab}^{pj} \qquad C_{ab}^{pj} = ||\langle C_{ab} \rangle||^{pj}$$

B. J. Frei *et al.*, JPP **86**, 905860205 (2020)

Nonlinear GK Coulomb (Landau) Collision Operator

- To model collisional effects, nonlinear **GK Coulomb collision operator** is **important:** $\lambda_{mpf} k_{\|} \ll 1$ **and** $\gtrsim 1$, Full-F and $k_{\perp}\rho_{a,b} \sim 1$
- In $(\mathbf{x}, \mathbf{v})$ phase-space, **Coulomb collision** operator is defined by

$$C_{ab}[f_a, f_b] = C_{ab}(\boldsymbol{x}, \boldsymbol{v}) = \frac{\partial}{\partial \boldsymbol{v}} \cdot \left[\boldsymbol{A}_{ab} f_a + \frac{\partial}{\partial \boldsymbol{v}} \cdot (\boldsymbol{D}_{ab} f_b) \right]$$

$$\boldsymbol{A}_{ab} = -\left(1 + \frac{m_a}{m_b}\right) \frac{\partial}{\partial \boldsymbol{v}} \cdot \boldsymbol{D}_{ab}$$

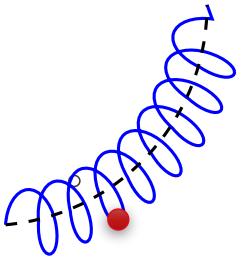
$$\boldsymbol{D}_{ab} = -\nu_{ab} \frac{\partial^2}{\partial \boldsymbol{v} \partial \boldsymbol{v}} G_b$$

$$\nabla_v^2 G_b = H_b$$

$$H_b = \int d\boldsymbol{v}' \frac{f_b(\boldsymbol{v}')}{|\boldsymbol{v}' - \boldsymbol{v}|}$$

$$G_b = \int d\boldsymbol{v}' f_b(\boldsymbol{v}') |\boldsymbol{v}' - \boldsymbol{v}|,$$

- **Gyroaveraged** of C_{ab} performed in $(\mathbf{R}, \mu, v_{\|}, t)$ phase-space



$$\boldsymbol{x} = \boldsymbol{R} + \boldsymbol{\rho}_a(\mu, \theta)$$

$$\langle C_{ab} \rangle (\boldsymbol{R}, \mu, v_{\|}) = \frac{1}{2\pi} \int d\theta C_{ab}$$

Finite Larmor radius (FLR) (important for $k_{\perp}\rho_{a,b} \sim 1$ and for energetic particles)

Nonlinear GK Coulomb in the Gyro-Moment Approach

$$C_{ab}^{pj} = \int d\boldsymbol{v} H_p(v_{\|}) L_j(\mu) \langle C_{ab} \rangle$$

- Perform the **gyro-average** of using a **spherical harmonic multipole** expansion in (ξ, v) of C_{ab}
- Integration in (ξ, v) coordinates using a **basis transformation** from $(\mu, v_{\|})$ to (ξ, v), $(T^{-1})_{pq}^{rsm}$

Full-F Nonlinear GK Coulomb Collision Operator

$$C_{ab}^{pj} = \nu_{ab} \sum_{\boldsymbol{k},\boldsymbol{k}'} \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \sum_{r,s,p,q,\ldots} \underbrace{\nu_{ab\ldots}^{lpj\ldots} (T^{-1})_{pq}^{rsm}}_{\text{Numerical coefficients}} \ldots \underbrace{K_s(b_a) K_q(b_b')}_{\text{FLR Kernels}} \underbrace{N_a^{rs}(\boldsymbol{k}) N_b^{pq}(\boldsymbol{k}')}_{\text{Gyro-Moments}},$$

- Arbitrary $k_{\perp}$, and m_a/m_b , T_a/T_b ratios

Jorge R., B. J. Frei and P. Ricci, JPP **85**, 905850604 (2019)

First Numerical Investigations: Linearized GK Coulomb

- Assuming $F_a = F_{aM} + f_a \quad (f_a \ll F_{aM})$

$$C_{ab} \simeq C^T_{ab} + C^F_{ab} \qquad \overset{\text{Test}}{C^T_{ab}} = C_{ab}[f_a, f_{bM}] \qquad \overset{\text{Field}}{C^F_{ab}} = C_{ab}[f_{aM}, f_b]$$

- Linearized **GK Coulomb Collision Operator** in the gyro-moment approach

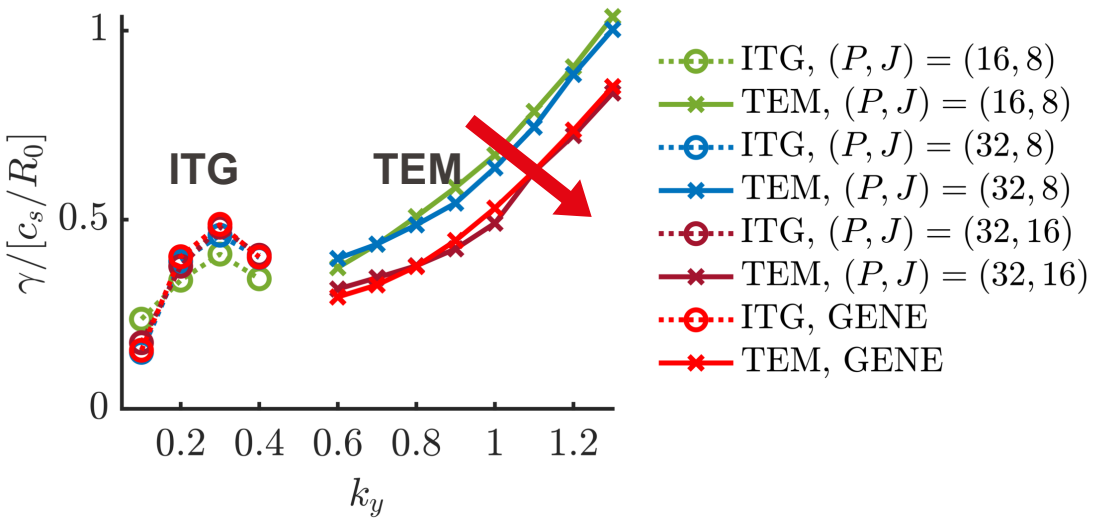
$$C^{Tpj}_{ab} = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \sum_{r,s,p,q,\ldots} \underbrace{\nu^{Tlpj}_{ab\ldots} \ldots \left(T^{-1}\right)^{rsm}_{pq}}_{\text{Numerical coefficients}} \overbrace{K_s(b_a)}^{\text{FLR Kernels}} \underbrace{N^{rs}_a}_{\text{Gyro-Moments}}$$

- Gyro-moment approach applied to different **simplified/*ad hoc*** linearized collision operators (e.g. GK/DK Sugama, GK/DK pitch-angle, GK/DK Dougherty)

B. J. Frei *et al.*, JPP **87,** 905870501 (2021)

Collisionless Microinstabilities in agreement with GENE

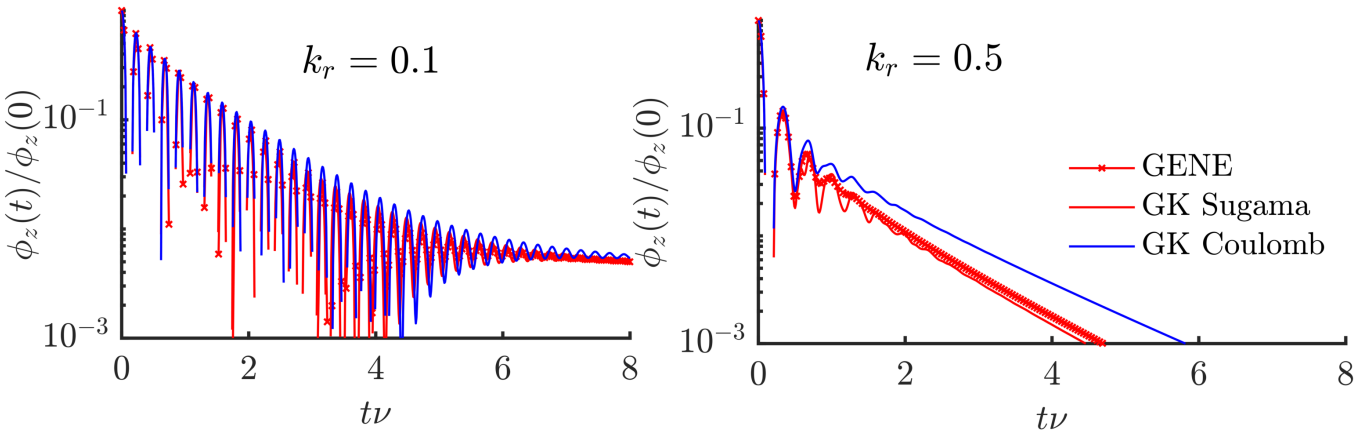
- Linearized Gyro-Moment Hierarchy equation implemented in flux-tube code
- Retrieve **ITG to TEM** transition (cyclone base case parameters) with $(P, J) \simeq (32,16)$



- Retrieve **KBM** transition at finite β_e, with $(P, J) \simeq (32,16)$
- Strong kinetic physics (e.g. trapped/passing boundary) **<u>require larger</u>** (P, J)

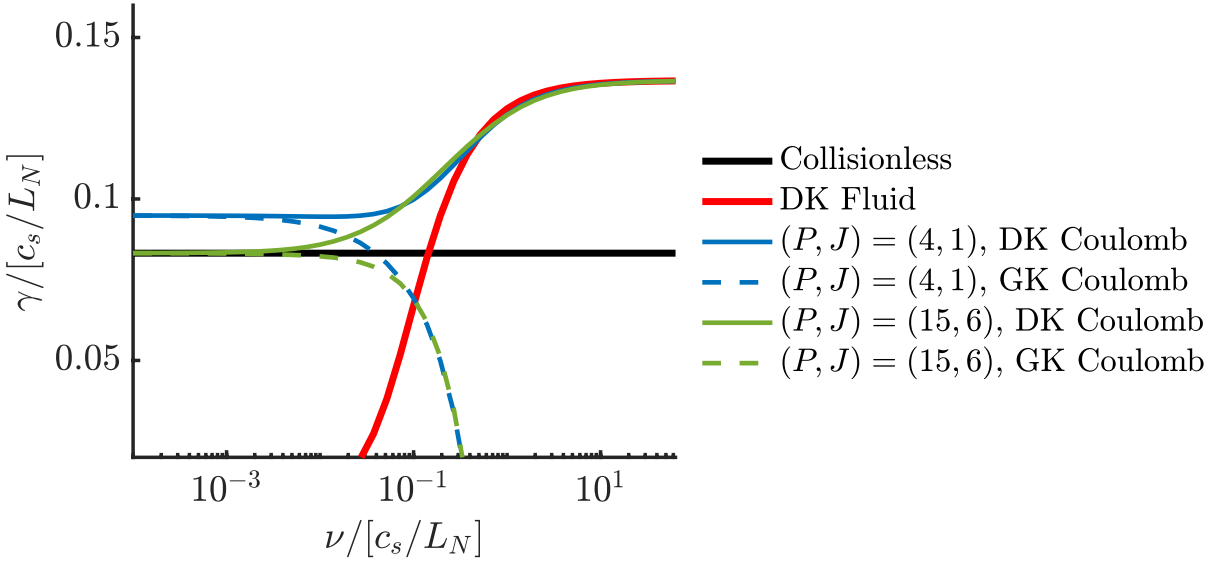
Deviations between GK Coulomb and Other Collision Models

- Simplified/*ad hoc* collision operator are often considered ⇒ possible deviation with GK Coulomb operator
- **GK Sugama** (good agreement with GENE) yields **stronger collisional zonal flow** (ZF) damping than **GK Coulomb** ($\nu_{i*} \simeq 3.1$)



- Collision operator models might affect turbulent level through ZF saturation mechanism

Bridging Kinetic and Fluid Limits of ITG



- **Collisionless limit retrieved** with $(P, J) \simeq (15,6)$ **<u>and</u> fluid limit retrieved** with $(P, J) \simeq (4,1)$
- ITG mode **damped/suppressed by collisional FLR effects**
- Number of gyro-moments **reduced with increasing collisionality ⇒Ideal for tokamak boundary modelling since** $\nu \sim nT^{-3/2}$

Conclusions

- **A new GK model** for the tokamak boundary based on **a Hermite-Laguerre polynomials basis** (gyro-moment expansion)
- **A nonlinear GK Coulomb collision operator** (arbitrary mass and temperature ratios) is derived; a **linear version derived and numerically implemented**; gyro-moment expansion of different **simplified/*ad hoc*** collision operator models (e.g., Sugama operator)
- Recover relevant **microinstability linear properties** (e.g.,TEM, ITG and KBM); in agreement with GENE
- Explore **deviations** between **GK Coulomb** and other collision operator models in linear cases (e.g., ZF damping and ITG)
- **Number of gyro-moment decreases with collisionality (SOL)** and **pressure gradients (edge)** $\Rightarrow$ optimal for edge and SOL modelling.
- Extend to nonlinear and realistic 3D equilibrium and Full-F conditions

Thank you for your attention

First Numerical Investigations: Linearized Gyro-Moment Hierarchy Equation

- Assuming $F_a = F_{aM} + f_a \quad (f_a \ll F_{aM})$

Landau Damping — Particle trapping — Magnetic Drifts

$$\frac{\partial}{\partial t}\mathbf{N}_a + \mathbf{H}_a \cdot \nabla_{\|}\mathbf{N}_a + (\mathbf{M}_a \cdot \mathbf{N}_a)\nabla_{\|}\ln B + \mathbf{D}_a \cdot \mathbf{N}_a$$

$$= \mathbf{S}_\phi \phi + \mathbf{S}_\psi \psi + \sum_b \left(\mathbf{C}^T_{ab} \cdot \mathbf{N}_a + \mathbf{C}^F_{ab} \cdot \mathbf{N}_b\right)$$

Background Gradient drive — **Linearized GK Collision Operators**

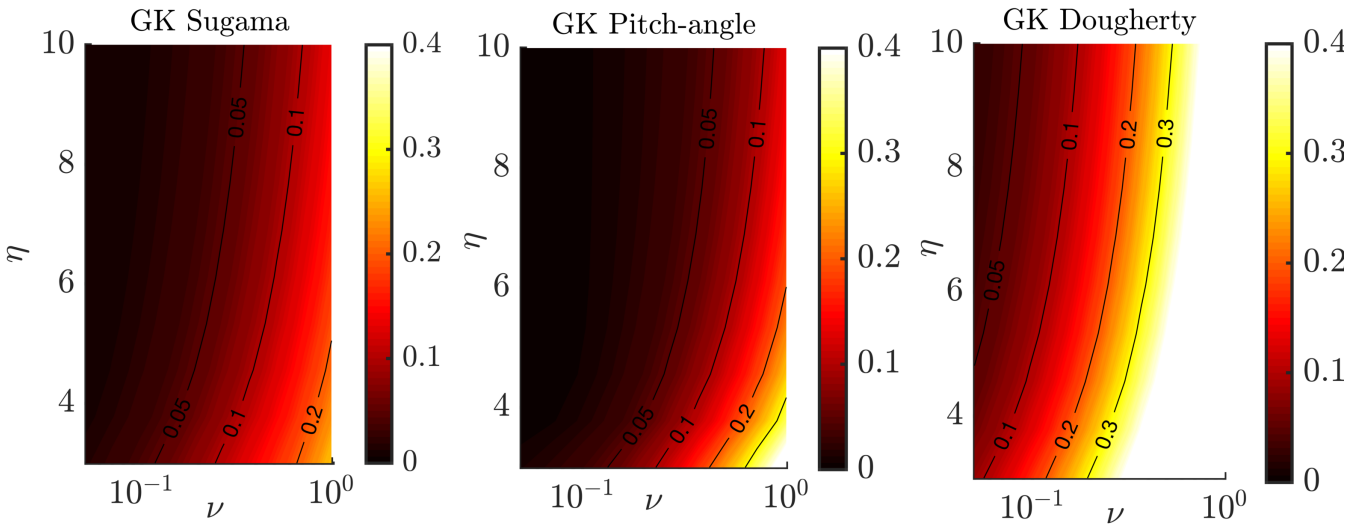
With $\delta\mathbf{E}_{es} = -\nabla\phi$, $\delta\mathbf{B}_\perp \simeq -\mathbf{b}\times\nabla\psi$ and $\mathbf{N}_a = [N_a^{00}, N_a^{01}, \ldots N_a^{10}, N_a^{11} \ldots N_a^{PJ}]^T$

- Closure at arbitrary collisionality available: $P = P(\nu, k_{\|})$, $J = J(\nu, k_\perp)$

- Implemented in a flux-tube code: $\mathbf{B} = B_0 \nabla x \times \nabla y$, $\mathbf{k}_\perp = k_x \nabla x + k_y \nabla y$

Deviations between GK Coulomb and Other Collision Models

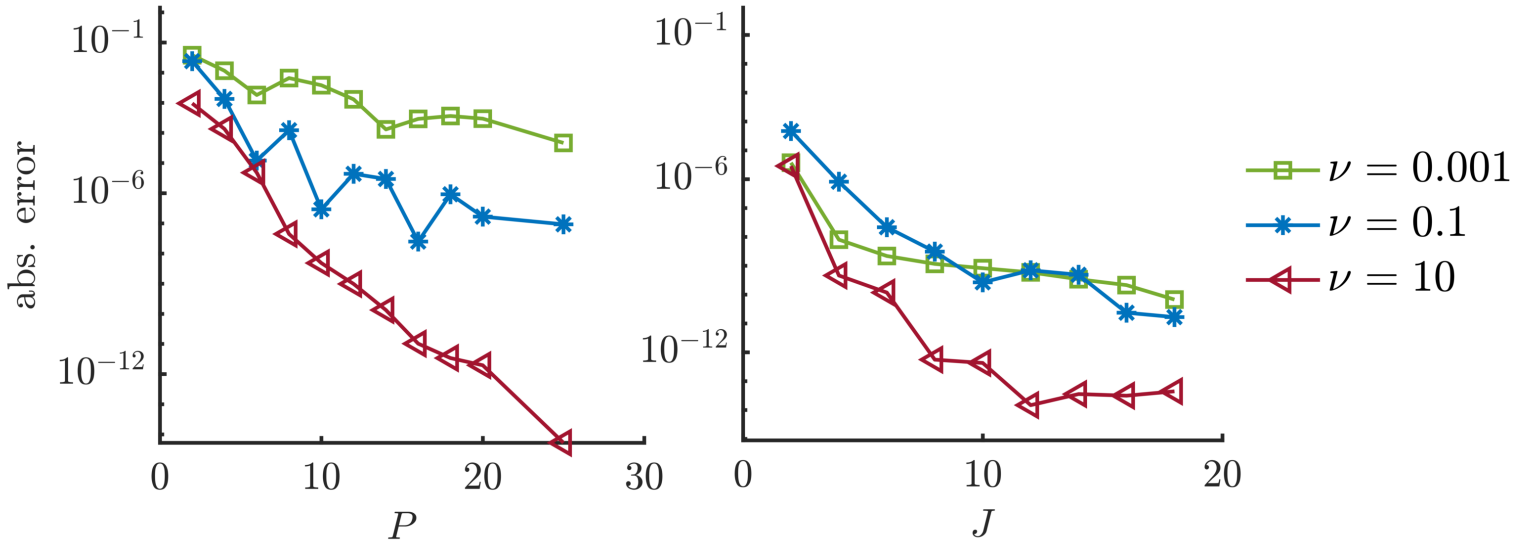
- Because analytically and numerically challenging, simplified/*ad hoc* collision operator are often considered ⇒ possible deviation with GK Coulomb operator
- Relative deviation of **ITG growth rate peak from GK Coulomb** as a function of collisionality, $\nu = \nu_{ii}/[c_s/L_N]$, and normalized temperature gradient, $\eta = L_N/L_T$



- Deviation up to $20\,\%$ with GK Sugama (benchmarked with GENE); largest deviation with GK Dougherty

Gyro-Moment Convergence improves with collisionality

- Convergence rate **increases with collisionality** ν on sITG growth rate peak

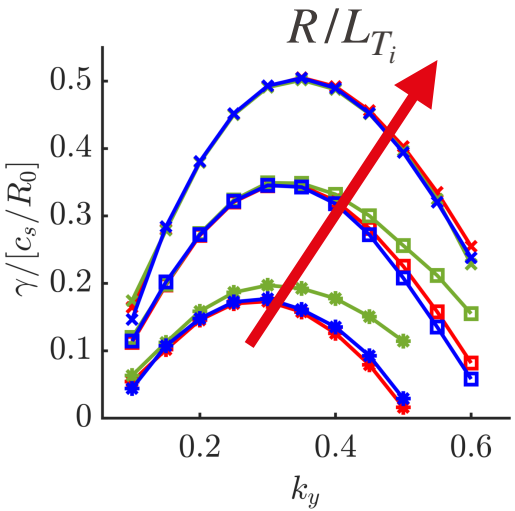


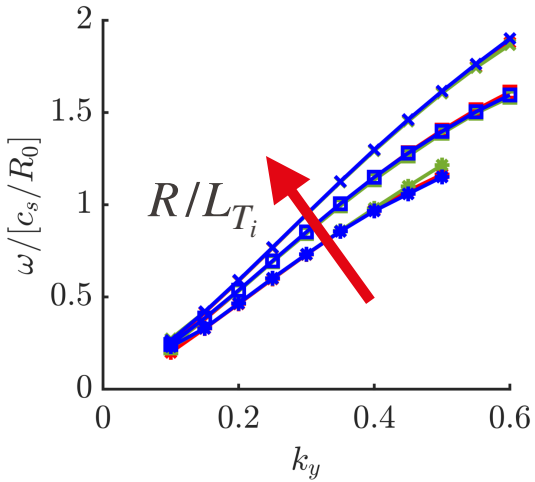
- **Ideal** for tokamak boundary modelling since $\nu \sim nT^{-3/2}$

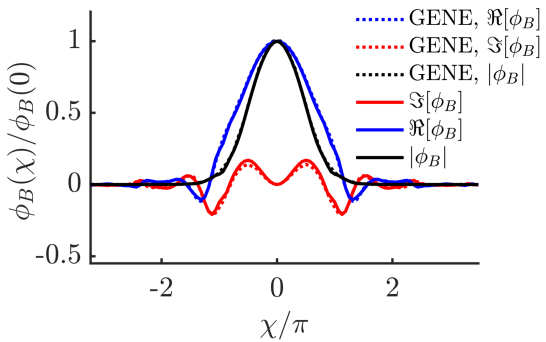
Cyclone Base Case in agreement with GENE

- **Collisionless** ITG with **adiabatic electron**:
  - $\mathbf{B} = B_0 \nabla x \times \nabla y$ and $\mathbf{k}_\perp = k_x \nabla x + k_y \nabla y$

$$R_{T_i} = \frac{R}{L_{Ti}} \simeq -R\partial_x \ln T_i$$



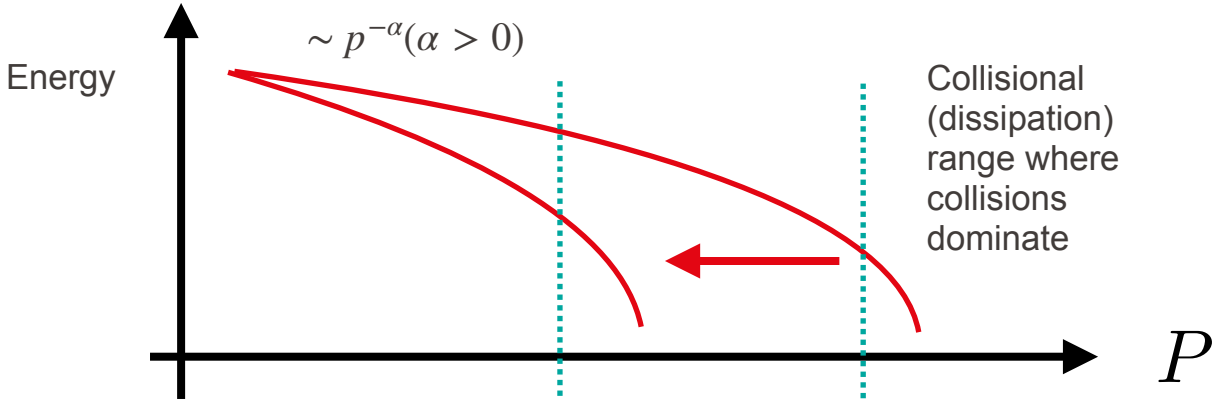




$R_{T_i} = 8$, GENE
$R_{T_i} = 10$, GENE
$R_{T_i} = 6$, GENE
$(P, J) = (16, 8)$
$(P, J) = (16, 8)$
$(P, J) = (16, 8)$
$(P, J) = (32, 16)$
$(P, J) = (32, 16)$
$(P, J) = (32, 16)$

Backup: Closures at Arbitrary Collisionality are Available

In practice, solve $\partial_t \mathbf{N}_a = \ldots$ up to $(P_c, J_c) \ll \infty$. How to properly choose (P_c, J_c) ?



$$P_c \simeq P_c(\nu, k_\parallel)$$

$$J_c \simeq J_c(\nu, k_\perp)$$

- At arbitrary collisionality, an asymptotic semi-collisional closure; $\mathbf{H}_a, \mathbf{M}_a, \mathbf{D}_a \sim O(\sqrt{p}, \sqrt{j})$ and $\partial_t \mathbf{N}_a \ll \mathbf{C}_{ab}^T \cdot \mathbf{N}_a \sim \nu p^\alpha \mathbf{N}_a$ assuming $(P_c, J_c) \gg 1$,
- In the high-collisional limit, Chapman-Enskog procedure:

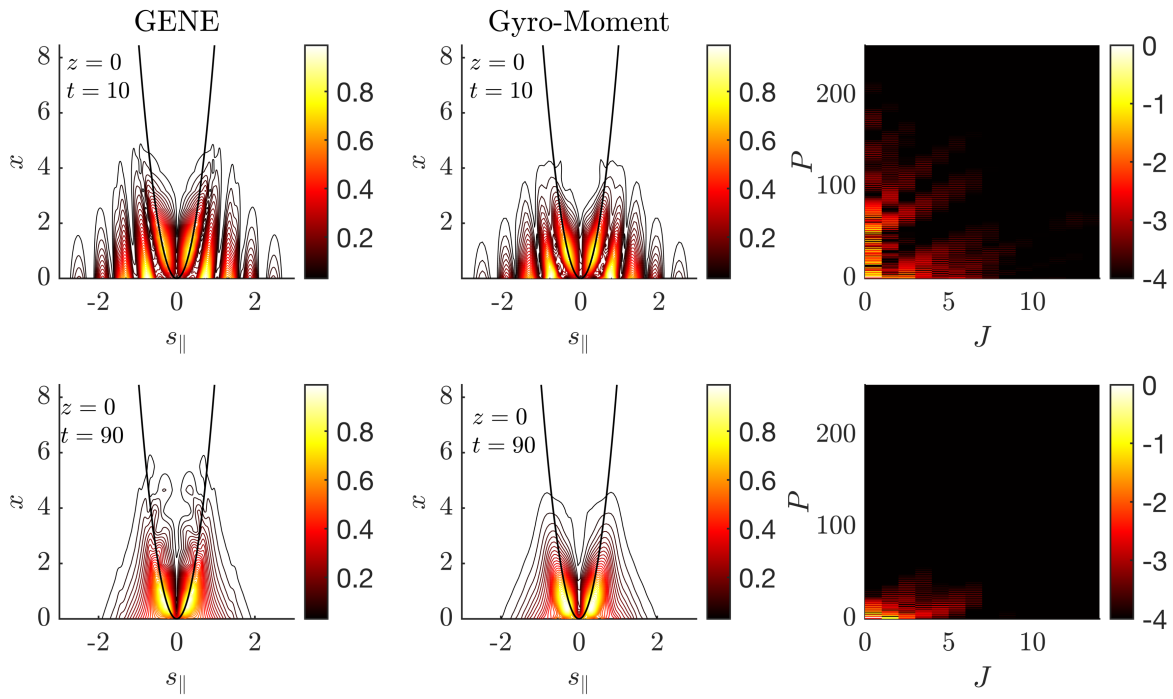
$$\mathbf{N}_a \simeq \mathbf{N}_a^{(0)} + \epsilon \mathbf{N}_a^{(1)} \qquad \epsilon \sim \lambda_{mfp}/L_\parallel \quad \Rightarrow \quad Q_\parallel \simeq -\chi_\parallel \nabla_\parallel T_\parallel$$

Backup: Zonal Flow Velocity-Space Structure

- Zonal flow ($k_y = 0$) residual is important in regulation of turbulent transport
- Ballistic response of passing particle creates fine $v_\parallel$ structures ($k_{v_\parallel} \sim k_\parallel t$)

- **In GENE, $k_{v_\parallel} \sim N_{v_\parallel}/v_T$; in MOLIX-X $k_{v_\parallel} \sim \sqrt{p}/v_T$.**

- **Velocity space structure comparison with GENE ($k_x = 0.05/\rho_s$ and $\nu_{i*} \simeq 0.03$)**

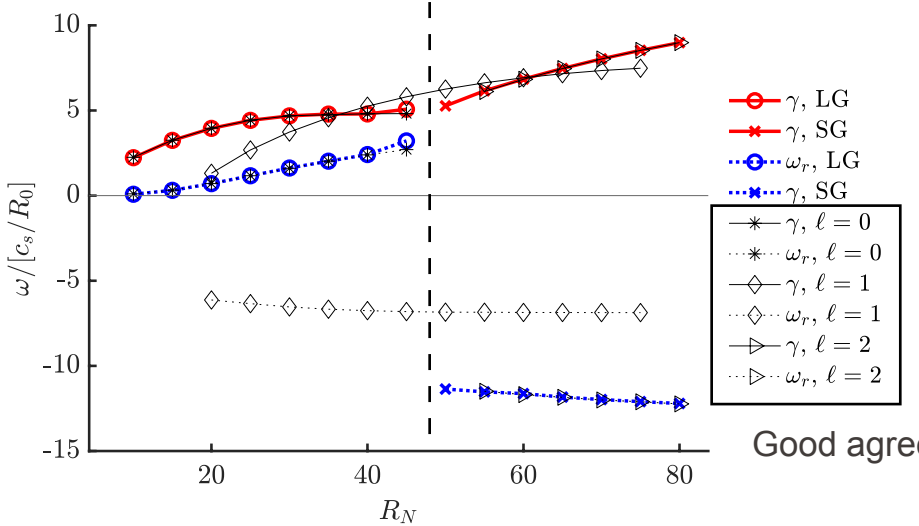


Steep Gradient H-Mode Conditions

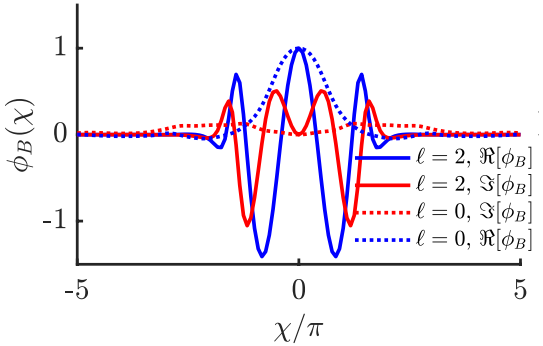
- **Steep pressure gradients** (e.g. mid-pedestal) alter linear properties of toroidal drift modes
- **Excitation of sub-dominant higher order** ($\ell > 0$) eigenstates above linear critical gradient.

$$R_N = \frac{R}{L_N} \simeq -R\partial_x \ln N \qquad R_T = \frac{R}{L_T} = R_N$$

Most unstable $\ell = 0$ Most unstable $\ell = 2$



Unconventional mode structure from $\ell = 0$ to $\ell = 2$



Good agreement with GENE Direct eigenvalue solver

B. J. Frei, PET 2021

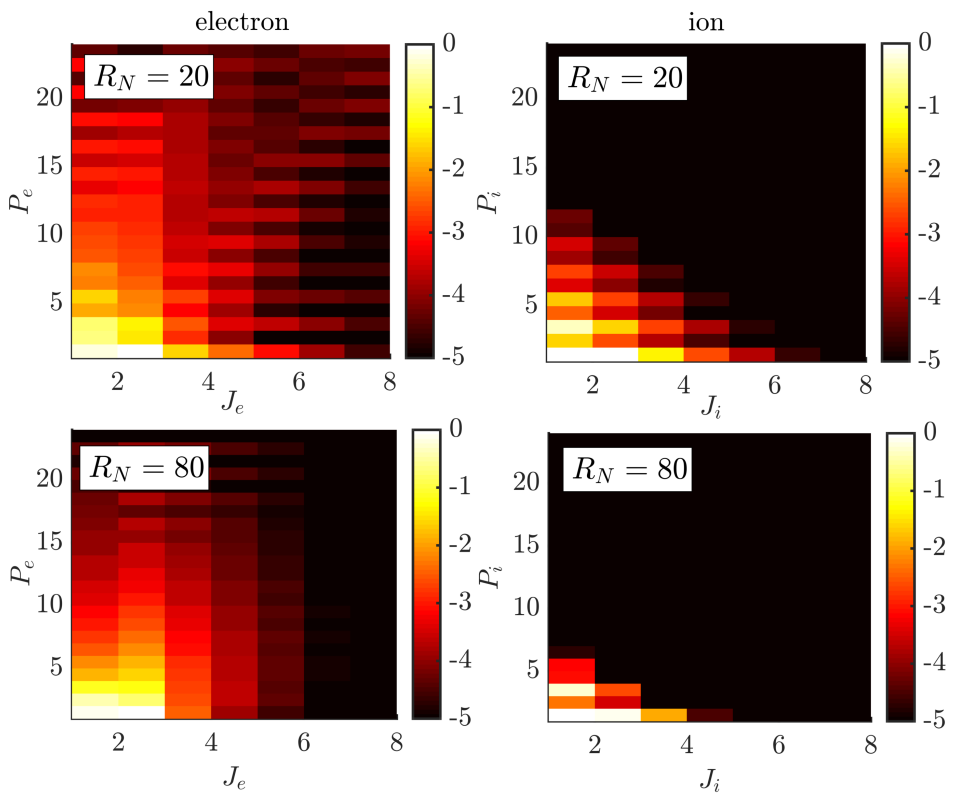
Steep Gradient H-Mode Conditions

- **Magnetic gradient drift resonance effects** become negligible, i.e.

$$\omega \sim \omega_* \gg \mathbf{k} \cdot \mathbf{v}_D$$

- **Hermite-Laguerre spectrum** well resolved with $(P, J) \lesssim (24,8)$

- **<u>Number of gyro-moments is reduced at steep gradients</u>** <u>(compared to core conditions)</u>, **<u>even in the collisionless case.</u>**



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