Application of Multi-Objective Optimisation to Process Measurement System Design

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Abstract

Multi-objective optimisation (MOO) has been used with an equation solver data reconciliation software to develop a tool for sensor system design based on modifying the sensitivity matrix of a simulated process. MOO enables searching for the best trade-off between two conflicting objectives: the cost of the system and the precision of key performance indicators (KPI) (variables that have to be measured or calculated). This methodology has been applied to design the sensor system of a two stage experimental air-water heat pump. Proper knowledge of modelling equations and constants helps to improve the estimation of the precision of variables, and lowers the cost of the system. Compared to single objective optimisation, the MOO strategy increases the number of solutions, yet the precision function still relates to different objectives for each KPI, and its formulation is shown to have an impact on the trade-off obtained.

Keywords: sensitivity matrix, reconciled precision, key performance indicators, population based evolutionary algorithm, Pareto optimal frontier.

1. Introduction

It is now standard to archive real time control system data which can in turn be used for process follow-up and simulation. A set of measurements, however, is rarely coherent due to process instabilities, unaccounted losses, sensor deviations, etc. Data reconciliation is essential to coherently transform raw data so as to determine the state of a process in operation. This requires redundant measurements, beyond what is strictly needed to solve a system of modelling equations. With insufficient (as is often the case of large scale processes) or unsuitably chosen measurement locations, the state of the process cannot be defined. The many benefits of online data reconciliation, e.g. improved plant production and operation, and early detection of equipment degradation,

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have been discussed by Heyen *et al*, (2000). From a financial viewpoint, priority may be given to significant process variables designated as key performance indicators (KPI).
A sensor system design methodology developed by Heyen *et al* (2002) combines the use of the equation solver type data reconciliation software VALI III (Belsim, 2001) and genetic algorithm programming. A cost function is minimised in a single objective optimisation by targeting specific precision values for KPI with a penalty function. This type of formulation tends to eliminate inexpensive solutions that do not match precision requirements, or conversely expensive solutions with better than required precision. This paper proposes the use of a queuing multiple objective optimisation (MOO) program (Leyland, 2002) with a non-dominated ranking scheme (Golberg, 1989) to emphasise the trade-off between precision and cost and broaden the array of solutions.

2. Methodology

The resolution procedure, is based on the one defined by Heyen *et al* (1996; 2002). Firstly, a data reconciliation model of a given process is built and solved for nominal operating conditions. The sensitivity matrix M of model A allows calculating reconciled variables and reconciled variances with the variance matrix P, provided as follows,

$$\begin{bmatrix} Y \\ \lambda \end{bmatrix} = \begin{bmatrix} P & A^T \\ A & 0 \end{bmatrix}^{-1} \begin{bmatrix} Py \\ -C \end{bmatrix} = M^{-1} \begin{bmatrix} Py \\ -C \end{bmatrix} \quad \text{with} \quad P = \begin{bmatrix} \frac{1}{\sigma_i^2} \end{bmatrix} \tag{1}$$

$$\operatorname{var}(Y_i) = \sum_{j=1}^{n} \frac{\left(M_{i,j}^{-1}\right)^2}{\sigma_j^2} \qquad \forall i = 1, n \tag{2}$$

The variance matrix P refers to position, type and precision of sensors that may be installed. The goal of MOO is to optimise P, by systematically selecting from a database the sensors that improve precision at a minimum cost. Unmeasured variables have close to or infinite variance, and for measured variables the variance becomes,

$$\sigma_i^2 = \left[\left(\sum_{j=1}^{s} \frac{\gamma_{i,j}}{\sigma_{sensor\_j}^2} \right) + \frac{1}{\sigma_{exist\_i}^2} \right]^{-1} \qquad \forall i = 1, n \tag{3}$$

In MOO evolutionary algorithms the definition of Pareto optimality for a vector (Pareto, 1896) is extended to a search space. If there are no new solutions after a certain number of generations, it is assumed that the trade-off curve coincides with the Pareto optimal frontier (POF). It should be noted that singular matrices need not be penalised.

3. Selection of the Objective Functions

The cost objective is the sum of capital costs of the system. Other factors could also be included (e.g. installation, maintenance, variations in cost with the number of sensors).
The precision objective has to be global, even if each KPI can be considered as a separate objective function. This is equivalent to a weighting problem. Though there is no theoretical limit to the number of objectives of evolutionary algorithms, Leyland (2002) has nevertheless pointed out that with a greater number of conflicting objectives,

- there are fewer dominated solutions, thus optimisation becomes meaningless,

- the required initial population is larger, thus calculation time is increased and,
- the interpretation of results becomes increasingly complicated.

Moreover, as individual KPI precisions are not always conflicting objectives, we formulated the problem with two objectives. The simplest formulation of the precision objective is to minimise the sum of reconciled variance amongst the list of KPI:

$$fprec = \sum_{i=1}^{n_{KPI}} \left(\frac{\sigma_i}{Y_i} \right)^2 \tag{4}$$

It can also be expressed as the least precise reconciled variance within the set of KPI:

$$fprec = \max_{i=1,n_{KPI}} \left(\frac{\sigma_i}{Y_i} \right)^2 \tag{5}$$

A third alternative, intended as a compromise between the prior two, is a modified Kreisselmeier-Steinhauser (KS) function (Raspanti *et al,* 2000), for which the variances of the KPI are summed while approximating the value of the least precise variance.

$$fprec = \frac{1}{\rho} \ln \left[\frac{1}{n_{KPI}} \sum_{i=1}^{n_{KPI}} \exp\left(\rho \left(\frac{\sigma_i}{Y_i} \right)^2 \right) \right], \quad \rho \in \Re^{+*} \quad fprec \to \max_{i=1,n_{KPI}} \left(\frac{\sigma_i}{Y_i} \right)^2 \text{ when } \rho \to \infty \tag{6}$$

4. Application Example

The method has been applied to design the sensor system of an experimental two stage air-water heat pump (Figure 1) (Iraburu, 2002). Data reconciliation was completed with a list of constant specifications relating to the pressure drops in the transfer lines, the composition of refrigerant and the geometrical characteristics of the compressors and heat exchangers. Other parameters were considered as process variables.

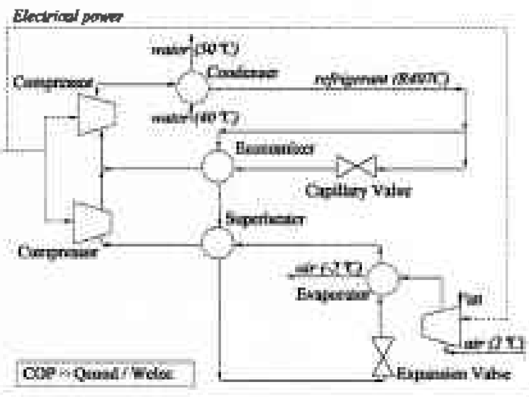

Figure 1. Air-water two stage heat pump

There were no installed sensors before system design optimisation. The available sensors are listed in Table 1. At each available sampling point, at most three sensors may be installed to perform a single type of measurement.

Table 1. Available sensors

Type[a]	Range	Error[b]	Units	Cost[c]	Type[a]	Range	Error[b]	Units	Cost[c]
P 1	0.1- 400	0.2 %	Bar	457	M 1	0- 0.02	0.2 %	kg/s	6 800
P 2	0- 0.2	0.2 %	Bar	756	M 2	0- 1.83	0.2 %	kg/s	10 000
dP	0- 0.05	0.2 %	Bar	400	V	10- 40	0.2 %	L/min	8 000
T 1	73- 1173	± 1.5	K	125	H	0- 100	0.8 %	%	640
T 2	73- 1173	± 0.3	K	200	E 1	0- 10	1.0 %	kW	2 000
T 3	173- 473	± 0.1	K	250	E 2	0- 10	0.3 %	kW	18 500

[a]abbreviations for types of measurements: P: pressure; dP: pressure drop; T: temperature; M: mass flow; V volumetric flow; H: relative humidity; E: electrical power.
[b]errors are relative except for temperature sensors. [c]costs are in CHF.

5. Results and analysis

5.1. Importance of adding redundancy by system balance equations

At first, a single KPI is selected: the coefficient of performance (COP) of the heat pump, i.e. the ratio of the condenser heat load to the electrical power consumption. Two models are compared: the entire flowsheet and another model in which the condenser and the electrical source are considered separately. The number of equations, constants, and variables, and the results for systems of minimal costs are given in Table 2.

Table 2. Optimisation for COP estimations

Model	Equations	Constants	Variables	Min cost (CHF)	STD (%)	Sensors
Cond./Elec.	60	12	67	10 957[a]	21.57	7
Flowsheet	165	31	184	11 571	0.326	25

[a]less expensive non singular solutions are found, but the KPI standard deviations exceed 100 %

The trade-off curves are shown in Figure 2, left. Since the problem involves integer variables, each point relates to a specific system, and the POF can be neither smooth nor continuous. Despite the greater number of sensors required for characterising the complete system, closed loops increase redundancy, which improves COP precision and lowers costs. For similar costs, the trade-off curve of the condenser and electrical source subsystem is entirely dominated. Figure 2, right shows one solution from the Pareto set (23,385 CHF, STD: 0.17 %). For the subsystem calculations, a solution with an equivalent precision would cost approximately 100, 000 CHF (cf. Figure 2, left).

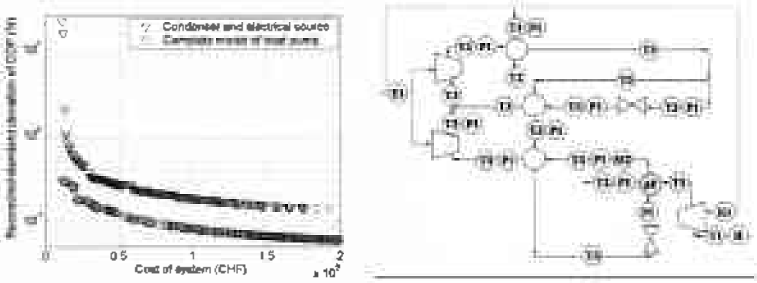

Figure 2. left: Trade-off curves for COP (after 100,000 evaluations), right: sample solution

5.2. Analysis of the formulation of the precision function

When several KPI are involved, the precision function may influence the resolution procedure and the results. Let us consider an example with four KPI: the COP and three pressure drop parameters (air side of the evaporator, capillary and expansion valves).

For equation 5 (Figure 3, left), the trade-off is located in the low cost/precision part of the search space. The least precise KPI is almost consistently the COP. The precision of the other KPI are equivalent, but sometimes worsen as costs increase. The precision objective can no longer be improved when, for lack of remaining sampling points, a plateau is reached for the evaporator air side pressure drop KPI. This limitation hampers exploration of the high cost/precision region.

Conversely, for equations 4 and 6 (Figure 3, right), it is essentially the high cost/precision region that is explored. The KPI precisions all improve somewhat more steadily with costs, but almost asymptotically, which is of lesser interest. Moreover, in the cost region where results overlap with those of equation 5, there is a wider spread in KPI precisions. Consequently, it is more costly to obtain a desired value of the least precise KPI (i.e. the COP). Nevertheless, a full coverage of search space can be assured only by these sum based functions. With equation 6 a compromise could be expected with a higher value of the ρ factor (set to 1000 in our calculations). However, the use of an exponential operator is an issue of concern in regard to machine precision. This type of function would be more robust should the logarithm operator be applicable first.

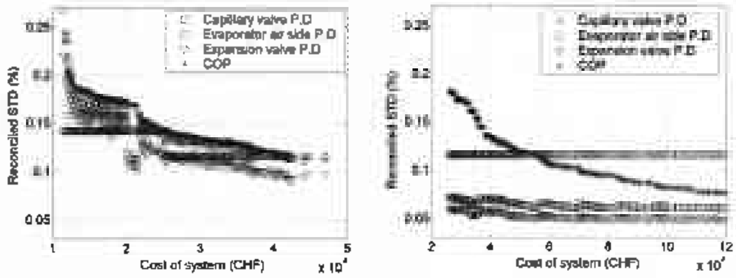

Figure 3. (left: Eq. 5, right: Eqs. 4 and 6) Trade-off curves for four KPI (100,000 evaluations)

With a satisfactory trade-off curve, the choice of a system for implementation can be based on targeted budgets and/or KPI precisions.

6. Conclusion

A method combining data reconciliation to a queuing MOO program has been proposed for the optimal design of sensor systems, and illustrated with the example of an experimental heat pump. The advantages of adding redundancy with balance and modelling equations has been demonstrated by comparing a model of the entire installation (which yields improved precision and costs) to a model that only includes the units which define the COP. However, the precision objective influences the trade-off obtained when there are multiple KPI. On one hand, minimising the least precise KPI appears to be the best strategy for minimising investments. On the other hand, the functions that can be expected to assure a fuller coverage of search space should be based on the sum of KPI. This issue may be solved with further changes to the modified KS function. With independent objectives, the MOO resolution approach has the benefit of yielding a trade-off of solutions with KPI precision as a function of costs. Feasibility criteria can then be used to finally select a system among the proposed solutions.

Nomenclature

A: Jacobian (incidence) matrix for measured and unmeasured variables
C: matrix of constants
fprec: precision objective function
k: number of sensors selected
M: sensitivity matrix
n: number of variables
n_{KPI}: number of key performance indicators
P: inverse variance-covariance matrix
s: number sensors available for one type of measurement
var (Y_i): reconciled variance of reconciled variable i
y: vector of measured variables
Y: vector of reconciled variables
$\gamma_{i,j}$: integer decision variable $\in \{0,1\}$
λ: Lagrange multipliers
ρ: weighting factor
σ_i: standard deviation of measured variable i
σ_{exist_i}: precision of sensor(s) already installed for measuring variable i
σ_{sensor_j}: precision of the additional sensor j

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