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**PAPER**

## Performing mathematical operations using high-index acoustic metamaterials

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**E-mail:** [romain.fleury@epfl.ch](mailto:romain.fleury@epfl.ch)**Keywords:** analog computing, acoustics, metamaterials, edge detection, slab waveguides

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**Abstract**

The recent breakthrough in metamaterial-based optical computing devices (2014 *Science* **343** 160) has inspired a quest for similar systems in acoustics, performing mathematical operations on sound waves. So far, acoustic analog computing has been demonstrated using thin planar metamaterials, carrying out the operator of choice in Fourier domain. These so-called filtering metasurfaces, however, are always accompanied with additional Fourier transform sub-blocks, enlarging the computing system and preventing its applicability in miniaturized architectures. Here, employing a simple high-index acoustic slab waveguide, we propose a highly compact and potentially integrable acoustic computing system and demonstrate its proper functioning by numerical simulations. The system directly performs mathematical operation in spatial domain and is therefore free of any Fourier bulk lens. Such compact computing system is highly promising for various applications including high throughput image processing, ultrafast equation solving, and real time signal processing.

### 1. Introduction

The idea of analog computation dates back to the early 19th century, when a number of mechanical and electronic computing machines intended for simple mathematical operations such as differentiation or integration were developed [1, 2]. Such analog computing devices were then totally overshadowed by the emergence of their digital counterparts in the second half of 20th century, as they could not compete with the speed and reliability of digital data processors [3]. The recent breakthrough in the seemingly unrelated field of metamaterials [4–20], however, is bringing them back into the competition as an important alternative computational approach at the hardware level, with highly promising applications in ultrafast equation solving [21, 22], real-time and continuous signal processing [23], and imaging [24].

In their inspiring proposal, Silva *et al* [25] proposed the first realization of computational metamaterials, performing various mathematical operations including differentiation, integration and convolution on electromagnetic waves. Such wave-based computational scheme then formed the basis for chains of ultrafast and highly efficient optical computing devices [26–47], all allowing one to go beyond the limitations of conventional analog computers.

In a general classification, all of the proposed metamaterial-based computing devices can be divided into two different categories according to the approach that is taken for the computation. In the first approach, the operator of choice is carried out by a thin planar metamaterial in Fourier domain. This approach, usually known as metasurface (MS) approach in the literature [25, 26], involves the use of two additional graded index (GRIN) bulk lenses applying Fourier transform (FT) on the input and output signals of the filtering MS. The inevitable use of GRIN lenses drastically increases the overall size of these types of computing systems, in turn hindering their applicability in compact architectures. The second approach, usually regarded as Green's function (GF) method, however, overcomes this limitation by not doing the computation in Fourier domain. In fact, as its name suggests, the approach relies on engineering the metamaterial block itself so that its GF follows that of the operator of choice. Although the available bandwidth when using this approach is often substantially reduced,

the resulting computing device is highly compact and potentially integrable as it avoids the usage of FT sub-blocks [25, 31].

Motivated by the renewed interest in optical analog computing, there has been a quest for similar acoustic systems carrying out mathematical operations on sound waves. As a matter of fact, some acoustic computational devices have already been demonstrated utilizing transmissive [48] or reflective MSs [49], offering platforms for efficient acoustic wave manipulation [50, 51]. The underlying computing principle in such systems, however, relies on FT method. Consequently, the filtering MS is always accompanied with additional focusing MSs taking the role of FT blocks, a fact which enlarges the resulting acoustic device as mentioned and, in turn, prevents its practical and flexible usage in miniaturized systems.

Here, we propose and demonstrate an acoustic computing system whose underlying working principle is based on GF method, meaning that the metamaterial block carries out the computation directly in spatial domain, without any need for additional Fourier lenses. We show how the proposed system is capable of performing different mathematical operations such as first or second order differentiation, and integration. We further demonstrate the possible usage of our computing device for edge detection of an image. Compared with the reported MS-based devices having overall lengths of about  $L = 10\lambda$  or more, our computing acoustic system is comparable in its length with the wavelength of operation, allowing its practical and flexible usage for the synthesis of highly efficient and compact architectures. Altogether, our results constitute a framework for various innovative acoustic applications comprising medical imaging, ultrafast equation solving, and real time signal processing.

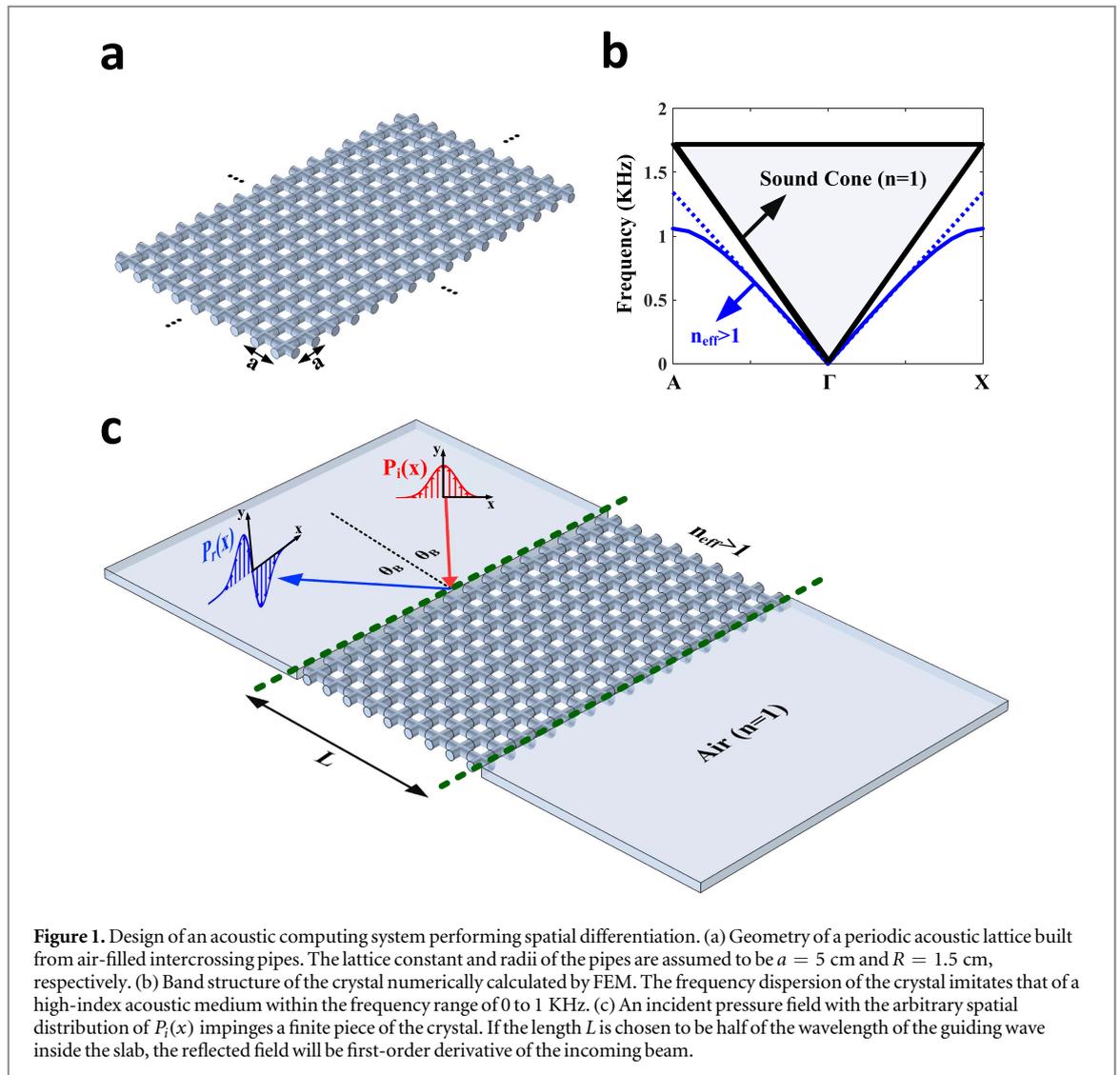
## 2. Methods

To start, consider the geometry of figure 1(a), depicting a metamaterial built from intercrossing air-filled acoustic pipes with the radius  $R = 1.5$  cm, arranged in a square lattice with the lattice constant  $a = 5$  cm. Such kind of metamaterial coils up the space and provides an internal geometrical detour for sound, reducing its effective travelling velocity [52–56]. Consequently, from a macroscopic point of view, the structure effectively acts as a high-index acoustic medium, a result which was already explored in [56]. For further assertion, however, we have calculated the band structure of the crystal using finite-element simulations and represented it in figure 1(b). Inspecting the obtained band structure reveals that the metamaterial under investigation indeed imitates a medium with the refractive index of  $n_{\text{eff}} > 1$ , as its dispersion curve falls below the sound cone and varies linearly within the frequency range of 0 and 1 KHz. Now, consider a finite piece of the crystal with length  $L$  placed in air (figure 1(c)). In this scenario, the high-index medium acts as an acoustic slab waveguide, guiding sound by total internal refraction as described in [56]. We first aim to perform differentiation, the most fundamental mathematical operation in science and engineering, making use of such simple slab. Assume an incident pressure field with the spatial distribution of  $P_i(x)$  is obliquely impinging on the slab as indicated in figure 1(c). Our goal is to engineer the slab in a way that it carries out differentiation in spatial domain, and provides the derivative of the incoming beam either by reflection or transmission. Considering the transfer function of an ideal differentiator ( $G(k_x) = ik_x$ ), one can argue that, in order to be able to differentiate the input field, the reflection or transmission coefficient of the slab is required to vanish at (at least) one incident angle. Similar to its optical counterpart, the transmission coefficient of this type of acoustic waveguide is non-zero for all incident angles. Nonetheless, its reflectance can, in principle, reduce to zero under certain circumstances. In fact, as outlined in [47], if the length  $L$  satisfies the following condition at some incident angle  $\theta_B$ , the reflection coefficient will attain a zero value:

$$k_0 n_{\text{eff}} \cos \left( \sin^{-1} \left( \frac{\sin(\theta_B)}{n_{\text{eff}}} \right) \right) L = v\pi, \quad (1)$$

where  $k_0$  is the wavenumber at the operation frequency and  $v$  is an integer. This is nothing but the usual Fabry–Pérot resonance condition, also known as longitudinal transmission resonance in acoustics. What actually happens at this incident angle is that the high-index slab mimics a half-wavelength transmission line, transferring all of the power by matching the characteristic impedances loaded to the input and output of the line. In the following section, we will describe how such a zero in the reflection coefficient of the waveguide can be leveraged for the realization of a spatial differentiation.

Intuitively, one may ask whether such simple waveguide is also able to perform other operators like integration, for example. To answer this question, we should refer to the GF of an ideal integrator in the Fourier domain, which is of the form  $G(k_x) = \frac{1}{ik_x}$ . Depending on whether we choose the transmitted or reflected field as the output, such form of transfer function necessitates the transmission or reflection coefficient of the slab to have a pole at one incident angle. In essence, the reflection coefficient of a slab waveguide cannot have a pole, neither in electromagnetism, nor here in acoustics. Nevertheless, its transmission coefficient can possess poles at



certain angles. More specifically, at the incident angles for which the wave-vector and frequency of the incoming beam match those of one of the guided modes of the waveguide, the incident wave strongly excites that mode, creating a pole in the transmission coefficient. This pole in the transmission coefficient of the waveguide can indeed be utilized to carry out spatial integration as we demonstrate in the results section.

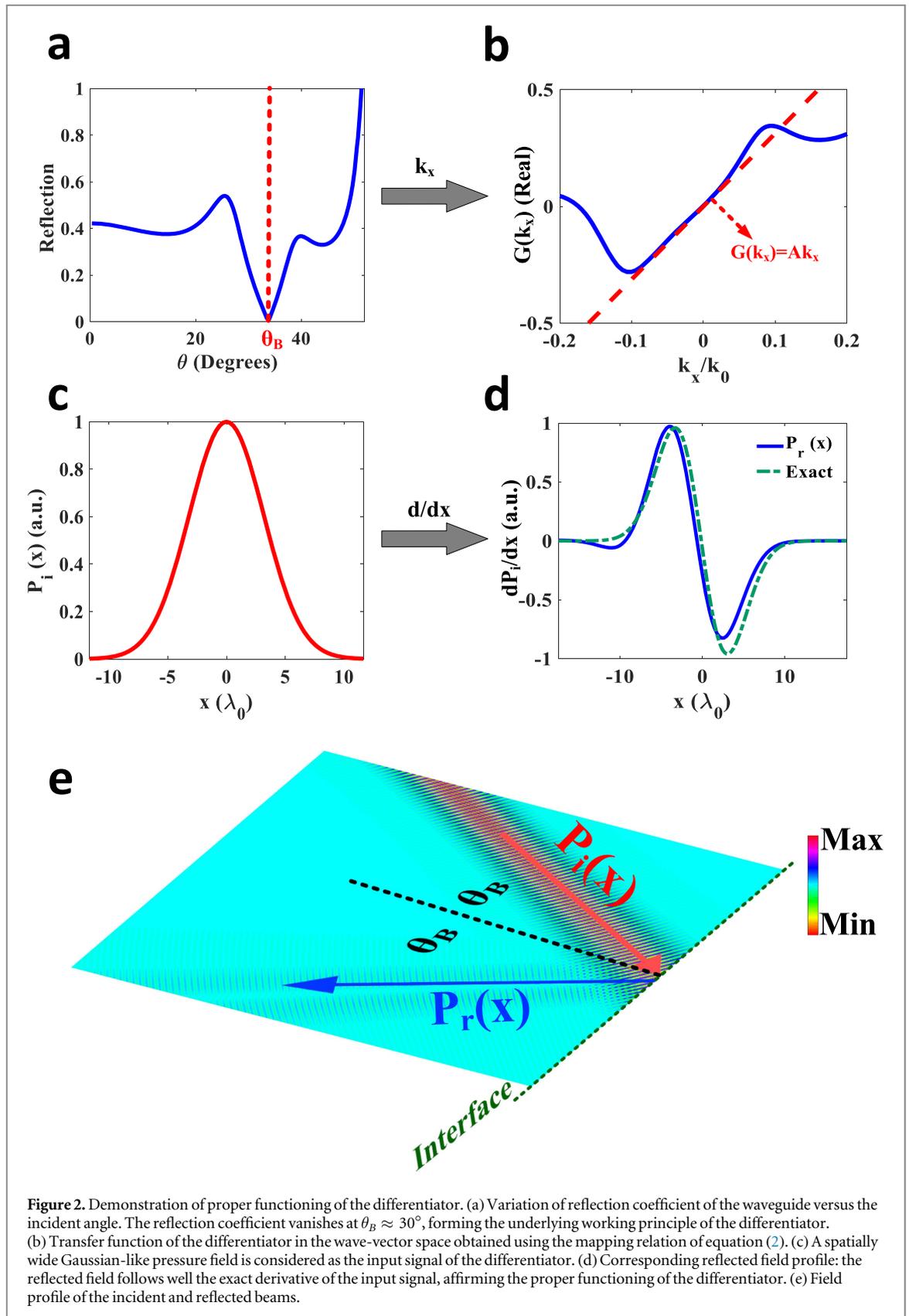
### 3. Results

#### 3.1. Spatial differentiation

Consider again the slab waveguide shown in figure 1(c), and suppose its length is chosen to be  $L = 9a$ . We perform finite element-based simulation using Comsol Multiphysics software to characterize the macroscopic behavior of the resulting metamaterial. To this end, we apply sound hard wall boundary condition to the edges of acoustic pipes by excluding their surroundings from the model. We then finely mesh the structure, excite it with a plane-wave type sound at the frequency of interest, and calculate the reflection coefficient making use of the resulting standing wave pattern. Figure 2(a) reports the corresponding reflection coefficient. As obvious, the reflectance has reduced to zero at  $\theta_B \approx 30^\circ$  for which the condition of equation (1) is fulfilled (notice that this angle can be easily adjusted by changing either the length of the slab or the frequency of operation). We now focus our attention on how the obtained reflection coefficient, calculated for various incident angles, can be mapped onto a transfer function in spatial Fourier domain. As described in [31], one can write the relation between the wave-vector  $k_x$  and incident angle  $\theta$  as

$$k_x = k_0 \sin(\sin^{-1}(|\sin(\theta)|) - \theta_B). \quad (2)$$

Employing the above equation, one may conveniently map the reflection coefficient onto its corresponding transfer function in wave-vector space. Shown in figure 2(b) is the real part of the transfer function. It is now



obvious that the transfer function of the slab is indeed zero at  $k_x = 0$ , which is key for the realization of spatial differentiation. We note that while the transfer function  $G(k_x)$  does not exactly match that of an ideal differentiator, it still can be well-estimated with a linear function near the origin as shown in the figure (the red dashed line). As a result, for the incoming fields  $P_i(x)$  having sufficiently wide spatial distribution (or equivalently having a small bandwidth in Fourier domain), the reflected field  $P_r(x)$  will be very close to the first-order derivative of  $P_i(x)$ . This anticipation can be easily confirmed by considering a spatially wide incident beam

having, for example, the Gaussian distribution of in figure 2(c), and calculating its corresponding reflected field (figure 2(d)). As observed, the reflected field is in perfect agreement with the exact derivative of the incoming field. The profiles of the incident and reflected wave fronts are further analytically calculated and sketched in the inset of figure 2(e), acknowledging again the proper performance of the differentiator. It should be noted that the amplitude of the reflected field is much lower than that of the incident field. This is actually normal and expected since the transfer functions of differentiators, even in their ideal case, have very low values near the origin.

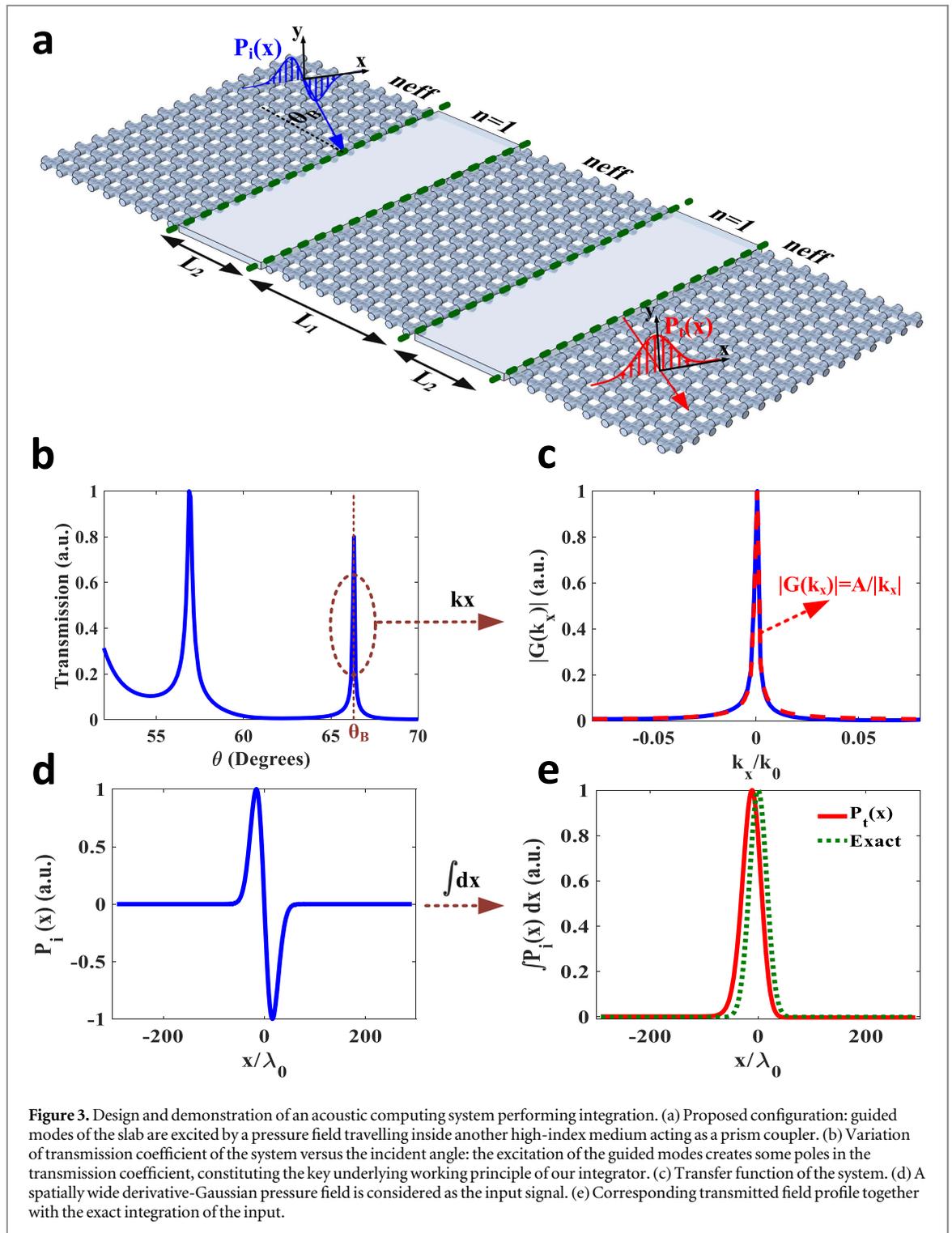
At this point, it is highly instructive to underline that while the proposed differentiator offers less bandwidth than its MS-based analogues, it is highly compact, a much-sought property allowing its practical usage in integrated architectures. To be specific, the overall length of the slab ( $L = \lambda/2$ ) is about one twentieths of that of MS-based differentiators in [48, 49]. Furthermore, while the MS-based structures are always accompanied with fabrication complexities due to the coupling responses between MSs, our high-index slab waveguide is quite easy to be implemented in practice [56].

### 3.2. Spatial integration

We next move onto designing an acoustic analog integrator. To this end, all we need is to excite one of the guided modes of the waveguide as explained before in the methods section. Unfortunately, however, there exists one problem towards achieving such functionality: in principle, it is not possible to excite the guided modes of the slab from air. This is a direct consequence of the phase matching criteria [57], and the fact that the effective refractive index of the slab is higher than its surroundings. In order to get rid of this problem, we use a well-known technique regarded as prism coupling [57, 58]. Consider the geometry represented in figure 3(a), where two additional high-index slabs, acting as the prism couplers, have been put before and after the primary waveguide. Suppose then an obliquely incident wave is travelling inside one of the prisms as schematically shown in the inset of the figure. It is worth noting that the fact that the sound trajectory does not follow the directions of the pipes should not cause any concern as our metamaterial derives its properties from its structure rather than its subwavelength composites. We assume  $L_1 = 4a$ , and  $L_2 = 9a$ , and calculate the transmission coefficient of the configuration via finite-element numerical simulations (figure 3(b)). The figure reveals the existence of two guided modes whose excitations at the incident angles of  $\theta_1 \approx 57^\circ$  and  $\theta_2 \approx 68^\circ$  has led to the peaks observed in the transmission. Here, without loss of generality, we focus on the second pole and assume the incident angle of the incoming beam to be  $\theta_B = \theta_2$ . Similar to the case of the acoustic differentiator, the obtained transmission coefficient can be readily converted to a transfer function in the wave-vector space employing the mapping of equation (2). This is actually done in figure 3(c), indicating the fact that the transfer function of the system can be well-estimated with that of an ideal integrator. We notice, however, that the transfer function is limited to one at the origin, rather than being infinite as it should be for an ideal integrator. This detrimental property, which stems from the leakage of the excited mode to the prisms, prevents the integrator from retrieving the zero harmonic (or the DC component) of the input field. However, our design still allows the efficient integration of spatially wide AC signals. For instance, suppose a pressure field having the Gaussian-derivative distribution depicted in figure 3(d) as the input of the system. Remarkably, not only is  $P_i(x)$  quite wide in the spatial domain but also it contains no DC component. The resulting transmitted field  $P_t(x)$  is plotted in figure 3(e). Comparing  $P_t(x)$  with the exact integration of  $P_i$  evidences the great functionality of the designed integrator.

### 3.3. Higher order operators

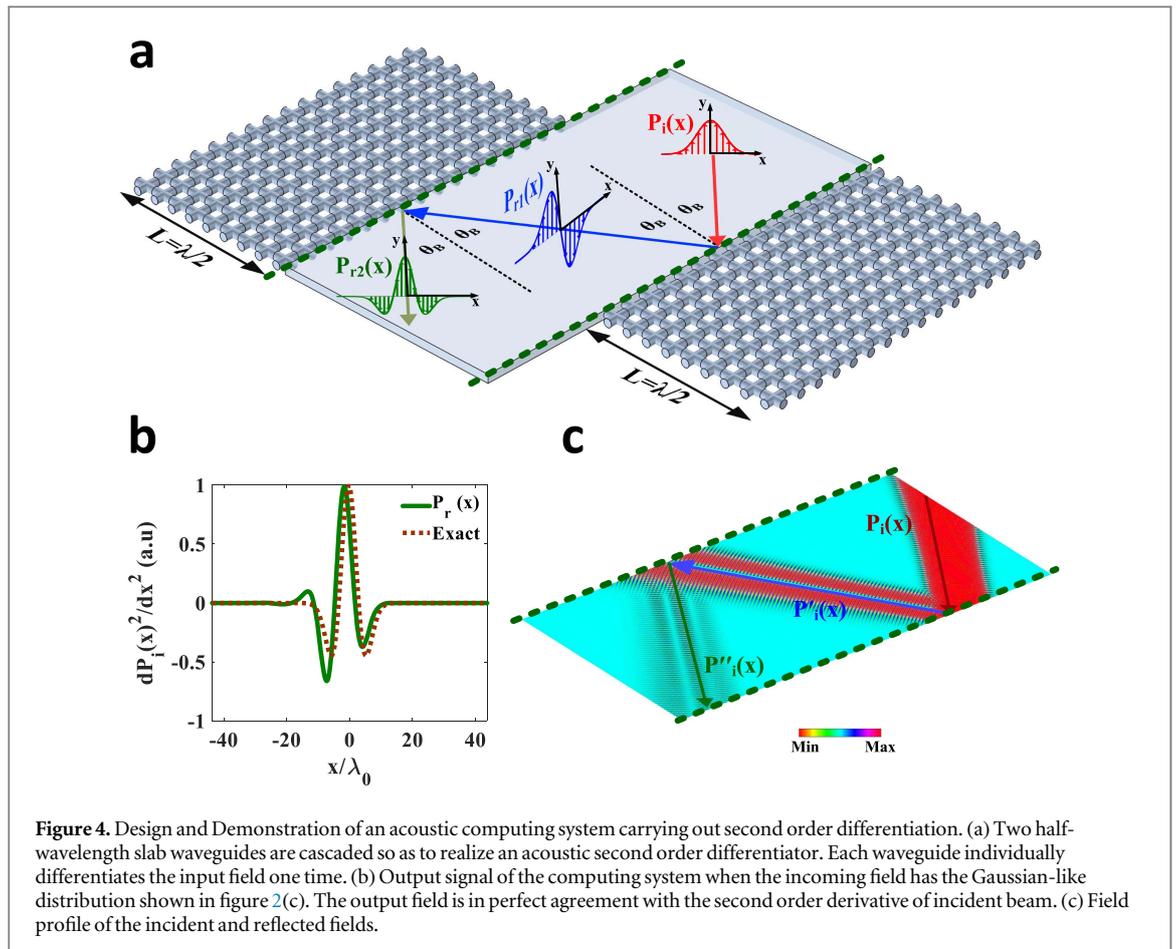
Now that we have successfully performed differentiation and integration, we study the possibility of performing more complex operators. Consider, for example, realization of second order differentiation, an operator which is diversely found in important partial differential equations like wave equation, Schrodinger equation and Euler equation. Our approach to realize this operator is quite simple: we cascade two half-wavelength acoustic slab waveguides, each differentiates individually the incoming pressure field one time. The cascading process has been conceptually represented in figure 4(a): an incident pressure field with the spatial distribution  $P_i$  coming in from air strikes the boundary of a half-wavelength slab waveguide. Then, the resulting reflected field  $P_{r1}(x)$ , which is in fact the spatial derivate of  $P_i$ , impinges another similar slab waveguide. The final pressure field  $P_{r2}(x)$ , reflecting back from the second slab will therefore be the second order derivative of  $P_i$ , provided that the input signal is sufficiently wide in spatial domain. To examine whether the designed system works well in performing second order differentiation, let us assume again the input field has the Gaussian distribution of figure 2(c). The corresponding output field  $P_{r2}(x)$  together with the exact second order derivative of the input field are reported in figure 4(b), confirming the proper functioning of the system. Further insight into the computation process can be obtained by exploring the calculated field profile illustrated in the inset of figure 4(c). It should be underlined that, when cascading differentiators to achieve a higher order one, the amplitude of the resulting output field drastically reduces such that it may happen to sink in noise. However, it is still the most straightforward and simplest fully passive approach to realize higher order operators.



Furthermore, by averaging the output signals, one can avoid the noise distortion even though this in turn reduces the speed of computation.

### 3.4. Edge detection

Finally, we examine the relevance of such acoustic computing devices for detecting the edges of an image. One common and established technique for edge detection, known as zero crossing technique [59], relies on calculating spatial derivative of the image in the direction(s) whose edges are intended to be detected. This can easily be carried out exploiting the acoustic computing setup schematically pictured in figure 5(a). The setup consists of a loudspeaker, a mask plane with locally engineered transparencies according to the shape of the desire image (shown in figure 5(b)), a half-wavelength acoustic slab waveguide, and a microphone. The underlying working principle of the setup is as follows. The sound generated by the loudspeaker is spatially



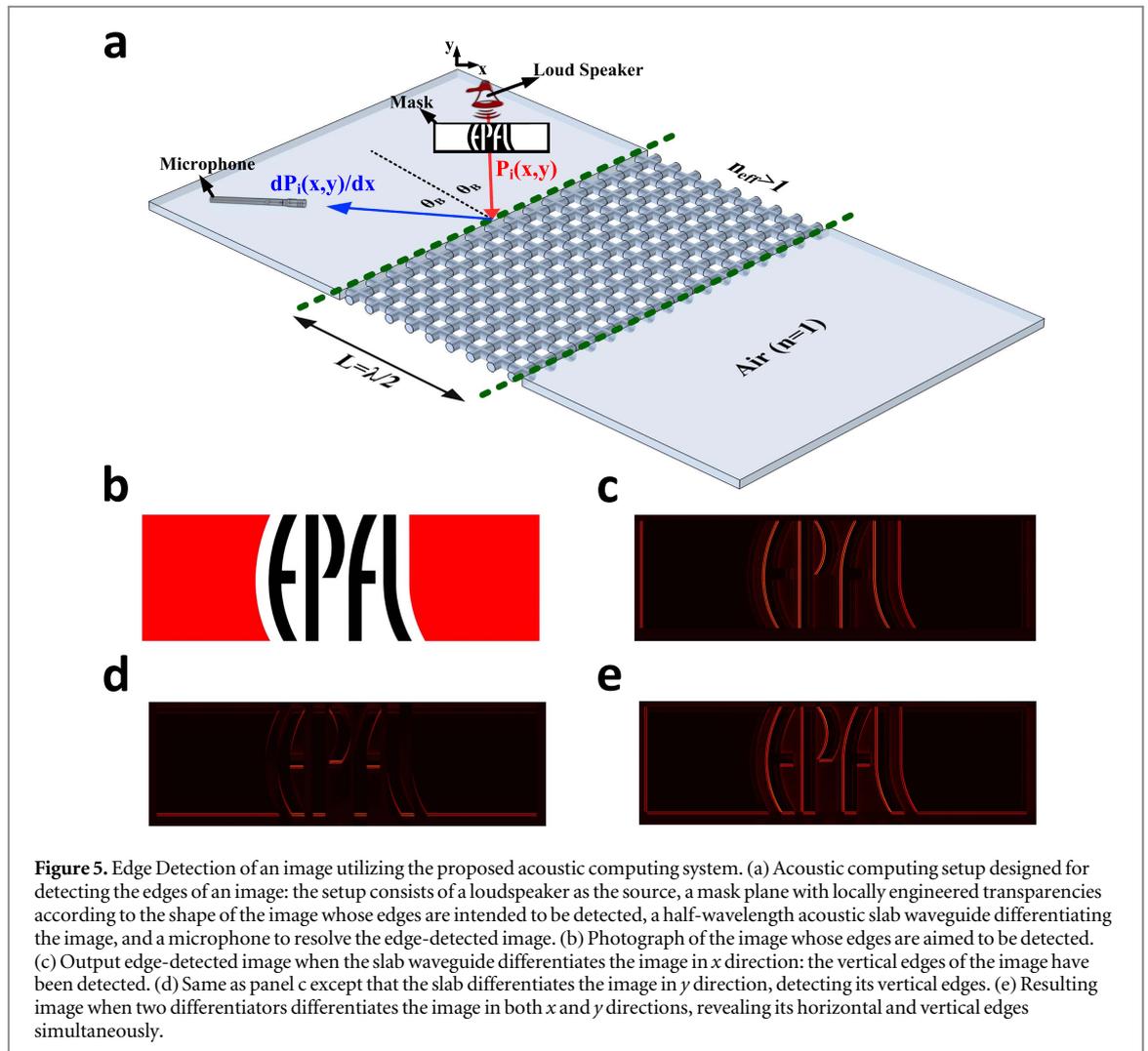
**Figure 4.** Design and Demonstration of an acoustic computing system carrying out second order differentiation. (a) Two half-wavelength slab waveguides are cascaded so as to realize an acoustic second order differentiator. Each waveguide individually differentiates the input field one time. (b) Output signal of the computing system when the incoming field has the Gaussian-like distribution shown in figure 2(c). The output field is in perfect agreement with the second order derivative of incident beam. (c) Field profile of the incident and reflected fields.

modulated by the mask plane, creating the appropriate spatial field distribution corresponding to the image. The resulting pressure field then impinges on the high-index acoustic slab and is spatially differentiated when it reflects. The reflected field is then detected by the microphone so as to create the edge-detected image. Figure 5(c) reports the output edge-detected image. Notably, since differentiation is carried out along  $x$  direction, only the vertical edges have been resolved. The horizontal edges can also be detected employing a similar setup, but with a slab waveguide carrying out differentiation along  $y$ . The resulting edge-detected image for the latter case is provided in figure 5(d). One may also consider two-dimensional edge detection by cascading two different differentiators; one differentiates the image along  $x$  whereas the other does that along  $y$ , and adding their resulting output fields. This way, both vertical and horizontal edges of the images can be appropriately detected in one measurement, as observed in the result of figure 5(e).

As a final remark, we note that in all of our simulations throughout the manuscript, we neglected the viscosity of air and assumed it to be lossless. This, however, should not cause any concern as the sound attenuation in this range of frequency is very low. To demonstrate this point, we repeated all simulations when air is accompanied with the realistic loss of  $5 \text{ dB km}^{-1}$  corresponding to the frequency of operation. We observed that this slight amount of attenuation affects the performance of none of the proposed computational devices.

#### 4. Discussion and conclusion

In summary, in this article, we proposed to achieve acoustic analog computing making use of high-index slab waveguides. These proposed devices are highly compact and potentially usable in miniaturized (subwavelength) signal processing systems. In particular, our computing devices are superior to their MS-based alternatives in that their overall lengths were comparable with the operational wavelength. This is because they directly performed the desired operation in spatial domain, and were needless of bulky Fourier lenses. We demonstrated how a simple, easy-to-fabricate acoustic slab enables realization of spatial differentiators and integrators. We further proved the relevance of such computing devices for detecting the edges of an image. Notice that the edge detection resolution level provided by our wave-based computing systems is inherently restricted by the diffraction limit [60–64], while it is not the case when using standard digital image processors. However, our



**Figure 5.** Edge Detection of an image utilizing the proposed acoustic computing system. (a) Acoustic computing setup designed for detecting the edges of an image: the setup consists of a loudspeaker as the source, a mask plane with locally engineered transparencies according to the shape of the image whose edges are intended to be detected, a half-wavelength acoustic slab waveguide differentiating the image, and a microphone to resolve the edge-detected image. (b) Photograph of the image whose edges are aimed to be detected. (c) Output edge-detected image when the slab waveguide differentiates the image in  $x$  direction: the vertical edges of the image have been detected. (d) Same as panel c except that the slab differentiates the image in  $y$  direction, detecting its vertical edges. (e) Resulting image when two differentiators differentiate the image in both  $x$  and  $y$  directions, revealing its horizontal and vertical edges simultaneously.

scheme allows one to process the entire image at the speed of sound, which is not achievable with digital data processors. This property can dramatically enhance the throughput of our analog image processor, eventually reducing the cost and energy required for the edge detection process. Moreover, such wave-based computing systems offer the opportunity to perform edge detection parallelly. One may also think of improving the design by considering metamaterial slabs supporting spoof acoustic plasmons, which would lead to devices not limited by diffraction and capable of subwavelength edge detection.

These computing devices have the potential to be employed as the building block for fast equation solving as well. For example, consider a very simple first-order differential equation as  $\frac{d}{dx}g(x) = f(x)$ . To find the unknown function  $g(x)$ , one should only employ the acoustic setup of figure 3(a), modulate the incident pressure field in accordance with the known function  $f(x)$ , and detect the transmitted field. More complex differential or integro-differential equations can also be solved exploiting the cascading method proposed in section 3.3. Such analog equation solvers have the advantageous property of doing calculation in real time.

The proposed computing systems can also be envisioned to be utilized in other promising applications including real-time signal processing, wave manipulation and optimization of acoustic neural networks. One may also do a further research on integrating these computing components into a union architecture, and making it reprogrammable utilizing analog acoustic switches.

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