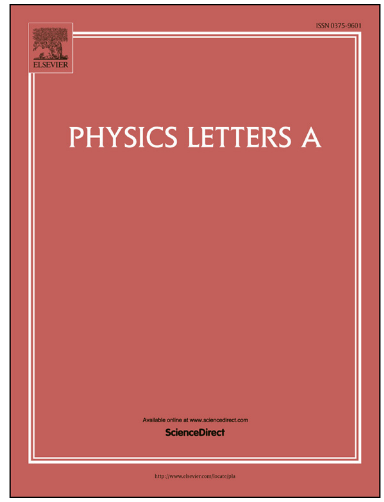

Spin Caloritronics, origin and outlook

Haiming Yu, Sylvain D. Brechet, Jean-Philippe Ansermet

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Highlights

- Thermodynamic description of transport: three-current model.
- Magneto-thermoelectric power and spin-dependent Peltier effects.
- Thermal spin-transfer torque.
- Anormalous Nernst effects and other Nernst-related effects.
- Spin Seebeck effect and magnetic proximity effect.

Spin Caloritronics, origin and outlook

Haiming Yu

Fert Beijing Institute, School of Electronic and Information Engineering, BDBC, Beihang University, China

Sylvain D. Brechet

Institute of Physics, station 3, Ecole Polytechnique Fédérale de Lausanne, 1015 Lausanne-EPFL, Switzerland

Jean-Philippe Ansermet

Institute of Physics, station 3, Ecole Polytechnique Fédérale de Lausanne, 1015 Lausanne-EPFL, Switzerland

Abstract

Spin caloritronics refers to research efforts in spintronics when a heat current plays a role. In this review, we start out by reviewing the predictions that can be drawn from the thermodynamics of irreversible processes. This serves as a conceptual framework in which to analyze the interplay of charge, spin and heat transport. This formalism predicts tensorial relations between vectorial quantities such as currents and gradients of chemical potentials or of temperature. Transverse effects such as the Nernst or Hall effects are predicted on the basis that these tensors can include an anti-symmetric contribution, which can be written with a vectorial cross-product. The local symmetry of the system may determine the direction of the vector defining

Email addresses: haiming.yu@buaa.edu.cn (Haiming Yu), jean-philippe.ansermet@epfl.ch (Jean-Philippe Ansermet)

such transverse effects, such as the surface of an isotropic medium. By including magnetization as state field in the thermodynamic description, spin currents appear naturally from the continuity equation for the magnetization, and dissipative spin torques are derived, which are charge-driven or heat-driven. Thermodynamics does not give the strength of these effects, but may provide relationships between them. Based on this framework, the review proceeds by showing how these effects have been observed in various systems. Spintronics has become a vast field of research, and the experiments highlighted in this review pertain only to heat effects on transport and magnetization dynamics, such as magneto-thermoelectric power, or the spin-dependence of the Seebeck effect, the spin-dependence of the Peltier effect, the spin Seebeck effect, the magnetic Seebeck effect, or the Nernst effect. The review concludes by pointing out predicted effects that are yet to be verified experimentally, and in what novel materials the standard thermal spin effects could be investigated.

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1. Introduction

The term "spin caloritronics" was coined to refer to all transport phenomena that involve spin and heat.1 2 This sub-field of spintronics has drawn considerable attention since 2009, when an international workshop launched this emerging field. Specialists have met annually ever since, as this theme of research attracts more and more researcher. 2016 has seen the seventh "Spin Caloritronics" workshop.3 The German Physical Society (DFG) launched a priority program to foster research in this field. 4 Recently, the US Department of Energy funded a UC-Riverside-based research cluster "SHINES"

(Spins and Heat in Nanoscale Electronic Systems) consisting 14 research groups from 7 institutions from coast to coast. Thus, spin caloritronics as an interdisciplinary field of magnetism, thermoelectrics and microelectronics will continuously attract global interests of cutting-edge scientific research.5

This review is structured as follows. The definition of some of the basic concepts of spin caloritronics are presented within a thermodynamic framework. Then, experiments on spin-dependent transport phenomena are reported, provided that the effects under study were driven by a heat current. Others have also attempted to review this field, which is bursting in all kinds of new directions.6 This paper does not seek to analyze the theoretical efforts carried out to account quantitatively for spin caloritronics phenomena.

A few examples can be mentioned to highlight the diversity of approaches. The magneto-thermoelectric power (MTEP) of magnetic tunnel junctions (MTJ) has been modeled using the Butticker-Landauer formalism.7 The key material properties of the magnetic electrodes has been identified by ab initio calculations.8 The details of spin transport across tunnel barriers have also been calculated. 9 10 Dissipative torques may affect magnetization dynamics, in conductors as well as in metals. These are often known as non-adiabatic torques or entropic torques. This type of torque, associated with the Dzyaloshinskii-Moriya interaction, was recently identified and its role in the dynamics of domain walls was analyzed.11 The method of master equations was used to describe a single quantum dot with spin-dependent electron temperatures due to its connection to ferromagnetic leads.12 The Kubo formalism of linear response theory has been applied to describe thermal spin orbit torques and the reciprocal effect, the heat current associated with magnetization dynamics.13 14 The existence of thermal spin orbit torques of electronic origin has been predicted using a Berry phase description.15 Magnonic contributions to the spin orbit torque have also been identified, 16 17 including thermally-driven torques.18 The effect of thermally-driven torque on domain wall motion has been simulated by using atomistic spin dynamics,19 or by considering the drift caused by thermal fluctuations in a temperature gradient.20 The phenomenon that has boosted the interest for spin caloritronics is the spin Seebeck effect. It has been accounted for by considering the dynamics of magnons in a temperature gradient.2122 The Boltzmann transport theory was also used to describe this effect.2324 A key consideration to understand this and other related effects is the non-equilibrium condition that may arise near an interface. 25 In particular, magnons and phonons may have different temperatures.26 The accumulation of magnons at an interface, due to presence of a temperature gradient, has been described in terms of a Bose-Einstein condensation.2728 A review on the theory of spin Seebeck effect can be found in 29.

1.1. Thermodynamic description of transport phenomena, definitions

In this section, we first recall that the thermodynamics of irreversible processes provides a framework in which to define Ohm's law 30 and the Hall effect,31 and in a similar fashion, the Seebeck and the Nernst effect. The transverse effects (Hall and Nernst) are expected merely on the basis of a symmetry argument. One should keep in mind that thermodynamics is a powerful way of establishing relations among various physical quantities, but thermodynamics does not provide quantitative estimates for any of the transport coefficients, as it does not address the underlying micromechanisms responsible for the transport phenomena that it predicts.

In thermodynamics, diffusive current densities are defined for each density of extensive variables. For example, entropy is an extensive variable and the current density $\boldsymbol{j}_s$ is defined. A lengthy development, which takes into account Newton's law and the first principle of thermodynamics, shows that the density of entropy source ρ_s is given by an expression of the generic form:32 3334

$$\rho_s = \frac{1}{T}\sum_i \boldsymbol{j}_i \cdot \boldsymbol{F}_i \tag{1}$$

The generalized forces $\boldsymbol{F}_i$ are the gradients of the intensive variables conjugated to the extensive state variables numbered by the index $i = 1..n$. Here we consider $(s, \{n_A\})$, the state variable densities for entropy and quantities of substance A, B ..., respectively. For a substance A, the associated generalized force is the gradient of its electrochemical potential, $\bar{\mu}_A = \mu_A + q_A V$, where V is the electrostatic potential and q_A the elementary charge of substance A. Thus, in the following, $\boldsymbol{F}_A = \boldsymbol{\nabla}\mu_A + q_A \boldsymbol{\nabla} V$. The generalized force associated with entropy is $-\boldsymbol{\nabla} T$.

The second principle of thermodynamics imposes that $\rho_s \geq 0$. This implies the following constitutive equations :

$$\begin{cases} \boldsymbol{j}_s = \mathsf{L}_{ss} \cdot (-\boldsymbol{\nabla} T) + \sum_B \mathsf{L}_{sB} \cdot \boldsymbol{F}_B \\ \boldsymbol{j}_A = \mathsf{L}_{As} \cdot (-\boldsymbol{\nabla} T) + \sum_B \mathsf{L}_{AB} \cdot \boldsymbol{F}_B \end{cases} \tag{2}$$

The Onsager matrix that underlies (2) must be positive definite in order to satisfy the condition $\rho_s \geq 0$. In all generality, the Onsager matrix elements such as L_{ss}, L_{sA} and L_{AA} are tensors. The Onsager-Casimir relations 35 36 imposes the following symmetry on the coefficients of (2) :

$$\mathsf{L}_{\alpha\beta}\left(s, \{n_A\}, \boldsymbol{B}\right) = \varepsilon_\alpha\, \varepsilon_\beta\, \mathsf{L}_{\beta\alpha}\left(s, \{n_A\}, -\boldsymbol{B}\right) \tag{3}$$

where, according to a result derived from statistical physics, the parameters $\varepsilon_\alpha = \pm 1$, $\varepsilon_\beta = \pm 1$. The parameters ε_α and ε_β are positive if the corresponding generalized forces $\boldsymbol{F}_\alpha$ and $\boldsymbol{F}_\beta$ are invariant under time reversal, and they are negative in the opposite case. For the vectorial quantities considered here and in the following sections, the parameters ε_α and ε_β are positive because the corresponding generalized vectorial forces $\boldsymbol{F}_\alpha$ and $\boldsymbol{F}_\beta$ are invariant under time reversal.

We consider now the particular case of an isotropic medium composed of one substance A, in the absence of chemical effect ($\mu_A = 0$), at a homogeneous temperature T but with an electric field applied to the system. The result (2) implies the tensorial expression of Ohm's law is :

$$\boldsymbol{j}_A = \frac{-1}{q_A} \boldsymbol{\sigma} \cdot \boldsymbol{\nabla} V \tag{4}$$

In all generality, the tensor $\boldsymbol{\sigma}$ can be decomposed in a symmetric tensor and an antisymmetric tensor, $\boldsymbol{\sigma} = \boldsymbol{\sigma}^s + \boldsymbol{\sigma}^a$. For the symmetric part, we will simply consider the isotropic case $\boldsymbol{\sigma}^s = \sigma \mathbb{1}$. The antisymmetric part provides richer physical insight. We can write $\boldsymbol{\sigma}^a$ in the form of a vectorial product, $\boldsymbol{\sigma}^a \cdot \boldsymbol{x} = \sigma_\perp \hat{\boldsymbol{u}} \times \boldsymbol{x}$, for any vector $\boldsymbol{x}$.[37] The particular physical properties of the system determine the direction and modulus of the vector $\sigma_\perp \hat{\boldsymbol{u}}$. Thus, the decomposition of the conductivity tensor into symmetric and antisymmetric parts allows us to write :

$$\boldsymbol{j}_A = -\frac{1}{q_A} \sigma\, \boldsymbol{\nabla} V - \frac{\sigma_\perp}{q_A} \left(\hat{\boldsymbol{u}} \times \boldsymbol{\nabla} V\right) \tag{5}$$

In particular, if a magnetic induction field $\boldsymbol{B}$ is applied to the system, the Onsager-Casimir relations (3) imply that $\sigma_{ij}^{a}(\boldsymbol{B}) = \sigma_{ji}^{a}(-\boldsymbol{B}) = -\sigma_{ji}^{a}(\boldsymbol{B})$, where the last equality is the expression of the antisymmetry of the tensor. Therefore, σ^{a} is an antisymmetric function of $\boldsymbol{B}$. For an isotropic medium, this means that $\hat{\boldsymbol{u}}$ must be in the direction of $\boldsymbol{B}$. Hence, (4) includes the Hall effect :

$$\boldsymbol{j}_A = -\frac{1}{q_A}\sigma\,\boldsymbol{\nabla}V - \frac{\sigma_\perp}{q_A}\left(\hat{\boldsymbol{B}} \times \boldsymbol{\nabla}V\right) \tag{6}$$

Furthermore, if we are near the surface of the medium, the isotropy breaks down and we have an axial symmetry about the normal $\hat{\boldsymbol{n}}$ to the surface. It is easy to show that the only way to have (5) with the symmetry about $\hat{\boldsymbol{n}}$ is to write :

$$\boldsymbol{j}_A = -\frac{1}{q_A}\sigma\,\boldsymbol{\nabla}V - \frac{\sigma_\perp}{q_A}\left(\hat{\boldsymbol{n}} \times \boldsymbol{\nabla}V\right) \tag{7}$$

Now, we consider transport effects due to the presence of a temperature gradient and in the absence of an electric field ($\boldsymbol{\nabla}V = \boldsymbol{0}$). From (2), we deduce :

$$\boldsymbol{j}_A = \frac{-1}{q_A}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\nabla}T \tag{8}$$

The notation has been chosen to keep track of the Seebeck effect and Ohm's law. Hence, we define $\boldsymbol{\sigma} = q_A^2\mathsf{L}_{AA}$ and $\boldsymbol{\varepsilon} = (1/q_A)\mathsf{L}_{AA}^{-1}\cdot\mathsf{L}_{As}$. The tensor $\boldsymbol{\varepsilon}$ can be decomposed in symmetric and antisymmetric parts. Following what we did for the conductivity tensor, we write $\boldsymbol{\varepsilon}\cdot\boldsymbol{x} = \varepsilon\boldsymbol{x} + \varepsilon_\perp(\hat{\boldsymbol{u}} \times \boldsymbol{x})$. Then (8) becomes :

$$\boldsymbol{j}_A = -\frac{\sigma\varepsilon}{q_A}\boldsymbol{\nabla}T - \frac{\sigma\varepsilon_\perp + \sigma_\perp\varepsilon}{q_A}\left(\hat{\boldsymbol{u}} \times \boldsymbol{\nabla}T\right) \tag{9}$$

Just as discussed regarding the conductivity, the unit vector $\hat{\boldsymbol{u}}$ can designate either the direction of the magnetic induction field or the normal to the surface.

The Seebeck effect and the Nernst effect are observed when the experimental conditions are such that no current flows in the medium. From (2), the conditions $\boldsymbol{j}_A = \mathbf{0}$ and $\boldsymbol{\nabla}\mu_A = \mathbf{0}$ imply,

$$\boldsymbol{\nabla} V = -\boldsymbol{\varepsilon} \cdot \boldsymbol{\nabla} T = \varepsilon \boldsymbol{\nabla} T - \varepsilon_{\perp} \left(\hat{\boldsymbol{u}} \times \boldsymbol{\nabla} T \right) \tag{10}$$

where we have made use in the last equality of the decomposition of the tensor in its symmetrical and antisymmetrical part, the latter being expressed as a vectorial product. The first term thus obtained expresses the ***Seebeck effect***.38 39 40 The second term corresponds to the ***Nernst effect***.41

1.2. The three-current model

The first use of thermodynamics to describe spin-dependent transport was the work of Johnson and Silsbee, who not only layed out a thermodynamic model, but also demonstrated spin-dependent transport experimentally, using two ferromagnetic contacts on an aluminum thin film.42

We apply the formalism of section 1.1 to the following model for conduction in ferromagnetic metals, first introduced by Mott, 43 and extensively used in spintronics.44 45 This model is based on the notion that the current is due to s-electrons and that the most likely collision is into d-states. In a ferromagnet, the d-states are split, causing the collision rates to be different for majority and for minority s-electrons. Hence, we are going to assume that our thermodynamic system has two types of charge carriers, the majority spins and the minority spins. We consider the current densities associated with these two types of charge carrier and we consider also the density of entropy current. Thus, we have three currents. Applying equations (2), we write :

$$\begin{pmatrix} \boldsymbol{j}_s \\ \boldsymbol{j}_\uparrow \\ \boldsymbol{j}_\downarrow \end{pmatrix} = \begin{pmatrix} \mathsf{L}_{ss} & \mathsf{L}_{s\uparrow} & \mathsf{L}_{s\downarrow} \\ \mathsf{L}_{\uparrow s} & \mathsf{L}_{\uparrow\uparrow} & \mathsf{L}_{\uparrow\downarrow} \\ \mathsf{L}_{\downarrow s} & \mathsf{L}_{\downarrow\uparrow} & \mathsf{L}_{\downarrow\downarrow} \end{pmatrix} \begin{pmatrix} -\boldsymbol{\nabla} T \\ -\boldsymbol{\nabla}\bar{\mu}_\uparrow \\ -\boldsymbol{\nabla}\bar{\mu}_\downarrow \end{pmatrix} \tag{11}$$

We recall that in our notation, $\bar{\mu}$ is an electrochemical potential and the substance currents $\boldsymbol{j}_{\uparrow(\downarrow)}$ are defined as currents of quantity of matter per surface area per unit time (without the charge). This model has been used for example to analyze the diffusive spin current emanating from a hot ferromagnet.46 It should be noted that others have chosen to distinguish the heat currents of each spin channels.4748 We proceed now to an identification of the physical meaning of the matrices in equation (11).

We define transport matrices for Ohm and Hall effects as well as for Seebeck-Nernst effects, as if we could control majority and minority currents individually. That is, for $\boldsymbol{j}_\downarrow = \boldsymbol{0}$ and $\boldsymbol{\nabla}\mu_\downarrow = \boldsymbol{0}$, we have :

$$\begin{pmatrix} \boldsymbol{j}_s \\ \boldsymbol{j}_\uparrow \end{pmatrix} = \begin{pmatrix} \mathsf{L}_{ss} & \mathsf{L}_{s\uparrow} \\ \mathsf{L}_{\uparrow s} & \mathsf{L}_{\uparrow\uparrow} \end{pmatrix} \begin{pmatrix} -\boldsymbol{\nabla} T \\ -\boldsymbol{\nabla}\bar{\mu}_\uparrow \end{pmatrix} \tag{12}$$

We could write similar expressions for the "down" spins. As Ohm's law and the Hall effect are defined for a system at homogeneous temperature and no chemical effect, we see that we have $\boldsymbol{j}_\uparrow = \boldsymbol{L}_{\uparrow\uparrow}(-q\,\boldsymbol{\nabla} V)$, and since the current of "up" spins would be defined as $q\,\boldsymbol{j}_\uparrow$, we can defined :

$$\boldsymbol{\sigma}_\uparrow = q^2\,\mathsf{L}_{\uparrow\uparrow} \qquad \boldsymbol{\sigma}_\downarrow = q^2\,\mathsf{L}_{\downarrow\downarrow} \tag{13}$$

Since we focus here on thermal effect, we only define these spin-dependent tensorial conductivities and don't give explicit expressions for the conductivity and the Hall conductance, but invite the reader to turn to the study of

Saslow, who has recently analyzed the ***spin Hall effect*** using a thermodynamic approach.49 It is customary to define a Seebeck and Nernst coefficients as if each spin channel could be controlled individually. Thus, starting from Eq. (12), the absence of current for this spin channel implies:

$$\boldsymbol{\varepsilon}_{\uparrow} = \frac{1}{q}\,\mathsf{L}_{\uparrow\uparrow}^{-1} \cdot \mathsf{L}_{s\uparrow} \tag{14}$$

The analog to Eq (12) for the down channel is

$$\boldsymbol{\varepsilon}_{\downarrow} = \frac{1}{q}\,\mathsf{L}_{\downarrow\downarrow}^{-1} \cdot \mathsf{L}_{s\downarrow} \tag{15}$$

The terms $\boldsymbol{L}_{\uparrow\downarrow}$ in (11) represent what is known in spin-dependent transport as spin mixing.50 This tensor determines the contribution to the current in one spin channel due to the generalized force of the other channel. It represents the effect of collisions that flip the spin of the electrons but do not change its momentum.51

Finally, we consider the Fourier law for $\boldsymbol{j}_Q = T\,\boldsymbol{j}_s$:

$$\boldsymbol{j}_Q = \boldsymbol{\kappa} \cdot (-\boldsymbol{\nabla} T) \tag{16}$$

Fourier law applies when there is no current flowing in the system, i.e. $\boldsymbol{j}_{\uparrow} = \boldsymbol{j}_{\downarrow} = \boldsymbol{0}$. Under these conditions, equation (11) yields a linear relationship between $\boldsymbol{j}_s$ and $\boldsymbol{\nabla} T$. The expressions simplify greatly when the contribution of spin mixing can be neglected. Using (13) and (14), equation (11) gives :

$$\mathsf{L}_{ss} = \frac{\boldsymbol{\kappa}}{T} + q\left(\boldsymbol{\sigma}_{\uparrow} \cdot \boldsymbol{\varepsilon}_{\uparrow}^{2} + \boldsymbol{\sigma}_{\downarrow} \cdot \boldsymbol{\varepsilon}_{\downarrow}^{2}\right) \tag{17}$$

In summary, equation (11) can be rewritten as :

$$\begin{pmatrix} \boldsymbol{j}_s \\ \boldsymbol{j}_\uparrow \\ \boldsymbol{j}_\downarrow \end{pmatrix} = \frac{1}{q^2} \begin{pmatrix} q^2 \mathsf{L}_{ss} & q\boldsymbol{\sigma}_\uparrow \cdot \boldsymbol{\varepsilon}_\uparrow & q\boldsymbol{\sigma}_\downarrow \cdot \boldsymbol{\varepsilon}_\downarrow \\ q\boldsymbol{\sigma}_\uparrow \cdot \boldsymbol{\varepsilon}_\uparrow & \boldsymbol{\sigma}_\uparrow & \boldsymbol{\sigma}_{\uparrow\downarrow} \\ q\boldsymbol{\sigma}_\downarrow \cdot \boldsymbol{\varepsilon}_\downarrow & \boldsymbol{\sigma}_{\uparrow\downarrow} & \boldsymbol{\sigma}_\downarrow \end{pmatrix} \begin{pmatrix} -\boldsymbol{\nabla} T \\ -\boldsymbol{\nabla} \bar{\mu}_\uparrow \\ -\boldsymbol{\nabla} \bar{\mu}_\downarrow \end{pmatrix} \tag{18}$$

These transport equations can be rewritten for the total current $\boldsymbol{j}$ and the ***spin current*** $\boldsymbol{j}_p$ (p stands for “polarization”, the index “s” is reserved for the entropy). This gives :

$$\begin{pmatrix} \boldsymbol{j}_s \\ \boldsymbol{j} \\ \boldsymbol{j}_p \end{pmatrix} = \frac{1}{q^2} \Sigma \begin{pmatrix} -\boldsymbol{\nabla} T \\ -q\boldsymbol{\nabla} V \\ -\boldsymbol{\nabla} \Delta\mu \end{pmatrix} \tag{19}$$

where the generalized conductivity Σ is defined as :

$$\Sigma = q \begin{pmatrix} q\,\mathsf{L}_{ss} & \boldsymbol{\sigma}_\uparrow \cdot \boldsymbol{\varepsilon}_\uparrow + \boldsymbol{\sigma}_\downarrow \cdot \boldsymbol{\varepsilon}_\downarrow & \boldsymbol{\sigma}_\uparrow \cdot \boldsymbol{\varepsilon}_\uparrow - \boldsymbol{\sigma}_\downarrow \cdot \boldsymbol{\varepsilon}_\downarrow \\ \boldsymbol{\sigma}_\uparrow \cdot \boldsymbol{\varepsilon}_\uparrow + \boldsymbol{\sigma}_\downarrow \cdot \boldsymbol{\varepsilon}_\downarrow & (\boldsymbol{\sigma}_\uparrow + \boldsymbol{\sigma}_\downarrow)/q & (\boldsymbol{\sigma}_\uparrow - \boldsymbol{\sigma}_\downarrow)/q \\ \boldsymbol{\sigma}_\uparrow \cdot \boldsymbol{\varepsilon}_\uparrow - \boldsymbol{\sigma}_\downarrow \cdot \boldsymbol{\varepsilon}_\downarrow & (\boldsymbol{\sigma}_\uparrow - \boldsymbol{\sigma}_\downarrow)/q & (\boldsymbol{\sigma}_\uparrow + \boldsymbol{\sigma}_\downarrow)/q \end{pmatrix} \tag{20}$$

We have used the conventional notation : $\Delta\mu = \mu_\uparrow - \mu_\downarrow$. We have assumed $\boldsymbol{\nabla}(\mu_\uparrow + \mu_\downarrow) = \mathbf{0}$ because we want to consider homogeneous systems. Hence, $\Delta\mu \neq 0$ only because of spin-dependent transport phenomena, but there is no chemical effect that would cause variation of the average chemical potential $(\mu_\uparrow + \mu_\downarrow)/2$. When $\Delta\mu \neq 0$, this means that the two spin populations are out of equilibrium. We expect this to arise near an interface through which current is flowing. At distances large compared to the spin diffusion length, we have $\Delta\mu = 0$. We note that is possible to derive an expression for the spin diffusion length within the formalism of irreversible processes.50 Experimental values have been obtained by studying transport perpendicular to the interfaces of multilayers. 5253

The physical implications of the three-current model expressed by (19) can be analyzed as follows. One can consider experimental conditions which are characterized by three of the set of six currents and generalized forces. Then the remaining three unknowns are determined by equations (19). The most important experimental condition in spin-caloritronics is the one where the temperature gradient is given and we consider what happens far away from an electrode or any another material in contact with the system under study. Then $\Delta\mu = 0$. Let us consider furthermore that no charge current is flowing, i.e. $\boldsymbol{j} = \mathbf{0}$. The second line of the matrix in (19) implies a Seebeck effect of the following form :

$$\boldsymbol{\nabla} V = -\left(\boldsymbol{\sigma}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow}\right)^{-1} \cdot \left(\boldsymbol{\sigma}_{\uparrow} \cdot \boldsymbol{\varepsilon}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow} \cdot \boldsymbol{\varepsilon}_{\downarrow}\right) \cdot \boldsymbol{\nabla} T \tag{21}$$

This result conforms with the well-known effective Seebeck coefficient for two materials set in parallel, when scalars are used instead of matrices.54 Since we have a tensorial relation between $\boldsymbol{\nabla} V$ and $\boldsymbol{\nabla} T$, we can carry out a similar analysis as was done to go from relation (4) to (5) and we can infer from (21) that we expect both a ***spin-dependent Seebeck effect*** and a ***spin-dependent Nernst effect***.

Furthermore, the third line in (19) implies that there is a spin current even if we set the charge current to be zero and also $\Delta\mu = 0$. It is given by :

$$\begin{cases} \boldsymbol{j}_p = \frac{1}{q}\left[\boldsymbol{\sigma}_{\uparrow} \cdot \boldsymbol{\varepsilon}_{\uparrow} - \boldsymbol{\sigma}_{\downarrow} \cdot \boldsymbol{\varepsilon}_{\downarrow} + \right. \\ \left. \left(\boldsymbol{\sigma}_{\uparrow} - \boldsymbol{\sigma}_{\downarrow}\right) \cdot \left(\boldsymbol{\sigma}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow}\right)^{-1} \cdot \left(\boldsymbol{\sigma}_{\uparrow} \cdot \boldsymbol{\varepsilon}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow} \cdot \boldsymbol{\varepsilon}_{\downarrow}\right)\right] \cdot \boldsymbol{\nabla} T \end{cases} \tag{22}$$

Since a temperature gradient is necessarily accompanied by a heat current (see eq. (16)), we can say that the three-current model predicts a ***heat-driven spin current***.

The Seebeck-Nernst effect expressed by (21) and the spin current predicted by (22) were derived under the condition that we are far from interfaces, so that $\Delta\mu = 0$. We can also find interesting consequences of the three-current model without this restriction, but assuming that no current is flowing at all, i.e. $\boldsymbol{j}_{\uparrow} = \boldsymbol{j}_{\downarrow} = \boldsymbol{0}$. These are the experimental conditions that characterize the ***Soret effect***.55 In this case, we have a given temperature gradient and the currents in (19) are set to zero, $\boldsymbol{j} = \boldsymbol{j}_p = \boldsymbol{0}$. A few algebraic manipulations yield the following relation between $\boldsymbol{\nabla} V$ and $\boldsymbol{\nabla} T$:

$$\begin{cases} \boldsymbol{\nabla} V = \\ \left[(\boldsymbol{\sigma}_{\uparrow} - \boldsymbol{\sigma}_{\downarrow})^{-1} \cdot (\boldsymbol{\sigma}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow}) - (\boldsymbol{\sigma}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow})^{-1} \cdot (\boldsymbol{\sigma}_{\uparrow} - \boldsymbol{\sigma}_{\downarrow})\right]^{-1} \\ \cdot \left[(\boldsymbol{\sigma}_{\uparrow} - \boldsymbol{\sigma}_{\downarrow})^{-1} \cdot (\boldsymbol{\sigma}_{\uparrow} \cdot \boldsymbol{\varepsilon}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow} \cdot \boldsymbol{\varepsilon}_{\downarrow}) - \right. \\ \left. (\boldsymbol{\sigma}_{\uparrow} + \boldsymbol{\sigma}_{\downarrow})^{-1} \cdot (\boldsymbol{\sigma}_{\uparrow} \cdot \boldsymbol{\varepsilon}_{\uparrow} - \boldsymbol{\sigma}_{\downarrow} \cdot \boldsymbol{\varepsilon}_{\downarrow})\right] \cdot (-\boldsymbol{\nabla} T) \end{cases} \tag{23}$$

Since we have a tensorial relation, we can once again argue that there can be a symmetric and an antisymmetric part to this tensor. We can reduce the symmetric part to an isotropic term and express the antisymmetric part with a tensorial product. Thus we find a relation of the form :

$$\boldsymbol{\nabla} V = -\varepsilon_{eff} \boldsymbol{\nabla} T - \varepsilon_{eff}^{\perp} \left(\hat{\boldsymbol{u}} \times \boldsymbol{\nabla} T\right) \tag{24}$$

Thus, we find that in the absence of any charge current, but out of equilibrium in the sense that $\Delta\mu \neq 0$, we find an apparent ***Seebeck effect*** and an apparent ***Nernst effect***. We expect this to take place near an interface, where the up and down spin populations are out of equilibrium because of the heat current. Equation (23) tells us that these two effects depend on the differences between the tensorial conductivities and thermoelectric matrices for the two spins channels. The thermodynamics of interfaces was first addressed by Johnson and Silsbee.565758 Saslow et al. further expanded on their work.59

We can invert equation (16) to write : $\boldsymbol{\nabla} T = -\boldsymbol{\kappa}^{-1} \cdot \boldsymbol{j}_Q$. We can then proceed to decompose the tensor $\boldsymbol{\kappa}^{-1}$ in its symmetric and anti-symmetric parts. Thus, following the similar steps taken to arrive at equation (5), we can write,

$$\boldsymbol{\nabla} T = -\kappa^{-1} \boldsymbol{j}_Q - R_{\perp} \left(\hat{\boldsymbol{u}} \times \boldsymbol{j}_Q \right) \tag{25}$$

where $R_{\perp}$ characterizes the strength of the Righi-Leduc effect.60 61 By considering the expression of $\boldsymbol{\kappa}$ deduced from (17), we could work out an expression for the spin-dependent Righi-Leduc coefficient in terms of the spin-dependence of the conductivity and the Seebeck coefficient.

1.3. Thermodynamics with magnetization as a state field

It is possible to include magnetization as a state variable in the thermodynamic description of irreversible processes. This was first done in the seminal paper of Johnson and Silsbee.42 Saslow analyzed magnetization dynamics in an inhomogeneous insulating ferromagnet.62 In this and the following two sections, we summarize our own contributions to this topic.63

We assume that each substance A has a dipole moment per unit of substance, $\boldsymbol{m}_A$ and the magnetization is $\boldsymbol{M} = \sum_A n_A \boldsymbol{m}_A$ where n_A is the quantity of substance A per unit volume. Since the magnetization $\boldsymbol{M}$ is an extensive quantity per unit volume, there is a continuity equation for it :

$$\dot{\boldsymbol{M}} + \boldsymbol{\nabla} \cdot \mathrm{j}_{\boldsymbol{M}} = \sum_A \left(\gamma_A n_A (\boldsymbol{m}_A \times \boldsymbol{B}) + \boldsymbol{\Omega}_A \times \boldsymbol{m}_A \right) \tag{26}$$

The right-hand side of (26) is the source term of this continuity equation. Its first term expresses the reversible evolution of the dipoles $\boldsymbol{m}_A$ under the action of the magnetic induction field $\boldsymbol{B}$. The second term is a dissipative term that accounts for the relaxation of the magnetization, as will become apparent in section 1.5. The left-hand side of (26) contains the magnetization current, $\mathsf{j}_{\boldsymbol{M}}$. It is a tensorial quantity. A thermodynamic argument yields :32

$$\mathsf{j}_{\boldsymbol{M}} = \sum_A \boldsymbol{m}_A \odot \boldsymbol{j}_A \tag{27}$$

where $\odot$ here means that the tensorial product has been made symmetric by taking the sum $(1/2)(\boldsymbol{m}_A \otimes \boldsymbol{j}_A + \boldsymbol{j}_A \otimes \boldsymbol{m}_A)$. The expression (27) is the classical analog to the quantum mechanical spin current introduced by Stiles and Zangwil.64 In this approach, each substance is characterized by a magnetic dipole density, which is a vectorial quantity. Thus, this approach is more general than the "up" and "down" spins of the three-current model, section 1.2. We draw from the expression (6) for the current $\boldsymbol{j}_A$ an expression for the magnetization current and find that it can take the form :

$$\mathsf{j}_{\boldsymbol{M}} = -\frac{\sigma_\perp}{q_A} \boldsymbol{m}_A \odot \left(\hat{\boldsymbol{B}} \times \boldsymbol{\nabla} V\right) \tag{28}$$

Thus, we predict a tensorial spin Hall current. At the surface of an isotropic medium, the orientation of which is given by the unit vector $\hat{\boldsymbol{n}}$, we expect a spin current of similar form :

$$\mathsf{j}_{\boldsymbol{M}} = -\frac{\sigma_\perp}{q_A} \boldsymbol{m}_A \odot (\hat{\boldsymbol{n}} \times \boldsymbol{\nabla} V) \tag{29}$$

Likewise, we expect from (9) a tensorial thermally-induced spin Hall current :

$$\mathsf{j}_{\boldsymbol{M}} = -\frac{\sigma \varepsilon_\perp + \sigma_\perp \varepsilon}{q_A} \boldsymbol{m}_A \odot (\hat{\boldsymbol{u}} \times \boldsymbol{\nabla} T) \tag{30}$$

where the unit vector $\hat{\boldsymbol{u}}$ designates either the direction of a magnetic field $\boldsymbol{B}$ or the normal to a surface $\hat{\boldsymbol{n}}$.

We consider a single substance A consisting of conduction electrons in the usual configuration where the magnetic moments $\boldsymbol{m}_A$ are orthogonal to the current density $\boldsymbol{j}_A$, i.e. $\boldsymbol{m}_A \cdot \boldsymbol{j}_A = 0$. In this case, the ***spin current vector in a conductor*** is given by,

$$\boldsymbol{j}_S = \boldsymbol{m}_A^{-1} \cdot \mathrm{j}_{\boldsymbol{M}} \tag{31}$$

Taking into account the expressions (7) and (9) for a current density $\boldsymbol{j}_A$ orthogonal to the electric potential gradient $\boldsymbol{\nabla} V$ and to the temperature gradient $\boldsymbol{\nabla} T$ respectively, and using the expression (31) for the spin current $\boldsymbol{j}_S$, the contraction of the tensorial relation (29) yields the ***spin Hall effect***,

$$\boldsymbol{j}_S = -\frac{\sigma_\perp}{q_A}\left(\hat{\boldsymbol{n}} \times \boldsymbol{\nabla} V\right) \tag{32}$$

and the contraction of the tensorial relation (30) yields the ***thermally-induced spin Hall effect***,

$$\boldsymbol{j}_S = -\frac{\sigma\varepsilon_\perp + \sigma_\perp\varepsilon}{q_A}\left(\hat{\boldsymbol{u}} \times \boldsymbol{\nabla} T\right) \tag{33}$$

The inversion of the spin Hall effect (32) yields the ***inverse spin Hall effect***,

$$\boldsymbol{\nabla} V = \frac{q_A}{\sigma_\perp}\left(\hat{\boldsymbol{n}} \times \boldsymbol{j}_S\right) \tag{34}$$

and the inversion of the thermally-induced spin Hall effect (32) yields the ***thermally-induced inverse spin Hall effect***,

$$\boldsymbol{\nabla} T = \frac{q_A}{\sigma\varepsilon_\perp + \sigma_\perp\varepsilon}\left(\hat{\boldsymbol{u}} \times \boldsymbol{j}_S\right) \tag{35}$$

When a heat current produces a transverse temperature gradient, we speak of a Righi-Leduc effect. Here it is the spin current $\boldsymbol{j}_S$ that drives the effect. Therefore, the effect predicted by (35) could be called a ***spin currrent-induced Righi-Leduc effect***. Of course, this temperature gradient can manifest itself as a Seebeck voltage in a contact that would be deposited at the surface of the thermodynamic system. This voltage would be a ***spin current-induced Seebeck voltage***.

1.4. Magnetic Seebeck and Magnetic Nernst effect

Since we have now added magnetization as a new state variable, the expressions for the generalized forces is modified. It includes a magnetic term : 32

$$\boldsymbol{F}_A = -\boldsymbol{\nabla}\mu_A - q_A\boldsymbol{\nabla}V + \boldsymbol{m}_A\boldsymbol{\nabla}\boldsymbol{B} \tag{36}$$

This new term corresponds to the force that a magnetic moment $\boldsymbol{m}_A$ experiences in the present of an inhomogeneous magnetic induction field $\boldsymbol{B}$, as in the Stern-Gerlach experiment.

In analogy with the derivation from (2) of the Seebeck effect and the Nernst effect, we impose the condition $\boldsymbol{j}_A = \boldsymbol{0}$ in (2) and consider the magnetic contribution (36) to the generalized force $\boldsymbol{F}_A$. The condition $\boldsymbol{j}_A = \boldsymbol{0}$ may also characterize insulators. Thus, we have :

$$\boldsymbol{m}_A\boldsymbol{\nabla}\boldsymbol{B} = \mathsf{L}_{AA}^{-1} \cdot \mathsf{L}_{AS} \cdot \boldsymbol{\nabla}T \tag{37}$$

In terms of the magnetization, given by $\boldsymbol{M} = n_A\boldsymbol{m}_A$, we have :

$$\boldsymbol{M}\boldsymbol{\nabla}\boldsymbol{B} = n_a k_B \,\mathsf{\Lambda} \cdot \boldsymbol{\nabla}T \tag{38}$$

where Λ is a dimensionless tensor, $\Lambda = \mathsf{L}_{AA}^{-1} \cdot \mathsf{L}_{AS}/k_B$. We illustrate the consequence of this result for two different experimental situations : in the first one, we assume that plane magnetization waves are excited; in the second, we assume the presence of a field of vortices or skyrmions.

For the first example, we consider that the magnetization dynamics is described as a superposition of plane waves. Then, it is possible to work out what induced **B** field is induced by the temperature gradient. 65 The effect is proportional to the inverse of the wavelength. So, in the following, we keep only the smallest k value, as it dominates this effect. This thermally induced field contributes to the relaxation of the magnetization. Its effect can be expressed as a ***thermal spin torque*** of the form :

$$\boldsymbol{\tau} = k_T \hat{\boldsymbol{M}} \times \left(\hat{\boldsymbol{M}} \times \boldsymbol{j}_S \right) \tag{39}$$

where k_T is proportional to the temperature gradient and has the units of wave numbers, and

$$\boldsymbol{j}_S = \frac{\mu_0 M_S}{k} \boldsymbol{m}_k \tag{40}$$

where $\boldsymbol{m}_k$ is the magnetization of mode of wave vector k, M_S is the magnetization at saturation. Thus, the thermodynamics of irreversible processes implies a thermally induced relaxation term in the Landau-Lifshitz equation. This torque can be expressed in such a way as to bring out a spin current $\boldsymbol{j}_S$. The expression (40) applies for ***spin currents in insulators*** and should not be confused with the expression (31) obtained for spin currents in conductors. (Recall that we arrived at the expression (40) for the torque under the assumption $\boldsymbol{j}_A = \mathbf{0}$).

For the second example, we decompose the tensor Λ into a symmetric part, which we assume to be isotropic, and an asymmetric part, writing :

$$\Lambda \cdot \boldsymbol{\nabla} T = \lambda \boldsymbol{\nabla} T + \lambda_\perp \hat{\boldsymbol{u}} \times \boldsymbol{\nabla} T \tag{41}$$

Vector calculus manipulations of the left-hand side of equation (37) allow us to write it as : 32

$$(\boldsymbol{\nabla} \times \boldsymbol{M}) \times \boldsymbol{B} = n_A k_B \lambda \boldsymbol{\nabla} T + n_A k_B \lambda_\perp \hat{\boldsymbol{u}} \times \boldsymbol{\nabla} T \tag{42}$$

We apply this expression to a medium filled with vortices of axes all pointing in the same direction. Then, the bound current $\boldsymbol{\nabla} \times \boldsymbol{M} = j_M \, \hat{\boldsymbol{v}}$ is a uniform vector field and the scalar j_M characterizes the strength of the vortex. The vector $\hat{\boldsymbol{u}}$ in (41) was inferred from general principles of symmetry of the tensor Λ, which is material property. Now, if we take $\hat{\boldsymbol{u}}$ to be $\hat{\boldsymbol{v}}$, then we have :

$$\boldsymbol{B} = \frac{\lambda_\perp n_A k_B}{j_M} \boldsymbol{\nabla} T \tag{43}$$

Thus, we predict a magnetic induction field induced by a temperature gradient. By analogy with the Seebeck effect, we can call this the ***magnetic Seebeck effect***. Consider now a situation in which $\lambda_\perp = 0$, then, by multiplying eq. (42) by $\hat{\boldsymbol{v}} \times$, we get :

$$\boldsymbol{B} = \frac{\lambda n_A k_B}{j_M} \left(\hat{\boldsymbol{v}} \times \boldsymbol{\nabla} T \right) \tag{44}$$

Since in this case the induced field is perpendicular to the temperature gradient, this effect can be called a ***magnetic Nernst effect***. This would be a first-order effect, predicted here for a field of magnetic vortices. A second-order magnetic Nernst effect can be expected in more general circumstances.66 As always, thermodynamics predicts the existence of cross-effect linked for example temperature gradients and electromagnetic fields. But it does not provide the strength of the effect, e.g. here, the value of the coefficients λ and $\lambda_{\perp}$. A variational argument could lead to a quantitative estime of λ, by considering $\boldsymbol{\nabla} \times \boldsymbol{M}$ as a variable of the energy density.67

1.5. Cross-effects on magnetization dynamics

When introducing the magnetization as one of the state variables defining the state of the system, we had to modify the expression of the entropy source density ρ_s, as follows : 32

$$\rho_s = \frac{1}{T} \sum_i \boldsymbol{j}_i \cdot \boldsymbol{F}_i + \boldsymbol{\Omega}_A \cdot (\boldsymbol{m}_A \times \boldsymbol{B}) \tag{45}$$

In order to satisfy the second principle of thermodynamics, which implies that $\rho_s \geq 0$, we must have the following relation between pseudo-vectorial quantities :

$$\boldsymbol{\Omega}_A = \mathsf{L}_{AM} \cdot (\boldsymbol{m}_A \times \boldsymbol{B}) \tag{46}$$

where the tensor L_{AM} is positive definite. We will consider an isotropic medium, so $\mathsf{L}_{AM} = L_{AM}\mathbb{1}$. Then, in the absence of magnetization current and for one substance A, equation (26) becomes the well-known Landau-Lishitz equation :

$$\dot{\boldsymbol{m}}_A = \gamma_A \boldsymbol{m}_A \times \boldsymbol{B} - \beta_A \boldsymbol{m}_A \times (\boldsymbol{m}_A \times \boldsymbol{B}) \tag{47}$$

where $\beta_A = n_A^{-1} L_{AM}$. This thermodynamic derivation of the Landau-Lifshitz damping was also obtained by Saslow.62 In the presence of a magnetization current J_M given by (27) and the current given by (9), the continuity equation

(26) for the magnetization yields a modified Landau-Lifshitz equation with :

$$\begin{cases} \dot{\boldsymbol{m}}_A = \gamma_A \boldsymbol{m}_A \times \boldsymbol{B} - \beta_A \boldsymbol{m}_A \times (\boldsymbol{m}_A \times \boldsymbol{B}) \\ \qquad + \dfrac{\sigma\epsilon}{q_A n_A} (\boldsymbol{\nabla} T) \cdot \boldsymbol{\nabla}\, \boldsymbol{m}_A \\ \qquad + \dfrac{\sigma\epsilon_\perp + \sigma_\perp \epsilon}{q_A n_A} (\hat{\boldsymbol{u}} \times \boldsymbol{\nabla} T) \cdot \boldsymbol{\nabla}\, \boldsymbol{m}_A \end{cases} \tag{48}$$

While thermodynamics cannot predict the value of the coefficients that link the generalized currents and the generalized forces, it reveals the present of cross effects such as the Seebeck effect, which links the thermal gradient and the electrostatic potential gradients. Thus here, we predict that the temperature gradient affects the magnetization dynamics and that the coefficient that determines this coupling can be deduced from other experiments, in which the Ohm, Hall, Seebeck and Nernst coefficients would be determined. Berakdar et al. posited the existence of a spin current proportional to a temperature gradient in a conductor, and analyzed theoretically its effect on magnetization dynamics.68

An argument based on linear transport theory has shown under what conditions a diffusive transport of magnons can be derived.69 In this case, a chemical potential can be applied to magnons and a thermodynamic approach may apply. For example, a Wiedemann-Franz law for magnons can be derived.70 An analysis of the magnon mean-free path has also been conducted in a simple kinetic gas theory approach.71

2. Spin-dependent Seebeck and Peltier coefficients

The spin dependence of the Seebeck coefficient can be tested in experiments which are the heat-equivalent of giant magnetoresistance (GMR).

It is often referred to as ***magneto-thermoelectric power***, MTEP. When MTEP is measured with a current in-plane configuration (CIP),7273 74 scattering at interfaces plays an essential role, just as in CIP-GMR. The interpretation of MTEP data is simpler in the case of heat flowing perpendicular to the plane (CPP-MTEP) of the interfaces.75 In this case, the thermodynamic model (Eq. 21) can be applied to account for bulk effects.76 A study of MTEP using current-in-plane (CIP) was reported recently for Co/Cu multilayers.77

The MTEP and the magneto-Peltier effects have been studied for magnetic tunnel junctions, both theoretically, using Green's function-based ab-initio calculations,78 and experimentally.79 80 A giant MTEP effect has been reported for junctions with MgO barriers.81 82 A domain wall contribution to MTEP has also been identified experimentally.83

A novel effect was observed when magnetoresistance was measured in the presence of a temperature gradient. Local temperature gradients develop, because of the Peltier effect. This has been observed in multilayers for which the CPP condition applied.84 In a typical textbook description of the Peltier effect, a current is driven through a junction between two materials with different Peltier coefficients, and, if the structure is at a uniform temperature, this electrical current implies differing heat currents in each material, and thus, a heat current must flow in or out of the junction. If the junction cannot be thermalized this way, then a temperature gradient develops in each layer, as a simple deriviation shows. (76) Hence, in a Co/Cu multilayer, a temperature gradient develops in each ferromagnetic layer "F" and in each normal layer "N" with,

$$\boldsymbol{\nabla} T_F = \frac{\Pi_F - \Pi_N}{\kappa_F + \kappa_N}\, j = -\boldsymbol{\nabla} T_n \tag{49}$$

where κ_F and κ_N are the heat conductivities, Π_F and Π_N the Peltier coefficients of the "F", respectively, "N", layers, and j the charge current density flowing across both layers. We recall that the Peltier and the Seebeck coefficients are linked by the relation $\Pi = T\varepsilon$, where T is the temperature of the material. The local gradients imply that a voltage develops across this pair of layers, which is proportional to $(\varepsilon_F - \varepsilon_N)^2$. Thus, if there are many pairs of such bilayers, the effect is cumulative and can becomes quite noticeable with a simple transport measurement scheme. It was evidenced experimentally through measurements of the temperature derivative of stacks of such bylayers, made in the form of multilayered nanowires (figure 1). 85 The existence of such local gradients has also been deduced from a Boltzmann modeling of transport in granular structures.86 A theoretical description has also been developed for granular systems, which includes spin-mixing effects. 87 This Peltier effect accounts for the surprising observation of a bias in the I-V characteristics of nanopillars.88

B.J.Van Wees and his group managed to obtain a direct measurement of the Peltier effect in a spin-valve nanopillar.89 It was direct in the sense that they were able to measure the cooling or heating at the interface, using local thermocouples. Measurements at the first and at the second harmonic of the frequency of the applied current allowed them to distinguish Joule heating and the Peltier effect. This group and others undertook the task of verifying experimentally the Onsager-Casimir reciprocity relations 3.9091

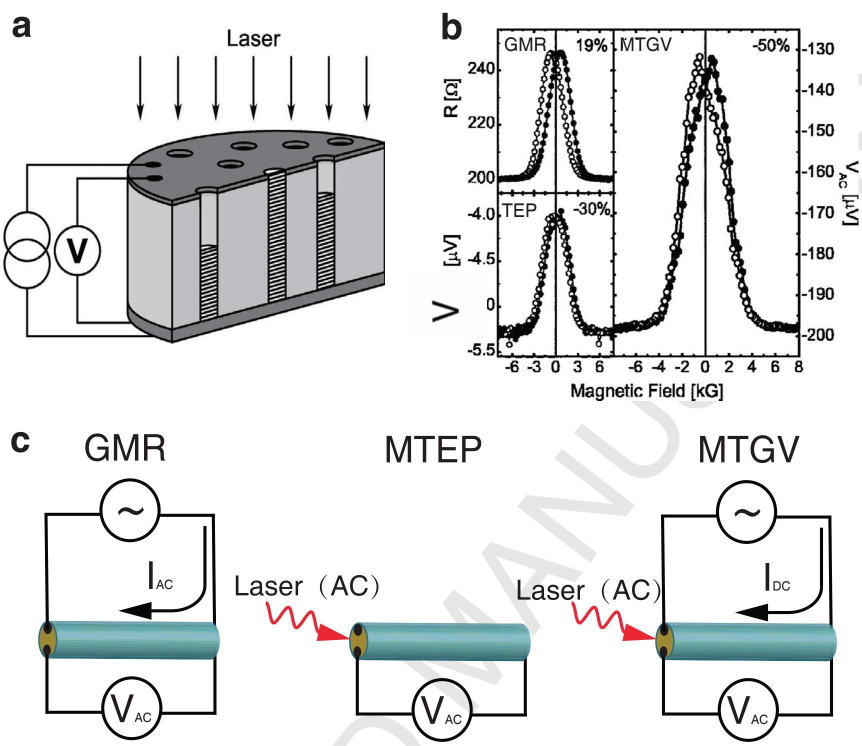

Figure 1: **GMR, MTEP and MTGV in magnetic nanowires a**, Magnetic nanowires of Co/Cu mulilayers fabricated by electrodeposition in porous polycarbonate membrane. Oscillating laser heating at one end of the nanowires at low frequency (10 to 500 Hz). **c**, three different measurement protocols on magnetic nanowires, i.e. Giant magnetoresistance (GMR), magneto thermopower (MTEP) and magnetic thermogalvanic voltage (MTGV). The typical results of these measurement on multilayer nanowires are presented in **b**.

3. Thermal spin-transfer torque

The effect of spin-transfer torque (STT) was first predicted by Slonczewski 92 and Berger 93 about two decades ago. Nowadays, this finding has been successfully applied in the next generation of magnetic memories

(STT-MRAM) 94. The conventional spin-transfer torque is to use electrical current to generate a torque on the magnetization (e.g. in spin valves or magnetic tunnel junctions). If the current density reaches a critical value, switching of magnetization occurs, which can be used for switching a memory bit between its "0" and "1" states. It is notorious that our microprocessors must evacuate a lot of heat and it would be great to make use of this heat before releasing it into the environment. It was first proposed theoretically in 2007 by Hatami et al. 95 that a heat current can also generate a torque and even switch magnetization just as a charge current would, owing to the STT. This torque is called thermal spin-transfer torque (TST). The total torque at a junction between two ferromagnets comprises two terms :

$$\boldsymbol{\tau} \propto P\Delta V + P\prime S\Delta T \tag{50}$$

where P and $P\prime$ characterize the spin asymmetry of the conductivity and of the Seebeck coefficient S, respectively, ΔV is the electrostatic potential drop across the junction, and ΔT the temperature drop across it.

About three years after this prediction, the effect of TST was first demonstrated experimentally in metallic spin valves embedded in nanowires.96 It was shown that the switching field of Co/Cu/Co spin valves is affected by a locally applied thermal gradient (fig. 2), meaning that there was a thermally-induced torque exerted on the magnetization. Soon after the first experimental evidence of TST in metallic spin valves, one of the inventors of STT, Slonczewski, predicted that TST can also arise in insulating heterostructures through thermal magnon transport 97. it was soon implemented in a prototype memory bit and the effect was confirmed.98 99

It has been shown also that a spin current can be induced by heating with ultrafast laser pulses 101102. Of course, this spin current leads to a thermal spin torque that can, for example, flip the magnetization of a spin valve.103 Likewise, very recently, the group of Stuart Parkin at IBM has demonstrated a TST effect on magnetization switching in magnetic tunnel junction 104. Further studies investigate the comparison between TST with STT105, observe TST using ferromagnetic resonance 106 or demonstrate thermally-driven domain wall motion, 107 in particular by magneto-optical Kerr imaging.108

In parallel to these experimental investigations, more theoretical studies on TST-related effects are conducted in magnetic tunnel junctions and magnetic insulators.109110111112113 Very recently, theoretical aspects are further developed 114 67 and some predictions are yet to be explore experimentally, such as that of heat-driven spin currents in junctions between silicene nanoribbons.115

4. Experimental studies on Nernst-related effect

4.1. Anormalous Nernst effect

The Nernst effect was discovered almost 100 years in non-magnetic materials, with a magnetic field applied normal a temperature gradient. The electric field was found normal to the both the applied field and the temperature gradient and proportional to the magnetic field strength. The anomalous Nernst effect (ANE) describes the induced electric field $E_{\mathrm{ANE}} = -N_{\mathrm{ANE}}\,\boldsymbol{M} \times \boldsymbol{\nabla} T$, perpendicular to both the temperature gradient $\boldsymbol{\nabla}$ and the magnetization $\boldsymbol{M}$.

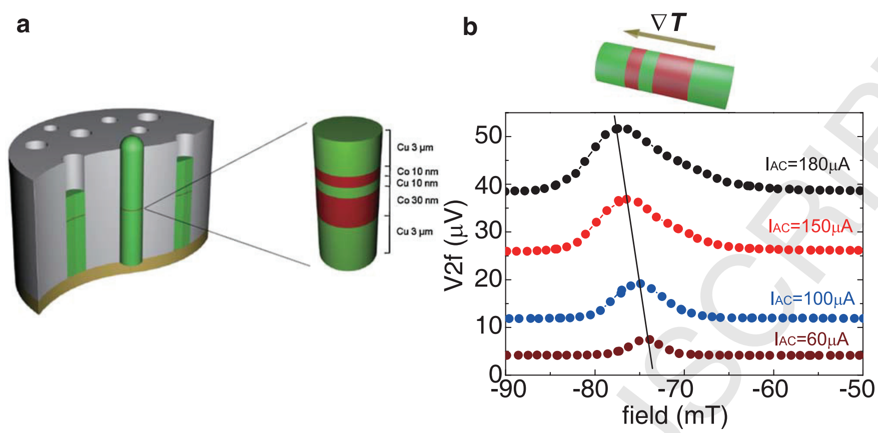

Figure 2: **Evidence for thermal spin-transfer torque in spin valves.** **a**, Nanowires containing Co/Cu/Co spin valve fabricated by electrodeposition inside of porous polycarbonate membrane. **b**, Asymmetric spin valve with a thermal gradient acting on the switching field of magnetization.

Most experiments are conducted with an out-of-plane temperature gradient and in-plane magnetization. Weiler et al. 116 (fig. 3) and von Bieren et al. 117 reported spatially resolved measurements of ANE using laser-heated Co_2FeAl and permalloy/gold microstructures, respectively. Note that Weiler et al. studied also platinum contact on yttrium-iron garnet and found evidence with such local, perpendicular temperature gradient, the longitudinal spin Seebeck effect, namely, a voltage due to the inverse spin Hall effect (Eq. 34) for a spin current coming from the YIG magnetic insulator. Reference 117 also presented time-resolved ANE measurement, and demonstrated the technique of ANE-based magnetic imaging of magnetization reversal. When the substrate beneath a Py strip is etched away chemically, it is possible to measure the ANE in-plane.118 There are also measurements done with in-plane temperature gradient and perpendicular magnetization, which require either relatively large fields to saturate the magnetization to the out-of-plane orientation, or films that have an innately perpendicular magnetized anisotropy.119

4.2. Planar Nernst effect

Another type of Nernst-related effect is the so called planar Nernst effect (PNE)120121, where both the magnetization and temperature gradient are in the film plane. The measured PNE voltage depends on the angle between the temperature gradient and the magnetization, with $E_{\mathrm{PNE}} = 2A_0 sin(\phi)cos(\phi) + c$. Therefore, the PNE voltage reaches a maximum (or a minimum) at a 45 degree angle. This PNE effect essentially originates from the anisotropic magnetothermopower (AMTEP), as shown in section 1.1. Pioneer experimental work was conducted by Pu et al.122 in 2006, studying PNE in ferromagnetic semiconductors with temperatures ranging down to 6K. Schmid et al.123 measured both ANE and PNE in permalloy thin films, using different substrate of MgO, GaAs and membrane (fig. 3). ANE is studied in various materials, e.g. perpendicularly magnetized ordered-alloy119, multilayers124 and oxides125, and it is often compared to the anormalous Hall effect, because it has a similar geometry.

4.3. Perspective for other Nernst-related effects

Apart from the ANE and PNE, there are quite some studies on the spin Nernst effect (SNE). Cheng et al. predicted the existence of SNE in two dimensional electron systems. 126 SNE can persist in the absence of magnetic field.127 Extrinsic spin Nernst is also studied by first principle.128 With the rapid development of thin film growth of novel alloys, there are many different materials to explore. For example, Nernst effect was measured in antiferromagnetically copupled superlattices of LaSrMnO$_3$ and SrRuO$_3$.129

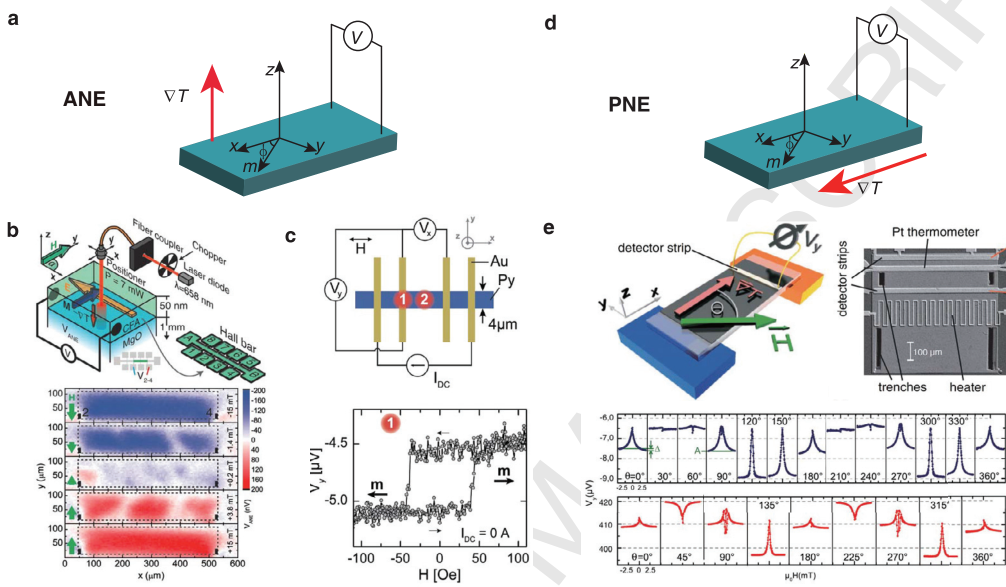

Figure 3: **a**, Schetch of the anormalous Nernst effect. **b**, experimental study from Weiler et al.116 of magnetic imaging using ANE. **c**, ANE studies from von Bieren et al.117 on suspended permalloy strip. **d**, Sketch of the planar Nernst effect. **e**, experimental results from Schmid et al.123

5. Thermal gradient effect in magnonics

5.1. Spin Wave amplification

In the 1960s and 70s, there was an interest for various forms of parametric amplification of waves. For example, instabilities were predicted and observed, notably by forcing a large current through a high-mobility semiconductor like InSb, which could possibly explain the observation of sub-THz waves emission when this material is subjected to a strong current. 130 131 The basic theory had been laid out by Pines and Schrieffer.132 The case of spin waves was considered also. Comstock considered the parametric coupling of magnetic and elastic waves, and observed the phenomenon in YIG spheres.133 134 Schloemann considered the compelling of surface magnetostatic waves with drifting charge carriers, and parametric pumping due to the coupling of magnons and phonons 135136137138139140. The proper understanding of this phenomenon was controversial, with alternative models by Gurevich,141 142 and Haas.143

Spin wave amplification by interaction with a charge current has been discussed and data reported in several papers. Baryakhtar et al. predicted an instability of spin waves in YIG when a beam of charges would pass near it.144 Trivelpiece et al. also considered an electron beam passing by a ferrite rode filling a coaxial line. 145 Instead of a an electron beam flying by, Schloemann realized that surface magnetostatic modes in a ferromagnetic insulator like YIG could be amplified thanks to a current running in a semiconductor laid on top of the YIG crystal.146 The magnetization induces a field in the conductor in which a DC current flows, thus inducing a Hall current. This current has a back action on the precessing magnetization in the ferromagnet, and amplification may arise. Spin wave amplification due to their interaction with a charge carrier current inside a semiconduting ferromagnet was considered by Rezende et al.147 It is interesting to note that these considerations took place a decade before the discovery of giant magneto-resistance.148 One way to think of this amplification is in terms of a Cerenkov effect, as amplification occurs when the drift velocity of the carriers is greater than the phase velocity of the wave.149

Spin wave amplification by interaction with a heat current was first suggested by Tenan and Miranda.150 They consider a ferromagnetic semiconductor, not an electron beam passing by a ferromagnet. They analyse the effect of the interaction of electrons with magnon in a Botlzmann theory of transport under the approximation $k\ell > 1$, where k is the magnon wave vector and ℓ the charge carrier mean free path. The system they have in mind is Ag-doped $CdCr_2Se_4$. They find that amplification is possible if the wave propagates from cold to hot. The value of the temperature gradient at the threshold for amplification is estimated at 10^5 K/cm. This can easily be achieved in nanostructures. We presume that the experiment was never attempted because this paper has never been referenced, so far. Since acoustic waves can excite spin waves, as was reviewed above, the thermal amplification of acoustic waves is an important ingredient in discussing the thermal amplification of spin waves. Thermal amplification of acoustic waves was predicted in the 70s. A temperature gradient can lead to the excitation of sound waves in semiconductors.151 152 The phenomenon has been observed in Nickel.153 154

A six-fold enhancement of the voltage detected at a Pt contact on a YIG strip was observed when a temperature gradient of a mere 20 K/cm was applied in plane.155 This voltage was detected at the cold end of the YIG strip. It was attributed to a constructive combination of spin pumping and spin Seebeck effects. In order to focus on magnetization dynamics and avoid interfacial spin phenomena, one of the present authors and his group launched wave packets from one end of YIG strip and detected them inductively at the other. The YIG strip was subjected to an in-plane temperature gradient. A change in the damping of the wave packets was observed, proportional to the temperature gradient.156 This effect was analyzed in the framework of the thermodynamics of irreversible processes, as described in section 1.4.

Amplification of spin waves has been obtained by the group of Azevedo and Rezende, launching spin waves from a microstrip at one end of a YIG strip, and detecting them with another microstrip at the other end. The temperature gradient was set normal to the plane of the YIG strip (fig. 4a,b).100 The emitter and receiver were 12 mm apart, and the working frequency in the range of 1 to 2 GHz. The authors interpreted their observation as arising from the spin current induced by the temperature gradient.157

Meanwhile, Lu et al.158 found that spin transfer in Pt/YIG bilayer system can control the relaxation of spin resonance, which is consistence in principle with the observed spin wave amplification, since the damping is countered by the thermally induced spin transfer torque.

5.2. Magnetic Seebeck effect

As the spin Seebeck effect (section 6) is defined as a voltage measured at the end of contacts laid on a magnetic system subjected to a temperature gradients, it was perceived that it may be a contact effect. In order to contribute to a clarification of the modeling of this intriguing effect, some groups engaged in the study of magnetization dynamics of a system subjected to a temperature gradient. This was done theoretically,157 159 and experimentally.160 A time-resolved measurement, consisting of sending wave packets up or down a temperature gradient applied to a YIG strip showed clear evidence that these wave packets were more strongly attenuated when propagating from hot to cold then cold to hot. This phenomenon was interpreted by exploiting the expression 37 of the magnetic Seebeck (fig. 4c,d).156 The reader should not be confused with the terminology. The "magnetic Seebeck" refers to the prediction of a magnetic field induced by temperature gradient in presence of an inhomogeneous magnetization. The "k" vector of the magnetic wave plays the role in the magnetic Seebeck effect that the charge plays in the (normal) Seebeck effect. This induced $\boldsymbol{B}$ field gives rise to a dissipative torque in the Landau-Lifshitz equation for the magnetization. Thus, the effect can be thought of as heat-driven torque for insulators, while the spin currents described in section 3 applied to conductors. We have yet to see an experimental verfication for the theoretical prediction that skyrmions diffuse from cold to hot regions.161

5.3. Spin wave heat conveyer

In the above section, we saw that a temperature gradient or a heat current can affect the propagation of spin waves. Inversely, when spin waves propagate, there is heat current associated with it, therefore spin waves or magnons can convey heat. It was shown by An et al (fig. 4e).162 that spin waves propagating in a 30 μm thick YIG film can convey heat in the propagation direction. As in the experiments reporting on this effect, the magnetostatic surface waves or the Damon-Eschbach (DE) modes are excited, the spin waves propagate on the upper surface or the bottom surface of the YIG film, depending on the applied field orientation. An infra-red thermal camera is then used to detect the heat current conveyed by the propagating spin waves. A theory for such effect is then provided by Adachi et al. 163 and micromagnetic simulations were conducted by Perez et al. 164. Very recently, experiments were done using 200 nm-thick YIG thin film and proved that a unidirectional spin wave conveyer can be made, owing to the non-reciprocity of DE mode spin wave. 165 To the contrary, this was not observed in the backward volume mode spin wave, which is reciprocal.

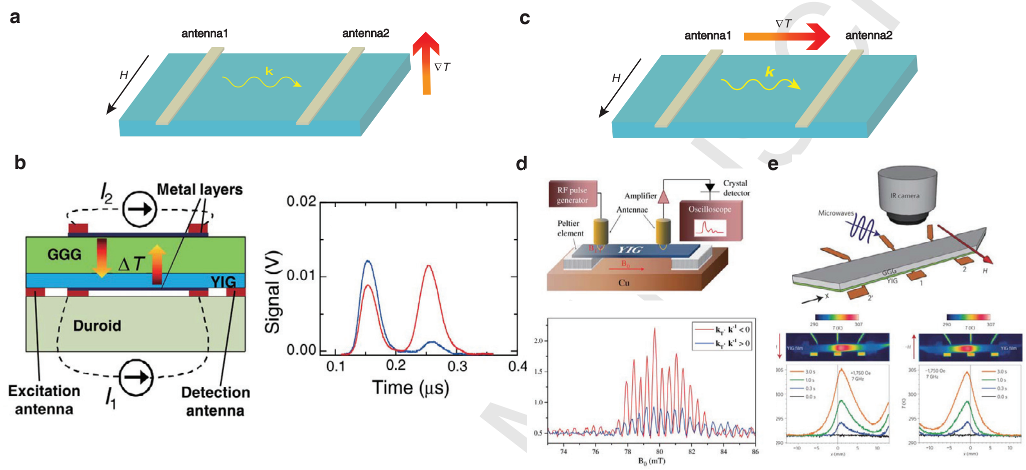

Figure 4: **a**, Sketch for spin wave propagation with an out of plane temperature gradient. **b**, Spin wave amplification experimental setup and key results for the effect from ref. 100 **c**, Sketch for spin wave propagation with an in-plane temperature gradient. **d**, Experimental setup for the observation of the magnetic Seebeck effect and key results.156 **e**, Setup and key results for the spin wave heat conveyor experiments.162

6. Transverse and longitudinal spin Seebeck and proximity effect

In 2008, the observation of the spin Seebeck effect (SSE) is announced by Uchida et al (fig. 5a).166. Later on, what was measured then would be called the transverse spin Seebeck effect, meaning that a temperature gradient is applied in the film plane and a spin current diffuses into the Pt detection layer and generates an inverse spin Hall voltage. Thus, the inverse spin Hall effect acts as a detector of the spin current. The effect was also observed using Pt contacts on an insulator, YIG,167 and on a ferromagnetic semiconductor, GaMnAs.168 A giant spin Seebeck effect was also observed by Jaworski et al.gi169 in a non-magnetic material, InSb.

However, the SSE is not without controversies and complications. One aspect of the SSE that was surprising at first was that it was observed in macroscopic structures, i.e. on a scale of millimeters, while other spin dependent processes were observed over a scale of the order of the spin diffusion length, which is only on the scale of tens or hundreds of nanometers. In addition, in magnetic systems, it is hard to eliminate the role of out of plane temperature gradient while applying an in-plane thermal gradient. If there is an out-of-plane gradient, a longitudinal spin Seebeck effect (LSSE) is expected,170.It was observed with YIG (fig. 5b), 171 and with hexagonal ferrite.172 A detailed calculation is made by Schreier et al. 25 and addressed the temperature profiles in the transverse spin Seebeck effect.

If the magnetic material is an insulator, the regular ANE no longer exists. Nevertheless, due to the magnetic proximity effect (MPE), the Pt layer on top of the YIG film becomes magnetic as well, at lease for a few nanometers (fig. 5c).173 Then an ANE voltage is expected from the Pt layer. The separation of longitudinal spin Seebeck and MPE-induced ANE is not very easy, but proved to be possible.174175176171 One way is to use Au/Pt structure, where proximity effect is almost negligible. However, the strength of the detected intrinsic LSSE is less than 1μV, much smaller than the voltage response detected in Pt/YIG.177

Inhomogeneous magnetic fields influence the magnetothermoelectric voltages and set a stringent bound on possible transverse spin Seebeck effect in permalloy.178 The non-local behavior observed in the transverse geometry has been accounted for by the fact that the magnons that define the temperature and those that transport energy belong to different regions of the phonon spectrum.179

The spin Seebeck effect can also be understood by the three current model discussed in the theoretical part of this review and is also discussed theoretically by Wegrowe et al.180 Very recently, an enhanced spin Seebeck effect is found by Jiang et al.181 in topological insulator/YIG heterojunction.

7. Prospects

As Hoffman and Bader put it, there is a lot of "opportunities at the frontiers of spintronics".182 Spin caloritronics is one such frontier. The present review of thermodynamics applied to spintronics suggests that we may expect effects of heat flowing through interfaces akin to the effects observed when charges are transported through interfaces.176 Furthermore, one classical thermal effect has remained unexplored : the Righi-Leduc effect, i.e. the temperature difference expected at the contacts of a Hall bar due to a heat current flowing along the bar. Furthermore, the effect of heat on magnetization dynamics in conductive ferromagnet, as described by equation 48 is yet to be investigated. Borlenghi et al. have pointed out that the control over the generalized forces associated with energy and magnetization may lead to a thermo-magnonic diode effect.183184185 This and a negative differential magnon tunneling,186 have yet to be explored experimentally. New discoveries may also stem from looking at classical phenomena, but in an en-

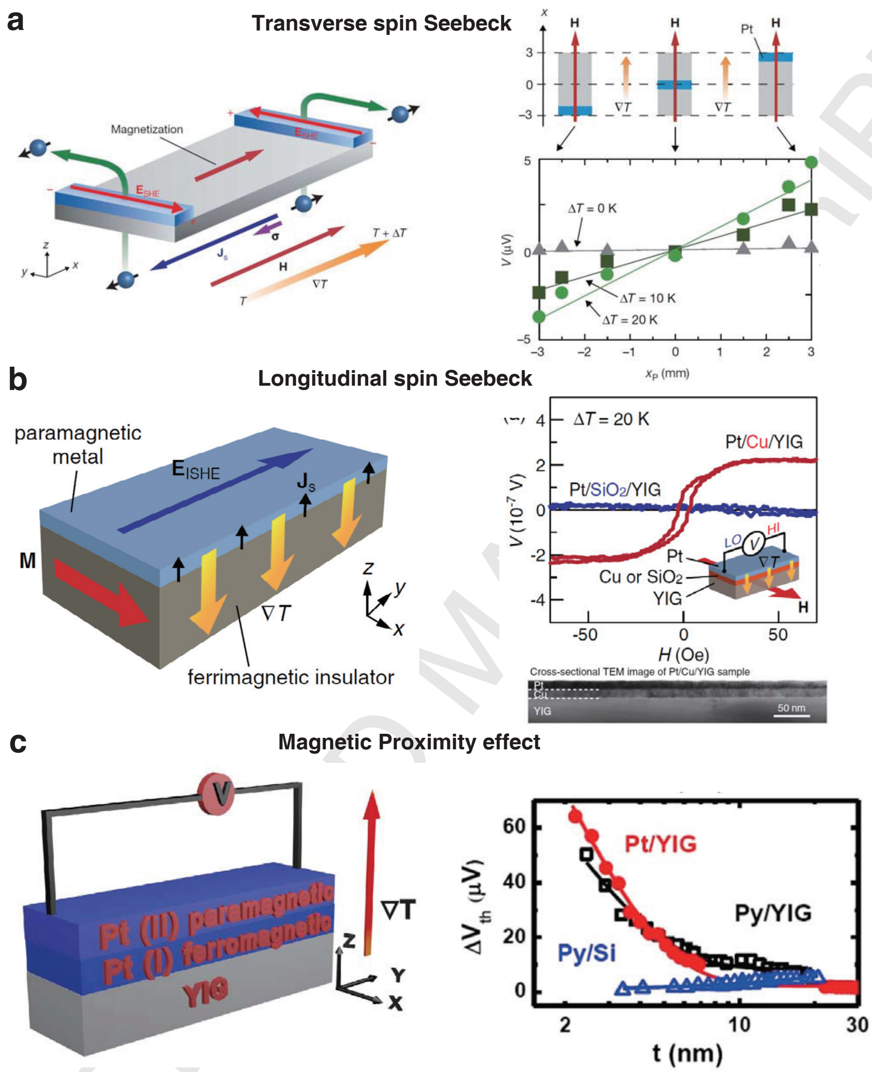

Figure 5: **a** Sketch of Transverse spin Seebeck and the typical experimental results.166 **b** Sketch of longitudinal spin Seebeck geometry and the measurement on Pt/Cu/YIG structure to separate LSSE from MPE-induced ANE voltage.176 **c** Sketch of MPE-induced ANE and experimental results of magnetic proximity effect in several sample structures.173

tirely new class of materials. This could happen by studying the anomalous Nernst effect in monoatomic layers of dichalcogenides,187 or the thermoelectric effect in magnetic silicene superlattices,188 or in graphene.189 190 191 192

The use of heat in quantum dots and qbits is still in its infancy. Nonetheless, it is encouraging that spin polarization of the conduction band of silicon can be induced by imposing a heat current to a ferromagnetic contact on silicon.193 We note also that it ought to be possible to pump spins by heat into quantum dots. 12 One might also control the spin Seebeck current by inserting a quantum dot (QD) between the metal lead and magnetic insulator.194 Lately, there are quite some advance in combining topological insulator (TI) with spintronics, e.g. TI/antiferromagnet heterostructures195. Most of TI are also excellent thermoelectric materials, such as Bi_2Te_3. Therefore, we expect further spin caloritronic research based on topological insulators.

Large scale conferences give a sense that one of the latest hot topics in spintronics is the exploitation of the properties of antiferromagnets for spintronics. The natural frequency of antiferromagnets is determined by exchange energies, thus, resonances are observed in the range of 0.3 to 3 THz, roughly.196197198199200 Thus, spintronics with antiferromagnets offer the prospect of creating compact, easily manufactured, spin torque oscillators at such frequencies. Magneto-optical manipulation of the magnetization open possible routes for applications, as evidenced by THz emission experiments.201 Most of the thermodynamic formalism reviewed here applies for conductive materials and could be applied to analyze phenomena observed in conductive antiferromagnets such as $\mathrm{Mn_2As}$, 202 or in semimetallic antiferromagnets like the Heusler alloy CuMnSb.203 Spin-orbit torques offer the prospect of fast switching.204 As for thermal effects in antiferromagnets, it has been shown already that laser-induced spin-reorientation phase transition are possible in orthoferrites, for example.205 206 The thermodynamics formalism (section 1.5), which leads to the magnetic Seebeck effect in ferromagnets,32 can also be applied to a system composed of the two sublattices of an antiferromagnet, which are subjected to a temperature gradient. Hence, a thermal spin torque induced by a thermal gradient can be expected in antiferromagnets. Lin et al. very recently shows that a thermally induced spin current can be enhanced by an additional antiferromagnetic layer.207 However, this finding is controversial, as others do not find that the presence of an antiferromagnetic layer enhances the spin Seebeck effect 208. The discrepancy may be due to differences in the exact conformation and composition of the interface, as interfacial spin dependent transport plays a critical role in the spin Seebeck effect. The longitudinal spin Seebeck effect using antiferromagnets has been observed by S. Seki et al. 209 and by S. M. Wu et al. 210.

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