SHEAR STRESS IN THE INSULATION AT THE CONDUCTOR ENDS OF A COIL

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1) INTRODUCTION

The tensile stress in a coil resulting from the hoop force is transferred through the conductor at the coil ends (connections) where it induces a shear stress in the insulation layer. A 1-D model is used to predict this shear and to derive equations to design the coil ends so as to maintain the shear within tolerable limits. With the design planned for the TCV coils, the maximum shear should not be larger than 15 MPa.

2) 1-D MODEL

![Diagram of 1-D model of the conductor end of the coil.]

Fig. 1 1-D model of the conductor end of the coil.

1 : Copper conductor of X-section = w h
2 : Epoxy-fiberglass insulation layer of thickness = t
3 : Main body of the coil

The main body of the coil (region 3) is under a constant tensile stress $\sigma_3$. The conductor end is glued over a contour of length $p$ (usually $p = w + h$) of constant thickness $t$. The stress $\sigma_3$ induces a stress $\sigma_1(x)$ in the conductor end (region 1) and a shear $\tau_2(x)$ in the insulation (region 2) near the end. $u_1(x)$ and $u_3(x)$ are the deformation of the conductor end and of the coil.
By definition, the stresses and the shear are given by:

\[ \sigma_1 = Y_c \cdot \frac{du_1}{dx} \]  

\[ \sigma_3 = Y_c \cdot \frac{du_3}{dx} \]  

\[ \tau_2 = G_e \cdot \frac{u_1 - u_3}{l} \]  

The tensile stress \( \sigma_3 \) is constant, therefore:

\[ u_3 = \frac{\sigma_3}{Y_c} \cdot x = \alpha \cdot x \]  

For a conductor with a constant \( X \)-section \( S \), the force balance on the element \( \Delta x \) gives:

\[ \frac{d\sigma_1}{dx} - \frac{\tau_2 \cdot p}{S} = 0 \]  

and from 3 the equation for \( \tau_2 \) is:

\[ \frac{d\tau_2}{dx} - \frac{G_e}{Y_c} \cdot \frac{(\sigma_1 - \sigma_3)}{t} = 0 \]  

The combination of the equations 1, 3, 4, 5 gives for \( u_1(x) \):

\[ \frac{d^2u_1}{dx^2} - k^2 \cdot u_1 = -k^2 \cdot \alpha \cdot x \]  

where \( k^2 = \frac{G_e \cdot p \cdot 1}{Y_c \cdot t \cdot S} \)  

The independent equations for \( \sigma_1 \) and \( \tau_2 \) are derived in the same way:

\[ \frac{d^2\sigma_1}{dx^2} - k^2 \cdot \sigma_1 = -k^2 \cdot \sigma_3 \]  

\[ \frac{d^2\tau_2}{dx^2} - k^2 \cdot \tau_2 = 0 \]  

The solution of the equation 9 with the boundary conditions:

\[ \sigma_1(x = 0) = \sigma_0 \quad ; \quad \sigma_1(x = \infty) = \sigma_3 \]
is:
\[ \sigma_1 = \sigma_3 - (\sigma_3 - \sigma_0) \cdot e^{-kx} \]  
(12)

where \( \sigma_0 \) is the residual stress at the end of the conductor. This stress is determined by the volume and geometry of the epoxy filling at the end.

From equation 5, \( \tau_2 \) is:
\[ \tau_2 = \frac{k \cdot S}{p} \cdot (\sigma_3 - \sigma_0) \cdot e^{-kx} \]  
(13)

The deformation \( u_1 \) is obtained from equations 3 and 4.
\[ u_1 = \frac{1}{k} \cdot \frac{\sigma_3 - \sigma_0}{Y_c} \cdot e^{-kx} + \frac{\sigma_3}{Y_c} \cdot x \]  
(14)

The maximum shear in the coil insulation is obtained from the equation 13 for \( x = 0 \).
\[ \tau_{\text{max}} = \frac{k \cdot S}{p} \cdot (\sigma_3 - \sigma_0) = \sqrt{\frac{G_e \cdot S}{Y_c \cdot p \cdot t}} \cdot (\sigma_3 - \sigma_0) \]  
(15)

This simple formula gives a criterion (\( \tau_{\text{max}} < 15 \text{ MPa} \)) to find out whether a coil needs a specific design of its connections to withstand the shear stress in the insulation.

2) APPLICATION TO THE TCV CENTRAL COLUMN

**COIL A**

\[ S = 491 \text{ mm}^2 \]
\[ h = 19.8 \text{ mm} \]
\[ p = 50 \text{ mm} \]
\[ Y_c = 1,2 \times 10^{11} \text{ Pa} \]
\[ \sigma_3 = 85 \times 10^6 \text{ Pa} \]
\[ G_e = 3 \times 10^9 \text{ Pa} \]
\[ w = 28.5 \text{ mm} \]
\[ t = 2 \text{ mm} \]
\[ \sigma_0 = 0 \text{ (This gives the maximum \( \tau \))} \]

\[ k = \sqrt{\frac{G_e \cdot p}{Y_c \cdot t \cdot S}} = 36 \text{ m}^{-1} \] \[ \text{and} \]
\[ \tau_{\text{max}} = 0.35 \sigma_3 = 30 \text{ MPa} ! \]

The region with a shear \( \geq 10 \text{ MPa} \) is only for \( x \leq x_c \) with
\[ x_c = \frac{1}{k} \cdot \ln \frac{\tau_{\text{max}}}{\tau} = 3 \text{ cm} ! \]
COIL E

\[ S = 168 \, \text{mm}^2 \]
\[ h = 13 \, \text{mm} \]
\[ p = 28 \, \text{mm} \]
\[ Y_c = 1.2 \times 10^{11} \, \text{Pa} \]
\[ \sigma_3 = 80 \times 10^6 \, \text{Pa} \]
\[ w = 15 \, \text{mm} \]
\[ t = 2 \, \text{mm} \]
\[ G_e = 3 \times 10^9 \, \text{Pa} \]
\[ \sigma_0 = 0 \] (This gives the maximum \( \tau \))

\[ k = \sqrt{\frac{G_e \cdot p}{Y_c \cdot t \cdot S}} = 46 \, \text{m}^{-1} \quad \text{and} \quad \tau_{\text{max}} = 0.28 \sigma_3 = 22 \, \text{MPa} ! \]

The region with a shear \( \geq 10 \, \text{MPa} \) is only for \( x \leq x_c \) with

\[ x_c = \frac{1}{k} \cdot \ln \frac{\tau_{\text{max}}}{\tau} = 1.7 \, \text{cm} ! \]

3) EQUATIONS WITH VARIABLE CROSS SECTION.

If the cross section \( S \), the cemented contour length \( p \), and the thickness of insulation \( t \) are variable along the coil ends \( x \leq \lambda \) then the equations 1 to 4 are still valid and equations 5 and 6 become:

\[ \frac{d \sigma_1}{dx} + \sigma_1 \cdot \frac{1}{S} \cdot \frac{dS}{dx} \cdot \frac{p}{S} \cdot \tau_2 = 0 \quad \text{(16)} \]

\[ \frac{d \tau_2}{dx} + \tau_2 \cdot \frac{1}{t} \cdot \frac{dt}{dx} \cdot \frac{G_e}{Y_c} \cdot \frac{(\sigma_1 - \sigma_3)}{t} = 0 \quad \text{(17)} \]

where \( S, p, \) and \( t \) are functions of \( x \) in the region \( x \leq \lambda \). \( t \) is constant over the contour length \( p \) to avoid stress concentration.

In general, these equations have to be solved numerically for \( x \leq \lambda \) with the boundary conditions:

\[ \sigma_1(x = 0) = \sigma_0 \quad ; \quad \sigma_1(x = \lambda) = \sigma_\lambda \quad \text{(18)} \]

The solution in the region \( x > \lambda \) are those of equations 9 and 10 with the boundary conditions:

\[ \sigma_1(x = \lambda) = \sigma_\lambda \quad ; \quad \sigma_1(x = \infty) = \sigma_3 \quad \text{(19)} \]

which gives:
\[ \sigma_1(x) = \sigma_3 \cdot (\sigma_3 - \sigma_\lambda) \cdot e^{-k(x - \lambda)} \quad 20) \]

\[ \tau_2(x) = \frac{k \cdot S}{p} \cdot (\sigma_3 - \sigma_\lambda) \cdot e^{-k(x - \lambda)} \quad 21) \]

The boundary value \( \sigma_\lambda \) is related to \( \tau_\lambda \) by:

\[ \sigma_\lambda = \sigma_3 - \frac{p}{k \cdot S} \cdot \tau_\lambda \quad 22) \]

where \( \tau_\lambda \) should be set to a value of the order of 10 MPa.

Remarks:
- For manufacturing reasons, it is preferable to look for solutions where \( t \), the insulation thickness, is constant along the conductor ends.
- The tensile stress in the insulation at the conductor ends (\( \sigma_0 \)) must also be kept at a reasonable level. This is particularly important if the insulation thickness increases toward the end.
- To include \( \sigma_0 \) in the solution of the equations 16 and 17 it is necessary to proceed by iteration. First the equations are solved with \( \sigma_0 = 0 \) and \( \psi_1(0) \) is calculated with the equations 3 and 4. From this deformation \( \sigma_0 \) is evaluated taking into account the geometry of the epoxy filling at the end. The equations are then solved again with this new \( \sigma_0 \). This procedure is repeated until a stable solution is found.
- To avoid large electric fields, the conductor ends must not have sharp edges.

4) CONCLUSION FOR THE TCV CENTRAL COLUMN

At the maximum tensile stresses evaluated for the TCV central column, the shear stresses reach high values along the last few centimeters at the ends of the conductors. It is therefore necessary to design the coil feeds so as to keep these shears below 10 to 15 MPa. This can be achieved by a specific tailoring of the copper conductor cross section and or the insulation thickness at the coil feeds.