Scale Effects in Modelling Two-phase Air-water Flows

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ABSTRACT: Beside analytical approaches, physical modelling represents probably the oldest design tool in hydraulic engineering. For free surface flows, the similitude according to a Froude similarity allows for a correct representation of the dominant forces, namely gravity and inertia. In parallel, fluid constants such as the surface tension and the viscosity might be incorrectly reproduced, affecting the air entrainment and transport capacity of a high-speed model flow. Consequently, small physical models operated under the Froude similitude systematically underestimate the air entrainment rate and air-water interfacial properties. To limit this deficit, minimal values of the Reynolds or Weber number have to be respected. The paper summarises the physical background of these limitations and their combination in terms of the Morton number. Based upon a literature review, these limits are listed and discussed, resulting in a series of more conservative recommendations in terms of air concentration scaling.

KEY WORDS: Air, Hydraulic structures, Physical modeling, Scale effects, Two-phase flow, Water.

1 INTRODUCTION

The spillway tunnel on the Arizona side of the Boulder Dam (USA) operated during four months in 1941 with a relatively small discharge. A routine inspection after this operation period indicated a 35 m long tunnel section which was completely destroyed as a consequence of cavitation. To repair the damage and to avoid similar phenomena in future, a re-lining combined with the installation of – then not yet common – aeration devices was recommended. Consequently, the US Bureau of Reclamation studied several ”air injection” devices in a 60:1 scale model (Bradley 1945), considering among others typical deflectors serving today as spillway aerators. This study certainly represents a pioneer work in chute aeration studies. Nonetheless the results seemed rather disappointing. Bradley concluded: “*The results of this investigation were negative in character. The plan for aeration of the Boulder Dam spillway by devices constructed on the tunnel invert* […] *does not appear encouraging*”. The statement derived from the fact that only very small, insufficient air concentrations were measured in the physical model designed based upon a Froude similitude. Bradley realized that “*the viscosity of the water and air are same in both model and prototype, and that entrainment of air from the surface will be much more pronounced in the prototype*”.

Air-water two-phase flows are observed in several hydraulic structures, such as hydraulic jumps, intakes, dropshafts, spillways, jets and plunge pools. The flow phenomena in these structures are challenging, and yet relevant to hydraulic design. Physical model testing is often applied to investigate the related flow characteristics. In order to keep the physical models within economic dimensions and to minimize the discharges to supply, Froude similitude and scale factors larger than 30:1 are often applied. These small models characteristically underestimate the air entrainment and transport in the fluid, because the effects of surface tension and viscosity are relatively over-represented in the model, given that water

is used as fluid in both model and prototype. This is illustrated in Fig. 1, showing a prototype opertion (Fig. 1a), two scale models at 10:1 and 25:1 geometric scale (Fig.1b) The literature describes mainly two approaches to combine the Froude similitude with a reasonable approximation of the rate of air entrainment, namely to (a) limit the model scale to maximally 10:1 (e.g. Chanson 1997, Boes 2000), or (b) respect minimum values of the Weber and/or Reynolds numbers (e.g. Pfister and Chanson 2012). Both approaches allow more realistic predictions of air entrainment and transport based upon the scaled model results. These limits are derived from model families or comparisons with prototype measurements.

Early studies of the bubble rise velocity in stagnant fluids highlighted the relevant dimensionless numbers, namely the Reynolds and Weber numbers. The results suggested some limiting value of them, above which the effect of fluid constants on the bubble motion is small. In these contributions, the introduction of the Morton number, as a link between the aforementioned numbers, allows for an expression of these limiting values as function of the Froude number. This concept is applied on the base of scaling limitations published in the literature, while the notion of scale effects is discussed in a broader context.

2 BUBBLE RISE VELOCTIY IN STAGNANT FLUID

The motion of a rising bubble in a stagnant fluid is dominated by the physical constants of the fluid and of the gas, namely:

ν = kinematic viscosity [m^2/s]

ρ = density [kg/m^3]

σ = surface tension [N/m]

The first two variables must be theoretically considered for both gas and fluid phases. For pressures which are far from the critical point, the forces within a gas bubble might be neglected as they are small compared to those within the liquid. Note that the critical point describes the condition at which the phase boundary between fluid and gas terminates. For water, the latter is at around 374 °C and 22 MPa. These conditions are unlikely in hydraulic engineering, and the gas (herein air) constants are of minor significance.

The motion of gas bubbles in a fluid is governed by buoyancy, resulting from the density difference between gas and fluid. If the gas density is negligible in comparison to that of the fluid, as for air and water, the buoyancy is a function of the pressure gradient $\partial P/\partial z$, hence the gravitational acceleration g, where z is the vertical elevation. Buoyancy depends further upon the bubble volume which is typically linked to an equivalent sphere diameter D. As soon as a bubble moves relative to the surrounding fluid, the latter generates a certain resistance, which is linked to the bubble (subscript b) rise velocity V_b. Schmidt (1934) conducted a dimensional analysis of the bubble motion in fluids. He concluded that the related processes can be described based on the following dimensionless numbers

bubble Reynolds number $$\mathsf{R}_b = \frac{V_b D}{\nu} \tag{1}$$

bubble Froude number $$\mathsf{F}_b = \frac{V_b}{\sqrt{gD}} \tag{2}$$

bubble Weber number $$\mathsf{W}_b = \frac{\rho V_b^{\,2} D}{\sigma} \tag{3}$$

The bubble rise velocity V_b was included in all numbers, and it could not be derived simply from dimensionless charts based upon R_b, F_b and W_b. A re-arrangement was thus proposed to give only one explicit term containing V_b, namely that of the bubble Reynolds number R_b. Schmidt suggested:

$$\mathsf{R}_b = \frac{V_b D}{\nu} \tag{4}$$

(a)

(b)

Figure 1 Comparison of air-water flows between prototype and laboratory model operations of stepped spillways; (a) Paradise Dam stepped spillway (Australia) in operation on 5 March 2013 (Photographs H. Chanson): $Q \approx 2{,}500$ m^3/s, $h_c/h = 2.9$, $\mathsf{R} = 7\times10^6$, $h = 0.62$ m, $\theta = 57°$ - View from downstream (left) and details of air-water flow structures next to the inception point of free-surface aeration (right); (b) Laboratory experiments (Courtesy of Mr P. Royet, IFSTTAR) – Left: $Q = 0.40$ m^3/s, $h_c/h = 3.2$, $\mathsf{R} = 2.6\times10^5$, $h = 0.060$ m, $\theta = 53°$; Right: $\mathsf{R} < 4\times10^4$, $h = 0.024$ m, $\theta = 63.4°$, 59° & 53° (from left to right)

$$\frac{\mathsf{W}_b}{\mathsf{F}_b^{\,2}} = \frac{D^2 \rho g}{\sigma} \tag{5}$$

$$\frac{\sqrt[3]{\mathsf{F}_b^{\,2}\mathsf{R}_b^{\,4}}}{\mathsf{W}_b} = \frac{\sigma}{\rho\sqrt[3]{g\nu^4}} \tag{6}$$

Equation (4) describes *a priori* the bubble characteristics including its diameter D and rise velocity V_b. Equation (5) includes the bubble diameter, fluid constants and gravity acceleration, while Eq. (6) contains exclusively fluid constants and gravity acceleration. Consequently the bubble rise velocity in a stagnant, infinite fluid volume was expressed by Schmidt (1934) as:

$$\frac{V_b D}{\nu} = f\left[\frac{D^2 \rho g}{\sigma}, \frac{\sigma}{\rho \sqrt[3]{g \nu^4}}\right] \tag{7}$$

Schmidt concluded his investigation with the remark that physical experiments had to be conducted to support the above mentioned hypotheses, in particular with different fluids. Habermann and Morton (1954) conducted such experiments related to the drag coefficient of freely rising air bubbles in different stagnant fluids under various temperatures. The following liquids were tested: water (6, 19, 21, and 49 °C), glim solution, mineral oil, varsol, turpentine, methyl alcohol, olive oil, syrup, different corn syrup-water mixtures, glycerin-water mixtures, and an ethyl alcohol-water mixture. Equation (6) was adapted by introducing the dimensionless parameter $\mathsf{M} = g\mu^4/(\rho\sigma^3)$ where μ the dynamic viscosity. Consequently, Eq. (6) was rewritten:

$$\mathsf{M} = \frac{\mathsf{W}^3}{\mathsf{F}^2 \mathsf{R}^4} \tag{8}$$

The tests of Habermann and Morton (1954) covered $0.2\times10^{-2} \leq \mathsf{M} \leq 0.3\times10^{-11}$.

With increasing bubble size, a change in rising bubble shape was observed, from spherical to ellipsoidal, and finally to spherical cap, for all liquids. The bubble volumes at which the transition occurred were a function of the fluid properties. Thus the dimensionless number M had to be a constant to generate a similar bubble behaviour. The reasoning may be applied for similar bubbles in different fluids, as well as for a single fluid including bubbles under different scale factors. The latter is a common situation in physical modelling of high-velocity free-surface flows in hydraulic structures, which encompass two-phase air-water flows. In these air-water flows, the fluid constants remain basically unchanged since air and water are used in both prototype and model, whereas other characteristic values linked to geometry and force ratios vary. Note however that Habermann and Morton (1954) concluded that the air bubble motion at the terminal velocity cannot be described solely by the dimensionless numbers presented in Eqs. (4) to (6), because the related drag coefficient was also a function of the gas phase motion within the bubbles.

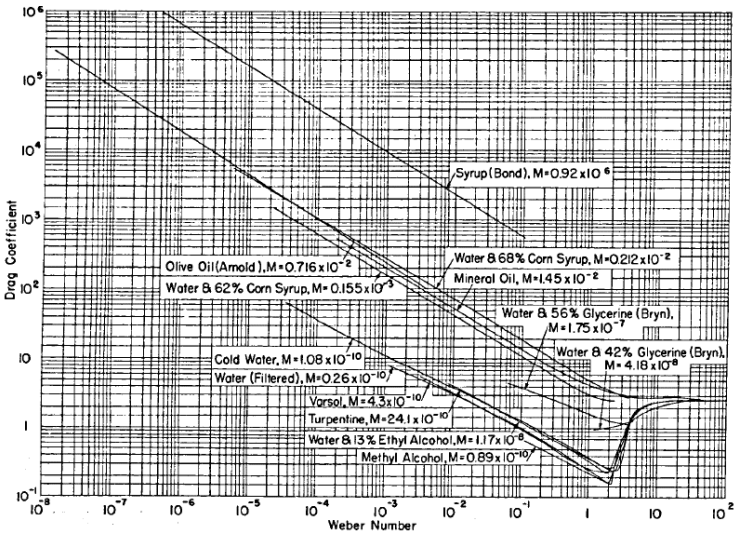

Figure 2 Drag coefficient as a function of the bubble Weber number W_b for a single bubble rising at terminal velocity in various stagnant fluids (Habermann and Morton 1954)

An example for the effect of M on the rise bubble drag coefficient is shown in Fig. 2. Although no clear trend is recognizable, "large" values of M (10^{-2}) tended to relatively large drag coefficients, whereas "small" values of M (10^{-11}) indicated rather small drag coefficients. Interestingly, however, for $\mathsf{W}_b > 40$, all drag coefficient curves seemed to collapse, independent of M. A similar trend was seen in terms of the bubble Reynolds number R_b, where the data collapsed for $\mathsf{R}_b > 3\times10^3$. The latter would imply that the

terminal bubble rise velocity of air bubbles is similar when the equivalent bubble diameter is larger than some 10 to 20 mm. In practice, the air bubble rise velocity in water tends to be constant for bubble diameters between 1 and 20 mm, with increasing rise velocity with augmenting bubble size for $D > 20$ mm (Comolet 1979).

3 CONCEPTUAL ANALOGY TO AIR-WATER MIXTURE-FLOW

The section 2 shows the first investigations into a description of the effect of fluid constants and bubble characteristics on their motion, although it did not attempt to reflect upon the current state-of-the-art on bubble motion in fluids. In physical modelling of hydraulic structures, the behaviour of a single air bubble in stagnant water is rarely of interest. Typically, the study focus is the flow characteristics of air-water mixture-flows as a continuum, for example in stepped spillways, hydraulic jumps, free water jets, steep chutes, and dropshafts. The knowledge of the air bubble entrainment and transport is essential to describe the flow properties, including the adequate free-board height, jet disintegration, friction losses, and air-water mass transfer rate (Rao and Kobus 1971, Wood 1991, Chanson 1997).

Most physical models are kept within economical dimensions, implying a Froude similitude with a geometric scale ratio of typically 30:1 to 60:1 (Novak and Cabelka 1981, Chanson 1999). The dynamic similitude used to derive model-scaling laws considers the ratios of forces acting on the fluid(s). The ratio of inertia to gravity forces results in the Froude number; the ratio of inertia to viscous forces gives the Reynolds number; the ratio of inertia to surface tension forces yields the Weber number. A true dynamic similarity of aerated flows require achieving identical Froude, Reynolds and Weber numbers in both prototype and model. This is physically impossible when the same fluids (air, water) are used in both prototype and model. As a consequence, small scale models based upon the Froude similitude may underestimate the air transport in the fluid, because the relative effects of surface tension and viscosity are over-represented (Wood 1991, Chanson 2009). Since a true dynamic similitude exists only at full-scale, the underestimation of scale model air entrainment and transport must be minimized in modelling practice by limitations in terms of W or R. These limits are derived from systematic model families and comparisons with prototype data.

Such limits represent a link between the above description related to the bubble rise velocity (section 2) and observations from model families (section 4). Following the results for single bubbles in a stagnant fluid (Schmidt 1934, Habermann and Morton 1954), a similar concept is proposed herein to adequately model an air-water mixture-flow continuum under a reduced geometric scale factor. The relevant dimensional numbers are:

Reynolds number $$\mathsf{R} = \frac{Vh}{\nu} \tag{9}$$

Froude number $$\mathsf{F} = \frac{V}{\sqrt{gh}} \tag{10}$$

Weber number $$\mathsf{W} = \frac{\rho V^2 h}{\sigma} \tag{11}$$

with V as flow velocity, and h as equivalent clear-water flow depth. Note that the dimensionless parameter M is denoted the Morton number. For air-water two-phase flows: $\mathsf{M} = 3.89\times10^{-11}$ using the fluid constants at 15 °C.

4 LIMITING FACTORS

A number of model families and comprehensive data sets were published to test scale effects in the modelling of air-water two-phase flows. Table 1 summarises a number of related literature, leading to some suggested limiting criteria of about $\mathsf{W}^{0.5} > 110$ to 170 and $\mathsf{R} > 1\times10^5$ to 3×10^5 when the relevant scaling parameter is the air concentration. For example, Pfister and Hager (2010a,b) identified a gross underestimate in terms of local bottom air concentration by up to one magnitude when $\mathsf{W}^{0.5}<140$. In Fig. 3, the abscissa corresponds to the streamwise normalizations f_D and f_S given by these authors, and the trend lines correspond to the best fit of all data from tests with $\mathsf{W}^{0.5}\geq140$.

Despite the relatively limited scope of the experimental investigations listed in Table 1, their results demonstrated unequivocally the limitations for the physical modelling of two-phase air-water flows. The findings of these systematic experimental studies highlighted that (a) the notion of scale effects must be defined in terms of some specific set of two-phase air-water characteristics, and (b) some aerated flow properties are more affected by scale effects than others, even in large-size facilities. The selection of the criteria to assess scale effects is critical: e.g., void fraction, turbulence intensity, bubble size. Any mention of scale effects must be associated with a list of tested parameters (Chanson 2009, Chanson and Chachereau 2013), and this is well-known in mono-phase flows (Schulz and Flack 2013). The experimental data show that some parameters, such as bubble sizes and turbulent scales, are likely to be most affected by scale effects (Chanson 2004b, 2009, 2013). It is noteworthy that no distorted physical modelling of air-water flows was considered yet, although the scale distortion may enable to achieve some similarity in terms of bubble rise velocity on chute spillways and inclined plunging jets.

Table 1 Limiting scale factors to prevent significant scale effects in two-phase air-water flows under Froude similitude, with a focus on air concentrations (also denoted as void fraction) for undistorted air-water scale models

Reference	*Criterion*	*Air-water flow parameter(s)*	*Application range*
Kobus (1984)	$R \geq 1.0\times10^5$	Air transport rate	Chute air entrainment
Koschitzky (1987)	$R \geq 1.0\times10^5$	Air demand flow rate	Aerator, particularly β
Rutschmann (1988)	$W^{0.5} \geq 110$	Air demand flow rate	Aerator, particularly β
Skripalle (1994)	$W^{0.5} \geq 170$	Air demand flow rate	Aerator, particularly β
Boes (2000)	$R \geq 1.0\times10^5$	Void fraction and interfacial velocity	Two-phase stepped spillway flow (30° & 50° chutes)
Chanson (2004a)	$R > 1.4\times10^5$ (*)	Void fraction, bubble count rate, bubble chord time, particle residence time	Dropshaft
Chanson et al. (2004)	$W^{0.5} \geq 32$	Void fraction, bubble count rate	Circular plunging jets
Chanson and Gonzalez (2005)	$R \geq 3\times10^5$ (*)	Void fraction, interfacial velocity, bubble count rate, turbulence intensity, bubble chord size	Two-phase stepped spillway flow (3.4° & 16° chutes)
Murzy and Chanson (2008)	$R > 1.0\times10^5$ (*)	Void fraction, interfacial velocity, bubble count rate, turbulence intensity, bubble chord time	Hydraulic jump
Felder and Chanson (2009)	$R > 2.5\times10^5$ (*)	Void fraction, interfacial velocity, bubble count rate, turbulence intensity, integral turbulent time scale, bubble chord size	Two-phase stepped spillway flow (22° chute)
Pfister and Hager (2010a)	$R \geq 2.2\times10^5$, $W^{0.5} \geq 140$	Void fraction	Chute aerator, C_b development
Chanson and Chachereau (2013)	$R > 1.3\times10^5$ (*)	Void fraction, interfacial velocity, bubble count rate, turbulence intensity, integral turbulent time scale, bubble chord size	Hydraulic jump
Felder (2013)	$R > 2.5\times10^5$ (*)	Void fraction, interfacial velocity, bubble count rate, turbulence intensity, integral turbulent time scale, bubble chord size, bubble clustering	Two-phase stepped spillway flow (9° & 26° chutes)

(*): incomplete criterion since an asymptotic result was not achieved

Self-similarity is another powerful tool in turbulent air-water flow investigations involving a wide spectrum of spatial and temporal scale. Self-similarity is closely linked to dynamic and kinematic similarities, and the existence of self-similar relationships may have major implications on the measurement strategy in experimental and physical modelling studies (Barenblatt 1996, Foss et al. 2007). Although it is nearly impossible to achieve a true dynamic similarity in air-water flows because of the too many relevant dimensionless parameters, a number of laboratory data showed a number of self-similar relationships that remain invariant under changes of scale. The results may provide a picture general enough to be used, as a first approximation, to characterise the air-water flow properties in similar hydraulic structures irrespective of the physical scale (Felder and Chanson 2009).

In addition to dynamic similarity and self-similarity, a further modelling approach may be based upon some theoretical developments leading to theoretically-based equations. An illustration is the analytical solution of the advection diffusion equation for air bubbles (Wood 1984, 1991, Chanson 1997, 2008). The existence of theoretical relationships may have some implications regarding the laboratory study approach and measurement methods. The existence of an analytical solution may allow a drastic reduction of the amount of measurements.

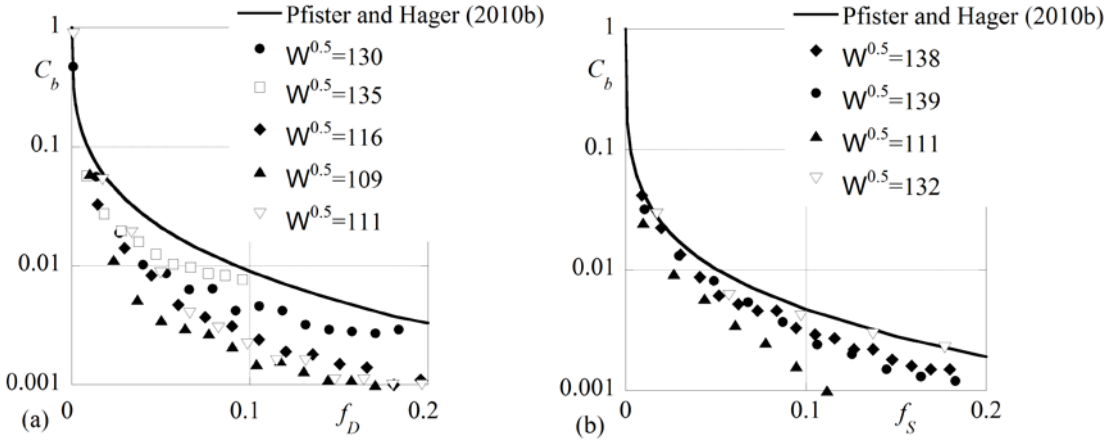

Figure 3 Bottom air concentration C_b curves versus normalization functions *f*, downstream of (a) deflector, (b) drop aerators, with trend line for unaffected tests (–) and symbols for tests affected by scale effects (after Pfister and Chanson 2012)

5 DISCUSSION

Two criteria are often applied relating to scale effects, i.e. limiting values for $W^{0.5}$ and R (Table 1). When the same fluids (air and water) are used in prototype and model, the two numbers depend on each other, beside F and the Morton number M (Eq. (8)). The use of the Froude similitude with water as liquid in both model and the prototype leads to

- M= constant (Eq. (8)), and
- F is identical in model and prototype.

For a given F, the product $MF^2 = W^3/R^4$ has to be identical in the model and the prototype flow. A transformation of Eq. (8) gives the direct relationship between R and W as

$$R = \left(\frac{W^3}{F^2 M}\right)^{1/4} \tag{12}$$

Inserting the limitations $W^{0.5}$ = 110, 140 and 170 into Eq. (12), Fig. 4 presents the results in the related F-R curves. For typical high-speed air-water chute flows with $5 \leq F \leq 15$, scale effects related to air concentrations are small when $W^{0.5} > 140$ or $R > 2\times10^5$ to 3×10^5. The limits are not sensitive to F in the aforementioned range, whereas more restrictive limitations in terms of R have to be applied for smaller values of F. Note that only one limitation in terms of R or $W^{0.5}$ has to be considered if applying Eq. (12), since the other is implicitly also respected.

Table 1 suggests that the proposed limiting relationships have become more restrictive over time. The trend might be linked to the development in measurement techniques, allowing for more precise and punctual (instead of cross-sectional) two-phase air-water flow measurements. Ultimately no scale effect is observed at full scale only, using air and water in prototype and model: i.e., in prototype flow conditions. But prototype observations are rare. The Aviemore Dam spillway investigations in New Zealand remain a key reference (Keller 1972, Cain 1978). A few prototype observations were conducted, mostly qualitative like at Dachaoshan Dam spillway (Lin and Han 2001). But even the Aviemore Dam spillway data sets might be challenged. The flow conditions corresponded to $R \approx 2\times10^6$, which is one to two orders of magnitude lower than the design flow conditions of very large spillway systems. A number of recent

air-water studies on dynamic similarity would suggest that the extrapolation of Aviemore Dam results could be subjected to some scale effects at larger Reynolds numbers. Figures 1 and 5 provide some comparative illustrations of prototype and laboratory air-water two-phase flows. Figure 1 presents some air-water skimming flow above a stepped spillway. The close-up photographs suggests that the turbulence next to the inception point of free-surface aeration differs significantly between prototype and models for a comparable Froude number. Figure 5 shows a hydraulic jump stilling basin in operation. In the prototype (Fig. 5a), the hydraulic power dissipated in the hydraulic jump as 6 MW per unit width, compared to 230 W/m in the laboratory model (Fig. 5b) for an identical Froude number. Again the surface turbulence appears to substantially different despite an identical Froude number.

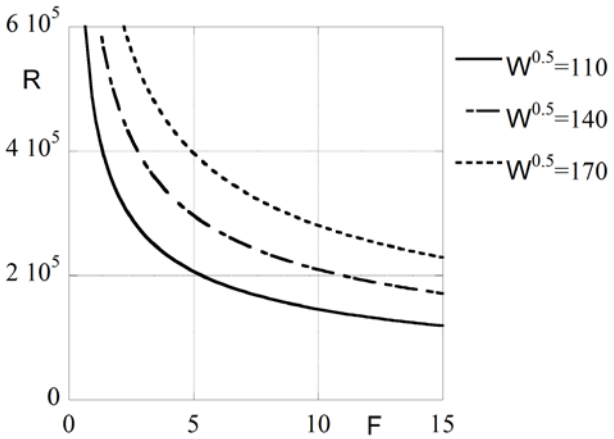

Figure 4 Relationship between R and F for different values $W^{0.5}$ (Eq. (12)) (after Pfister and Chanson 2012)

(a)

(b)

Figure 5 Comparison of air-water flow features between prototype and laboratory model operations of hydraulic jumps; (a) Hydraulic jump stilling basin downstream of Paradise Dam spillway (Australia) in operation on 30 December 2010 (Courtesy of B. Chanson): $Q \approx 6{,}300$ m^3/s, F = 8, R = 2×10^7; (b) Laboratory experiment: Q = 0.030 m^3/s, F = 8, R = 6×10^4

6 CONCLUSION

The physical modelling of air-water two-phase flows in hydraulic engineering would require the Froude number F, Weber number W and Reynolds number R to be identical in prototype and laboratory. But this is physically impossible. A re-arrangement of the dimensionless numbers, as proposed by Schmidt (1934), resulted in the introduction of the Morton number M. The results of previous investigations served as basis to the herein described scale effects in modelling high-speed two-phase flows, namely (a) the significance of the non-dimensional numbers F, W and R, (b) their combination to M, and (c) some limiting values of R and W to reduce the effect of the latter. Combining these considerations together with published limits to minimise scale effects in terms of air concentration, the outcome indicated that values of $W^{0.5} > 140$ and $R > 2\times10^5$ to 3×10^5 should be respected to avoid relevant scale effects in terms of air concentrations within $5 \leq F \leq 15$. If one limitation is considered, then the other is implicitly respected. For $F < 5$, these limits have to be selected more conservatively.

The results of recent experimental investigations emphasised further that the selection of the criteria to assess scale affects is critical. These results showed that some parameters, such as bubble sizes and turbulent scales, are likely to be affected by scale effects, even in relatively large-size laboratory models (e.g. 2:1 to 3:1). No scale effect is only observed at full scale using the same fluids in prototype and model. As final words, the present study emphasises (again) the needs for full-scale prototype data of two-phase air-water flows, typically observed in prototype hydraulic structures.

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