A transformable tensegrity-ring footbridge

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Summary

Tensegrity structures are spatial reticulated structures composed of cables and struts. A tension-compression equilibrium leads to lightweight systems that change shape through length changes in their members. Active members thus control several degrees of freedom simultaneously. Tensegrity-ring modules are transformable circuit-pattern modules. The linear combination of tensegrity rings has been shown to be viable for a footbridge application. Shape transformations of a ¼ scale four-ring-module tensegrity-footbridge system are studied in this paper. Transformations are obtained employing active continuous cables and springs in the tensegrity system to reduce the number of active elements. Obtaining a desired shape may involve independent actuation in several active elements. Independent actuation steps are found with the combination of a dynamic relaxation algorithm and a stochastic search algorithm.

Keywords: *tensegrity; transformable; footbridge; active control; stochastic search*.

1. Introduction

Tensegrity structures are spatial reticulated structures composed of cables and struts in a self-equilibrium. Although tensegrity was introduced in the 1950s, there are few examples in structural engineering [1-5]. Tensegrity systems are attractive transformable units for adaptive structures as structural elements and actuators can be combined. Both active struts and cables have been used to control tensegrity structures [6-9]. Sultan and Skelton [10] showed that appropriate cable actuation allows tensegrity systems to remain close enough to their equilibrium manifold to maintain stiffness during shape transformations, contrary to traditional deployable systems. The position and the number of active cables depend on the chosen topology and application. However, such deployment usually requires a large number of active cables. The number of active cables can be reduced with the use of continuous cables. Moored and Bart-Smith [11] showed that employing continuous cables changes tensegrity mechanics resulting in a more generalized formulation than with discontinuous cables. Furthermore, continuous cables may result in remote actuation schemes of tensegrity systems as actuation devices can be detached from the structure and placed at the supports [12].

This paper focuses on shape transformations of a multiple-degree-of-freedom ¼ scale tensegrity-footbridge system using continuous active cables and springs. Shape transformations such as deployment and shape corrections are conducted using the same remote actuation scheme. Due to the complexity of the shape-transformation task and the large solution space, a stochastic search algorithm is combined with a modified dynamic relaxation algorithm to identify the right actuation steps for each active cable in order to obtain the desired shape transformation.

2. The Tensegrity-Ring Topology

There is a wide variety of tensegrity units that can be used to develop structural systems, including modules and assembly forms. However, not all tensegrity modules are suitable for structural applications as neither all modules nor assemblies result in transformable systems. The proposed tensegrity-footbridge system is based on four pentagonal tensegrity-ring modules assembled in a linear way as shown in Figure 1. Pentagonal tensegrity-ring modules belong to the circuit-pattern topologies [13]. The “ring module” characterization originates from Motro et al [14] who studied their ring-like topology as well as their ability to fold based on cable-length changes. When ring modules are assembled together in a linear form, they create a structural system resembling a hollow-rope where the empty space in the center can be used as walking space with the addition of a deck [14]. The hollow-rope assembly results in a transformable system that changes shape or deploys through cable actuation [15]. In this study, the deployment of the footbridge system is assumed to be from both sides.

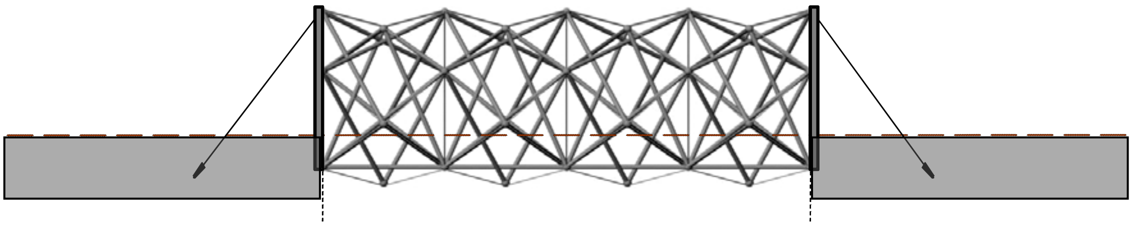


Figure 1: Illustration of the pentagonal tensegrity-ring footbridge

Pentagonal ring modules include a single strut circuit of 15 struts connected to 30 cables through 15 nodes. The nodes are arranged in three pentagonal layers. Figure 2 illustrates the topology of a pentagonal ring module. The strut circuit-pattern is thus composed of diagonal struts and intermediate struts. Diagonal struts connect the nodes on the external pentagonal layers of the module, while intermediate struts connect the nodes of the intermediate pentagonal layer with the external ones. Cables can also be grouped into two topology families: layer cables and x-cables. Layer cables are cables that connect the nodes of the external pentagonal layers. X-cables are the cables on the lateral faces of the module connecting the nodes of the intermediate pentagonal layer with the external ones. X-cables can be further distinguished in coplanar and non-coplanar cables according to their in-plane position in relation to diagonal struts.

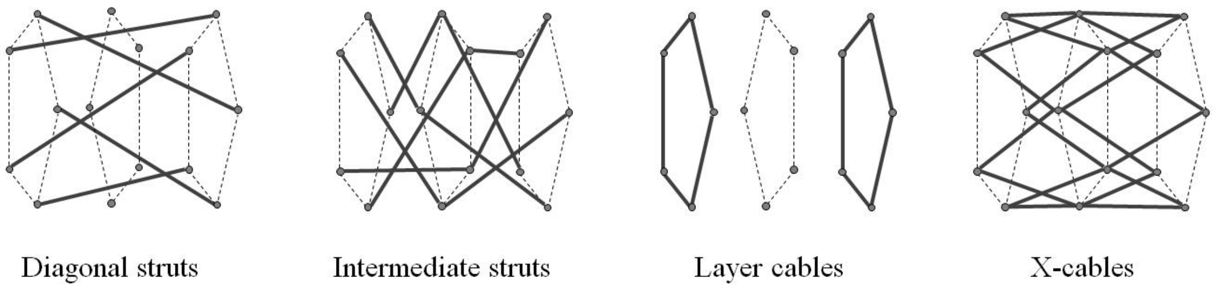


Figure 2: Pentagonal tensegrity ring module topology

Pentagonal ring modules are suitable for structural applications because they have no internal mechanisms and six self-stress states. Furthermore, the four-module tensegrity-system considered for the footbridge application has no internal mechanism and 36 self-stress states. The number of internal mechanisms and self-stress states is obtained through the study of the equilibrium matrix A defined as in [11] with rigid body movements constrained:

$$\bar{A} = C\tilde{g}\tilde{l}^{-1}S^{T}\tilde{\tilde{\bar{l}}} \tag{1}$$

where C is the connectivity matrix, g corresponds to the projection of element length in each Cartesian direction, l is the corresponding element length vector and S is the clustering matrix. The clustering matrix links continuous cables (clustered cables) to discontinuous cables. Hence, for traditional discontinuous systems it corresponds to an identity matrix with the size depending on the number of the elements in the system. The structural behaviour of the pentagonal tensegrity-ring module was studied in [16].

In this paper, continuous cables are applied only in non-coplanar x-cables. The 2 x-cables that are not coplanar with diagonal struts at each lateral face of the module are replaced by a continuous cable. In the linear “hollow-rope” system for the footbridge application, continuous cables run through two modules replacing thus 4 x-cables. Employing continuous cables affects the structural behavior of the system modifying the tensegrity equilibrium and reducing the kinematic constraints [11]. The study of the equilibrium matrix showed that the continuous pentagonal ring module has no internal mechanism and a single self-stress state. If continuous cables run in the half-footbridge system then the four-module hollow-rope system has no internal mechanisms and 6 self-stress states thus remaining suitable for structural applications. These results are summarized in Table 1.

Table 1: Mechanical characterization of the pentagonal tensegrity-ring module and footbridge system

	Module with discontinuous cables	Module with continuous cables	Footbridge system with discontinuous cables	Footbridge system with continuous cables
Rank of the equilibrium-matrix	39	39	129	129
N° of mechanisms	0	0	0	0
N° of self-stress states	6	1	36	6

3. Shape Transformations of the Tensegrity-Ring System

Tensegrity systems are stable systems and therefore require active elements to change their shape. If large shape transformations are required, cable actuation is usually more efficient than employing telescopic struts. The number of active cables required depends on the topology, the desired shape transformation and the actuation scheme applied.

The pentagonal tensegrity-ring module is deployable if cable lengths can be adjusted [14]. Deployment is defined as the transformation from a compact to an expanded form. Previous studies identified the contact-free deployment-path space of pentagonal ring modules as well as suitable actuation schemes [17]. The deployment motion involved is composed of a translation, a dilation and a rotation. The motion is similar to a helix elongation: when a helix is elongated, it twists and its internal space shrinks (Figure 3). Thus, during deployment the length of x-cables (lateral cables) increases while the length of layer cables (front and back side cables) decreases. Consequently, there is an inverse relationship between x-cables and layer cables with x-cables controlling folding while layer cables controlling unfolding. However, actions on both cable families are required to obtain stable configurations.

In order to simplify the design and control of the tensegrity system, continuous cables and spring elements are employed resulting in a 5-actuator system. Continuous active cables replace individually actuated x-cables thus reducing the number of actuators required. Furthermore, actuators can thus be placed on the supports since continuous cables run through the boundary nodes. Employing continuous active cables in the footbridge system thus eliminates actuator-related design constraints such as mass and volume of the actuation devices. Springs allow length changes without requiring any actuation devices. However, spring-length changes are driven by actions on other elements. In this study, springs replace layer cables so that their length changes are driven by actions of the continuous active x-cables as illustrated in Figure 3. Hence, springs are elongated in the folded configuration and contracted in the unfolded configurations allowing spring energy to be used for unfolding. Moreover, springs are thus in a low energy state in the unfolded configuration reducing the risk of energy bursts under service.

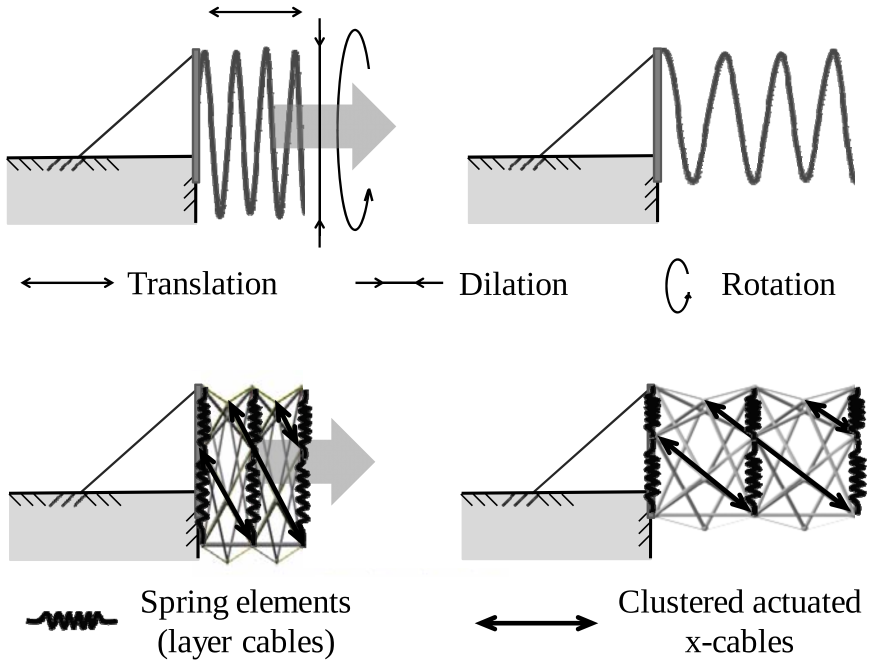


Figure 3: Deployment motion and actuation components of the tensegrity-ring footbridge-system

Although the shape transformation of deployment of the tensegrity-ring system involves modifying several degrees of freedom, it is found feasible with the same length change (actuation step) applied in all 5 active continuous x-cables. Consequently, a single actuator connected to the 5 active cables is sufficient for the deployment of each half of the tensegrity-footbridge system. However, single actuator configurations do not allow adjustments in the shape of the system unless the actuator controls individually every cable. Therefore, in order to allow a wider range of shape transformations, including shape corrections during deployment, the actuation scheme is composed of 5 independently active continuous x-cables for each half of the footbridge system. The feasibility of the deployment of the module with continuous active cables and springs was validated on a small scale physical model shown in Figure 4. For this prototype, deployment was successfully controlled using 5 hand cranks placed at the supports of the model.

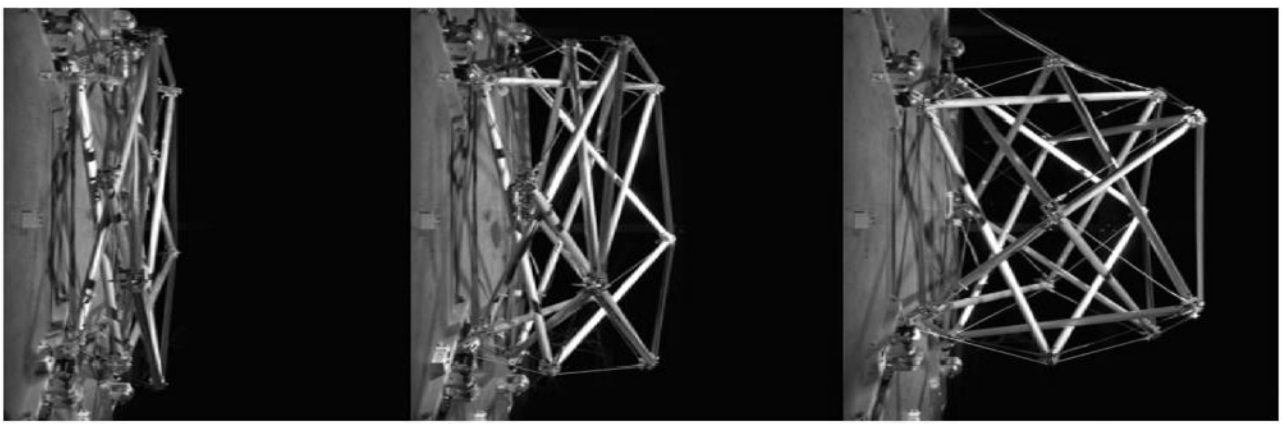


Figure 4: Snapshots from the deployment of a small scale ($^1/_{10}$) tensegrity-ring module

4. Analysis and Shape Control

In this paper, the structural analysis of the continuous tensegrity-ring system is performed using a modified dynamic relaxation algorithm that takes into account continuous cables [18]. The system studied next in this paper is a ¼ scale four-ring-module tensegrity footbridge made of steel hollow tubes, steel cables and springs as shown in Figure 5. The Young modulus is set at 210GPa and 115GPa for tubes and cables respectively. Details of the steel hollow tubes and steel cables are given in Table 2. Spring stiffness is set at 5% of cable stiffness with a rest length of 0.85m.

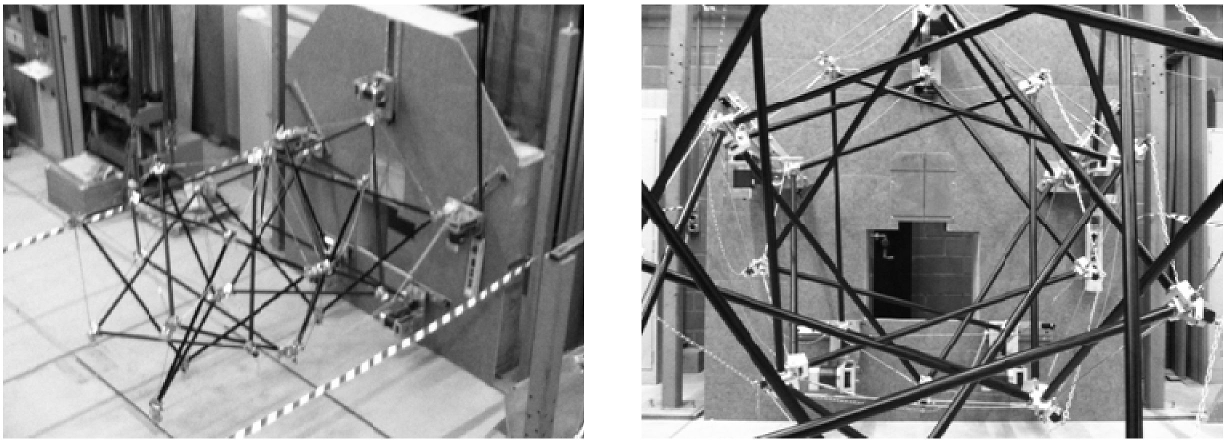


Figure 5: Pictures of the ¼ scale ring-module tensegrity footbridge system

Table 2. Details of the ¼ scale four ring-module tensegrity footbridge-model

Member type	Length [m]	Diameter [mm]	Thickness [mm]	Element strength [kN]	Self-stress [% element strength]
Struts	1.35	28	1.5	10.6	-
Cables	0.69	4	-	9	12.5

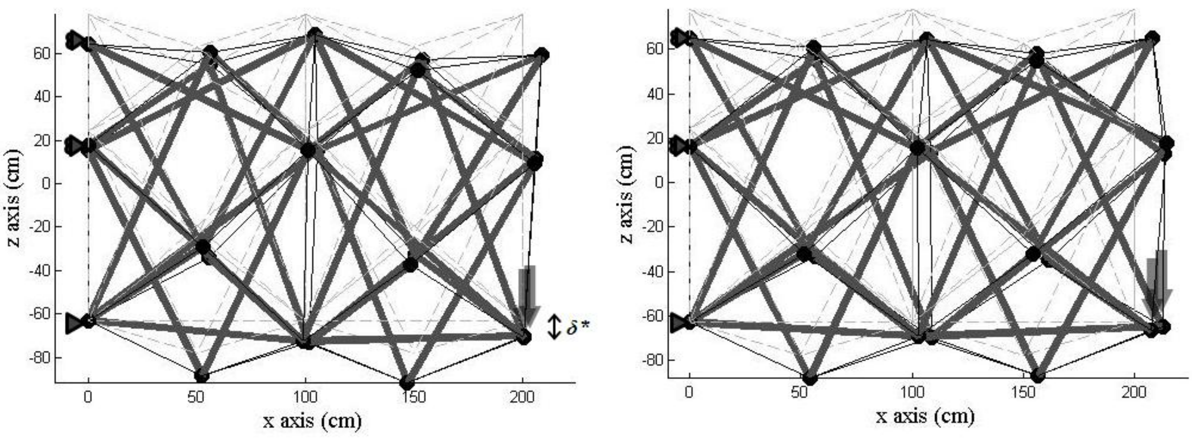
In this study, quasi-static actuation and frictionless motion are assumed during shape transformations. Quasi-static actuation implies that dynamic analyses of shape transformations can be approximated by a series of static analyses. Thus, shape changes are analyzed using an algorithm that integrates cable actuation and spring reaction in the modified dynamic relaxation algorithm. Actuation is implemented as a length change in active cables. After each actuation, a new equilibrium configuration is found. Hence, large shape transformations such as deployment are decomposed into a series of intermediate equilibriums thus remaining always close to the equilibrium manifold. The number of intermediate equilibriums depends on the actuation step. Large actuation steps may result into unstable configurations, while small actuation steps are computationally expensive. Although deployment of the tensegrity-ring system is found feasible using the same actuation step in all continuous active x-cables, obtaining the desired deployed shape or complex shapes requires the application of several actuation steps in the active cables.

The initial strategy employed to find the right actuation steps that lead to the desired shape is an iterative approach similar to a gradient-based search. Starting with a trial solution actuation steps are then applied depending upon constraint violations of element contact and element strength to arrive at a particular shape. It is thus assumed that the search space has a single minimum solution and that individually adjusting the active cables leads to this minimum. However, this assumption is seldom valid as actuation steps simultaneously affect multiple degrees of freedom and the objective function has multiple local minima. Furthermore, it is often of interest to generate a number of solutions so that there can be a selection using secondary criteria such as the energy required by the actuation devices. Consequently, a stochastic search algorithm is combined with the modified dynamic relaxation algorithm to identify steps hat lead to the desired shape.

Stochastic search algorithms replace gradient search methods when the objective function includes multiple minima. There are many tens of stochastic methods, such as Simulated Annealing, Genetic Algorithms and Particle Swarm Optimization. The stochastic search algorithm used in this study is Probabilistic Global Search Lausanne (PGSL). Its principal assumption is that sets of near-optimal solutions are near sets of good solutions [19]. The PGSL algorithm is based on a probability density function that is iteratively modified so that more exhaustive searches are made in regions of good solutions. It is suitable for objective functions with continuous parameters and was successfully used in other engineering tasks [20] as well as for the control of a tensegrity structure [21, 22]. In this study, cable-actuation steps are defined as the search parameters for this study. They are defined within a set of [l_0-10, l_0+10] with a 0.1cm precision where l_0 corresponds to the cable rest-length. Finally, the Objective Function (*OF*) consists of the sum of the absolute values of the differences in nodal coordinates between the desired shape and the current shape of the system in addition with a penalty cost:

$$OF = \sum^{N} \left| (n_d - n_c) \right| + \sum^{E} P(f_1, f_2) \tag{2}$$

where N is the number of nodes, n_d and n_c are the nodal coordinates for the desired and the current shape respectively. E is the number of elements and P is the penalty function accounted for constraint violations of element contact f_1 and element strength f_2. Element contact is estimated based on element geometry and position, while internal forces are estimated using the modified dynamic relaxation algorithm. The actuation-solution space depends on the number of actuators and their characteristics (range and precision). The space grows exponentially with the number of actuators. The size of the actuation space for the two module footbridge-system with the 5 continuous active x-cables is estimated at $100^5 = 10^{10}$ solutions.



BEFORE CORRECTION **AFTER CORRECTION**

Figure 6: Shape corrections on the half-footbridge tensegrity-system: shape before correction (*OF =5*), shape after correction using stochastic search (*OF = 0.5*)

* Note: Displacement δ is amplified for visualization

Figure 6 shows an example of shape correction due to a loading of 0.5kN applied at the two right-bottom end-nodes resulting to an average displacement δ of approximately 2.5cm of the deployed two-module system (*OF = 5*). The goal of the shape transformation is to correct the displacement of the right-bottom end-nodes while maintaining the structural integrity of the system under the applied loading. Therefore, only the right end-nodes are taken into account in the objective function along with the penalties for element contact and strength. PGSL provides a 90% correction of the deformed shape with respect to the actuation scheme applied (*OF = 0.5*) avoiding strut contact and element failure.

A better shape control can be obtained considering a larger number of active cables in the module as controllability of the shape depends on the number of active cables in the system. Therefore, a better shape correction is obtained if springs of the layer elements are replaced by discontinuous active cables. Figure 7 shows the shape correction for an average displacement δ of approximately 2.5cm (*OF = 5*) of the two-module system with continuous active x-cables and discontinuous active layer cables. The obtained correction shape is closer to the initial shape (*OF = 0.0*) due to a larger number of active cables.

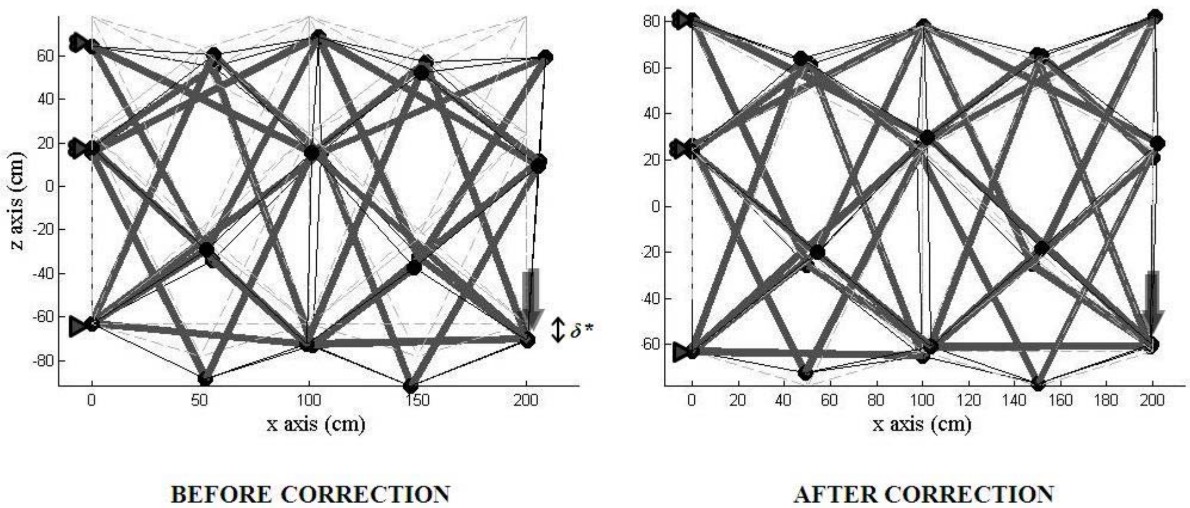


Figure 7: Shape corrections on the half-footbridge tensegrity-system: shape before correction (*OF =5*), shape after correction using stochastic search (*OF = 0.0*)

* Note: Displacement δ is amplified for visualization

5. Conclusions

This paper focuses on multi-degree-of-freedom shape transformations of a ¼ scale tensegrity footbridge. The conclusions from this study are as follows:

- Shape controllability increases with the number of actuators. Shape control of the pentagonal tensegrity-ring system is successfully conducted when actuators are replaced by springs.
- The actuation scheme with active cables and spring elements is applicable for both large transformations such as deployment as well as for small shape changes such as deployment corrections. Actuation steps differ according to the desired shape change.
- Actuation steps required for contact-free shape corrections while maintaining structural integrity are successfully identified through combining a stochastic search algorithm with a dynamic relaxation algorithm that has been modified for continuous cables.

The efficiency of using advanced computing methods for shape control of active deployable tensegrity systems has potential for shape control of other tensegrity systems.

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