

# Asynchronous variational integrators as a mechanical tool for dimensioning thin-shell structures.

Yves Weinand - Tudor Ratiu  
Section de Génie Civil, EPFL, ENAC, IS, IBOIS, Station 18  
Section de Mathématiques, EPFL, IMB, CAG, Station 8  
CH - 1015 Lausanne, Switzerland

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## 1 Introduction

This is an interdisciplinary research project. This proposal presents research directions in the mechanics of thin-shell and rod theory by using the formalism of discrete mechanics applied to the study of structures in civil engineering. Its aims are to combine the domains of mathematics (Prof. Tudor Ratiu EPFL/DMA) and civil engineering (Prof. Yves Weinand EPFL/Ibois). The major objectives of this interdisciplinary group are the search and the development of a practical tool to study irregular surfaces. Thin shells have been a major subject of interest over the last four years at the Laboratory for timber constructions (IBOIS, Director Prof. Yves Weinand) at the EPFL.

We use theory of AVIs (discrete mechanics) developed at Caltech by the team of J. Marsden and subdivision surfaces applied to irregular thin-shells in civil engineering to solve our problems. With these tools we shall study both dynamic and static phenomena conserving the symmetries of the problem. Moreover, the energy is conserved for non-dissipative systems.

## 2 State of research done in the chosen field

### 2.1 Study of the dynamics and statics of the Euler-Bernoulli beam

For the Euler-Bernoulli beam the assumptions of infinitesimal deformations were applied and the material was assumed to be hyperelastic and isotropic.

The AVI method was used to carry out dynamic and static two-dimensional simulations. For static simulations we considered a non-conservative system with dissipation to

find the equilibrium position and used the discrete version of the Lagrange-d'Alembert principle.

For numerical experiments, we used two kinds of shape functions: Hermitian cubic shape functions and cubic B-splines. We compared our experimental results to analytical results for several boundary conditions, different lengths of the beam, different thickness and materials. Simulated beams at equilibrium are very close to analytical results. The amplitude behavior showed a good conservation of energy by the AVI integrator. Several improvements were investigated.

(Ref 1 : F. Demoures, J. Nembrini, T. Ratiu and Y. Weinand. *AVI as a mechanical tool for studying dynamic and static Euler-Bernoulli beam structures*. Proceedings of the 1st EPFL Doctoral Conference in Mechanics, pp. 69-72, February 19, 2010.)

(Ref 2 : F. Demoures, J. Nembrini, T. Ratiu and Y. Weinand. *AVI as a mechanical tool for studying dynamic and static beam structures*. Proceedings of 1st International Conference on Structures and Architecture (ICSA2010), Guimarães, Portugal, July 21-23, 2010.)

## 2.2 Study of dynamic and static of thin shells based on Kirchhoff-Love constraints

We restrict our attention to the Kirchhoff-Love theory of thin shells and, accordingly, we constrain the deformed director  $\mathbf{t}$  to coincide with the unit normal to the deformed middle surface of the shell. We used the linearized model described by Cirak, Ortiz and Schröder (2000).

To ensure that the bending energy is finite we used bicubic and biquadratic uniform B-splines. To get equilibrium positions, we introduced a dissipative term. In the presence of forcing, the discrete Noether theorem holds, which allows us to obtain consistent results. Integration runs have provided concluding results in the sense that equilibrium positions can be reached very quickly. The second aim was to study the consistency of these results when locally modifying time-steps according to constraints are introduced. Once again, B-splines have displayed very good behavior when subjected to such modifications and, most particularly, cubic B-splines.

We considered also two thin shells of the same size, leaning against each other, so they form an edge. As previously, we get the equilibrium position by introducing a dissipative term. Here also, heterogeneous time-stepping has shown interesting results. This last part, devoted to plates and, most particularly, to the introduction of constraints, indeed calls for a more careful study, which would enable us to distinguish between the roles of the integration method and that of the physical phenomenon in the appearance of instabilities.

## 2.3 Study of dynamic and static of beams under large overall motions

This work is based on the Simo-Marsden-Krishnaprasad beam model and Lie group variational integrators developed by T. Lee, M. Leok, and H. McClamroch.

Lie group variational integrators fit well with the beam model of Simo-Marsden-Krishnaprasad which corresponds to a modification of the Cosserat model due to Antman. The configuration space is  $Q = \{\phi = (\phi_0, \Lambda) \mid C^\infty([0, L], \mathbb{R}^3 \times SO(3))\}$ , where  $\phi_0$  is a mid-line in  $C^\infty([0, L], \mathbb{R}^3)$ , and  $\Lambda \in SO(3)$ . Given  $\{\mathbf{E}_I\}$ , the standard basis in  $\mathbb{R}^3$ , the beam is explicitly described as  $x = \phi(X^1, X^2, X^3, t) := \phi_0(S, t) + \sum_{\alpha=1}^2 X^\alpha \Lambda \mathbf{E}_\alpha(X^3, t)$ .

We considered two temporal discretizations, associated to synchronized and asynchronous time variational integrators. We also took into account the conservation of the discrete momentum map.

The strain of the corresponding discrete model remains objective (frame-indifferent). This is a fundamental property of three-dimensional elasticity which can be violated by certain interpolations of rotations. The inherent property to preserve the symmetries allows us to properly define the equilibrium position.

The implementation of this Lie group variational integrator done by Sigrid Leyendecker (University of Erlangen-Nuremberg), and Sina Ober-Blöbaum (University of Paderborn) is currently in progress.

(Ref 3 : F. Demoures, F. Gay-Balmaz, J. Nembrini, T. Ratiu, and Y. Weinand. *Flexible beam in  $\mathbb{R}^3$  under large overall motions, and, Asynchronous Variational Integrators*. Proceedings of IABSE-IASS, London, September 20-23, 2011.)

## 2.4 Study of dynamic and static of plates under large overall motions

This work is based on the Simo-Fox shell model and Lie group variational integrators.

The configuration space is  $Q = \{\phi = (\phi_0, \Lambda) \mid C^\infty(\mathcal{A}, \mathbb{R}^3 \times S_{\mathbf{E}_3})\}$ , where  $\phi_0$  is a mid-surface in  $C^\infty(\mathcal{A}, \mathbb{R}^3)$ , and  $S_{\mathbf{E}_3}$  is the set of rotations with rotation axis normal to  $\mathbf{E}_3$ . Given  $\{\mathbf{E}_I\}$ , the standard basis in  $\mathbb{R}^3$ , the beam is explicitly described as  $x = \phi(X^1, X^2, X^3, t) := \phi_0(u, t) + \xi \Lambda \mathbf{E}_3(u, t)$ , with  $(u, \xi) \in \mathcal{A} \times [h^-, h^+]$ .

We get a first variational integrator for this model based on the previous work we did with the Simo-Marsden-Krishnaprasad beam model. A second one, based on the discrete Euler-Poincaré theory, was done by taking into account the symmetry of the plate.

We used a discrete double bracket dissipation to get an equilibrium position. This dissipation has the advantage to preserve the symmetries, in such way that the discrete Noether theorem holds.

## 2.5 Importants news

### 2.5.1 Masoud Sistaninia left the EPFL

Masoud Sistaninia collaborated on this project from December 1, 2009 to July 31, 2011. A publication announced in the preceding SNF scientific report 2010 which summarizes the results 2.1, and 2.2 presented above as well as the comparison of Masoud Sistaninia with FEM was planed. This publication had to be abandoned.

### 2.5.2 Collaboration

- **François Gay-Balmaz** (CNRS-Normale Supérieure, Paris); he is involved in several of these projects in the elaboration of the algorithms
- **Sigrid Leyendecker** (University of Erlangen-Nuremberg); implementation and achievement of benchmarks
- **Sina Ober-Blöbaum** (University of Paderborn); implementation and achievement of benchmarks
- **Mathieu Desbrun** (Caltech); he has been involved constantly in various aspects of some of the projects and has manifested a strong interest in a closer collaboration

## 3 Output

### 3.0.3 Conference

*Workshop discrete mechanics and integrators.* Centre interfacultaire Bernoulli - EPFL. 6 October 2011. Organisers : Tudor Ratiu, Francois Demoures and Yves Weinand.

### 3.0.4 Publications in progress

- F. Demoures, F. Gay-Balmaz, S. Leyendecker, S. Ober-Blöbaum, Tudor Ratiu, and Yves Weinand. *Geometrically exact beam model in  $\mathbb{R}^3$  under large overall motions, Lie group variational integrator, and AVI.*
- F. Demoures, F. Gay-Balmaz, S. Leyendecker, S. Ober-Blöbaum, Tudor Ratiu, and Yves Weinand. *Geometrically exact shell model under large overall motions, Lie group variational integrator, and AVI.*