Evaluation of characterization methods of Printer MTF

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ABSTRACT

We explore two recent methods for measuring the Modeling Transfer Function of a printing system[12]. We investigate the dependency on the amplitude when using the sinusoidal patches of the method proposed in[1] and show that for too small amplitudes the measurement of the MTF is not trustworthy. For the method proposed in[2] we discuss the underlying theory and in particular the use of a significance test for a statistical analysis. Finally we compare both methods with respect our application – the processing and printing of photographic images.

Keywords: printer modulation transfer function, characterization, printer, scanner, significance test

1. INTRODUCTION

With the growing demand for high quality reproduction of photographic images using ink jet printers, the interest in qualifying and quantifying their behavior has become more and more important. Our aim is to obtain a model of the printer able to predict its output for any given image at the input. In addition, this model could be used to qualify how well a printer is adapted to a certain purpose, since it contains the characteristics of the printer it models. The input of such a model is the original image and the output is an image simulating the printed result. To define this model it is necessary to know the printer modeling transfer function (MTF) which is based on the assumption that the printer is a linear system.

Different approaches to find such a modeling have been recently published[12]. In this article we will explore the methods proposed by Jang and Allebach[1] and by Hasegawa et al.[2]

Section 2 begins with a survey of the parameters a printer MTF could possibly depend on, in order to choose out of them an appropriate subset which sufficiently describes the printer behavior. Mainly the dependency on the direction is evaluated by analyzing the characteristics of ink droplets on paper.
In section 3 the approach proposed by Jang and Allebach[1] is explored and modifications are proposed primarily to reduce noise in the measurement. We have found that this can be achieved by increasing the amplitudes of the test patches, since the measured MTF values become stable when the amplitudes are chosen sufficiently high. However, too high amplitudes are not preferable either, since the obtained MTF values become not reasonable. On the printer that we have tested the MTF properties are different along the horizontal and the vertical axis and can be interpolated for intermediate angles.
Section 4 is dedicated to explore the method proposed by Hasegawa et al.[2]. After introducing this method we propose a simplification for the hypothesis test of the statistical analyze and discuss the choice of the significance level on a theoretical level. Furthermore we point out the importance of the scan resolution for the hypothesis test, since it influences strongly the test and should be consciously chosen.
Finally both methods are compared with respect to our needs which is the printing process of photographic images.

2. AN APPROPRIATE SETUP FOR THE EVALUATION

2.1 Printing system

It is necessary to choose and keep a fixed printing system for the whole evaluation in order to be able to analyze and compare obtained results. Our printing system is an *Océ TCS-500* large format ink jet printer which prints on standard *Océ* ink jet paper at 600 dpi and the ink values are coded with 8 bit.
The scanner used for the analysis is an *Epson Expression 10000XL* which has a much higher resolution (2400 dpi) than the printer. This is necessary to avoid aliasing problems when scanning the printout to measure the MTF.

2.2 Test images

For a fixed complete printing system an MTF can depend on different parameters related to the characteristics of the input images. Among them are:

- bias and amplitude of spatially periodic changes
- color
- direction of spatial modulations or contours (horizontal, vertical or intermediate angles)
- content of pixels printed just before (printed value could depend on previous print head actions)
- spatial position (a sequence of pixels could depend on it's position on the paper)

The bias level should be considered, since the halftoning process and ink flow on the paper depend on it (e.g. through the dot gain control). Under the assumption of a linear system, the amplitude should not matter. Unfortunately this is not absolutely true and will be discussed in section 3.3. The choice of the direction in which an MTF should be measured is important, since the complexity of the measurement could be significantly reduced. We discuss this problem in section 2.3. For reasons of simplicity we will consider the printer as having no memory effect and as being independent of the spatial position. Additionally we limit to the case with gray level images, since this paper discusses more basic effects where color plays at the moment a secondary role.

2.3 Choice of the optimal modulation directions

The common mechanical design of an ink-jet printer implies two major directions to investigate: the direction of paper transport (vertical) and the direction of the moving print head (horizontal). Each direction is build up with significantly different mechanics. The question is now whether the printer's characteristics in the horizontal and vertical directions are sufficient to investigate or whether intermediate angles have also to be taken into account. To answer this question we propose to analyze the shape of the ink droplets printed on the medium. For this purpose a very bright gray level patch (value ≈ 250 on a 8 bit range) so that the droplets do not overlap. It is printed at the printer's standard resolution (600 dpi) and scanned at 1200 dpi. An algorithm then extracts each single droplet (see figure 1(a)) and calculates the following properties:

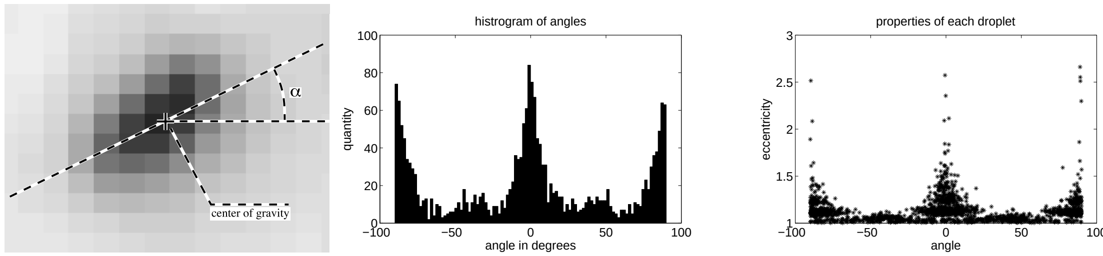
- its center of gravity by considering $w(p_{ij}) = \frac{100-Y_{ij}}{100}$ as the weight of pixel p_{ij}, where Y_{ij} is the Y value of pixel p_{ij} in CIE-XYZ color space, normalized to 100 % for a white diffuser reference under D50 diffuse lightning,
- its direction angle α calculated from the eigenvectors of the covariance matrix of the pixels p_{ij} weighted by $w(p_{ij})$,
- the eccentricity e of its elliptical shape equal to the square root of the ratio of the two eigenvalues of the covariance matrix ($e = 1$ for circular shape).

The results from the analysis of 2010 droplets on standard paper are shown in figure 1(b) and 1(c). In the histogram of the orientation angles α one sees that the droplets are mostly oriented in horizontal ($0°$) and vertical ($\pm 90°$) directions. The eccentricity e of the droplets is more important in horizontal and vertical direction as can be seen in figure 1(c). Since the horizontally and vertically oriented droplets are spatially equally distributed, the effective mean droplet shape is a cross. However the eccentricity is not too important ($e_{mean} = 1.1$). This droplet shape analysis and the printer mechanics are two reasons to justify the measurement of the MTF in horizontal and vertical direction.

2.4 Color characteristic curves

If the dot gain of the printer is not linearized, an initial linearization has to be achieved. Since the analysis in the articles[1] and[2] use the CIE-XYZ space we decided to linearize our printer with a luminance scale according to the measured Y values.

The test images are generated using Y values which lie in the printable interval $[Y_{low}, Y_{high}]$ and then transfered into numeric ink levels in the interval $[0 \ldots 255]$ with the help of a look up table (LUT). In our case $Y_{low} = 17.8$ and $Y_{high} = 85.6$.



(a) Example of an extracted droplet (scan at 1200 dpi)

(b) Histogram of the direction angle α

(c) Eccentricity e vs. direction angle α

Figure 1. Droplet shape analysis for a set of 2010 extracted droplets

3. SURVEY OF THE METHOD PROPOSED BY JANG AND ALLEBACH

3.1 Introduction to the method

The method proposed by Jang and Allebach[1] consists of printing patches with sinusoidal patterns and comparing their amplitudes with the amplitude between the values of constant tone patches, where the constant tone patches have a uniform value which are the maximum (max), the mean (bias) and the minimum (min) of the corresponding sinusoidal patches.

One row of the test image (see figure 2) consists of these three constant tone patches followed by nine sinusoidal patches oscillating between the min and max value with frequencies set to $\{10, 20, 30, 40, 50, 60, 80, 100, 150\}\frac{cycles}{inch}$ respectively. To measure the MTF with different biases, the test image consists of 19 rows. An example of one row of such a test image is illustrated in figure 2. The biases of the 19 rows are the tone levels in $\{\lfloor k \times 255 \rceil \,|k = 0.05, 0.10, 0.15, \ldots, 0.95\} = \{13, 26, 38, \ldots, 242\}$ where $\lfloor x \rceil$ is the nearest integer to x. The amplitude is set to 13, in order to avoid cropping the signal. The bias level varies across the different rows.

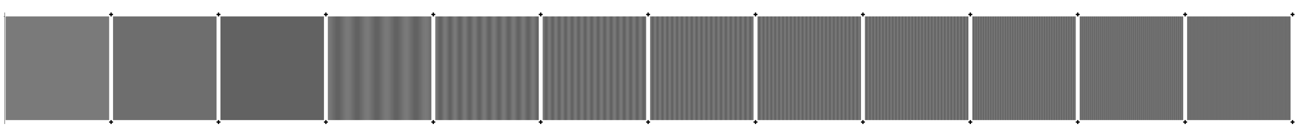


Figure 2. Example of one single row of Jang and Allebach's test images

The analysis is visualized in figure 3 and can shortly be described as follows.

Each row (constant mean gray level) is processed separately.

- Within one row the three constant tone patches are processed first. Their means are calculated in the CIE-XYZ space and converted into CIE-Lab values.
- The nine sinusoidal patches from the same row are extracted separately, averaged perpendicularly to the direction of modulation. The mean converted into CIE-Lab space and projected on the line which connects the upper and lower mean values of the constant patches.
- Then, for all the projected points on the line the ΔE distance to the lower mean value of the constant patches is calculated. The result is a vector of scalar ΔE-values which is Fourier transformed in the next step and the amplitude of the corresponding main frequency of the patch is read.
- The amplitude is compared with the ΔE distance between the constant tone patches and it should normally be smaller. Since the scanner is not compensated at this point, their ratio is not yet the printer MTF. It is the MTF of the system composed by both the printer and the scanner.
- For the scanner compensation we use the scanner MTF which has been separately measured with specific engraved patterns on a physical chart.[3] We then estimate how much the scanner attenuates a signal which oscillates between the two outer constant tone patches. This scanner ratio should be between 1 and the ratio calculated in the previous step. Dividing the first calculated combined printer & scanner ratio by the above scanner ratio gives the compensated printer MTF.

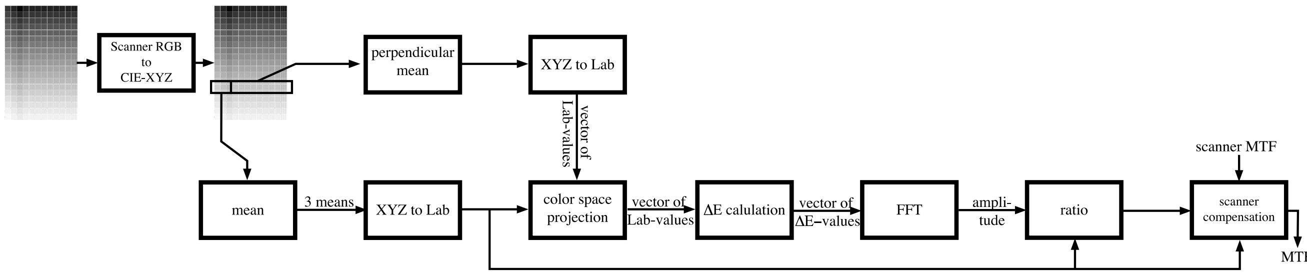


Figure 3. Flowchart of Jang and Allebach's algorithm

3.2 Analysis

This method offers the possibility to measure the MTF for different bias levels, orientations (e.g. horizontal and vertical) and colors. With these parameters, there are many different varieties of measurements possible which is a priori interesting. In addition to that it includes a method for the compensation of the scanner MTF, in order to calculate the MTF of the printing system only.
The possibility of color measurements is essential for a complete printer model. However we have not made use of that in this paper, since our interest is to evaluate the basic principle of the method. The fact that the amplitude ratio is analyzed in CIE-Lab space is very important, since this is a psychometric space which has been designed to have euclidean distances proportional to perceptual color differences.
This method does not yet take into account the halftoning and noise introduced by the printing system. By taking care of them, one could gain more reliable measured values. We first obtained MTF values significantly greater than 1. Since this is principally impossible, it showed a need to further investigate. We discuss the noise and its influence on the measured MTF values in the following section 3.3.

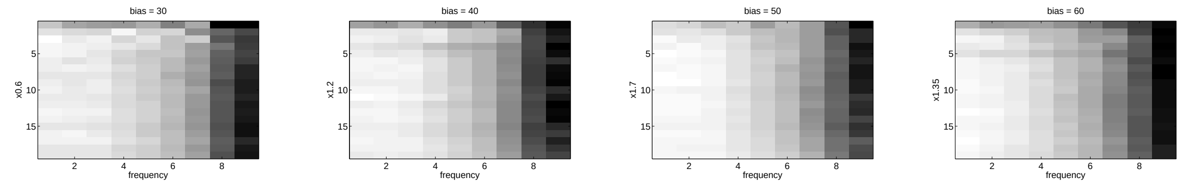
3.3 MTF values greater than 1 – amplitude dependency

In our first attempt to reproduce Jang and Allebach's experiments, several measures of the MTF were greater than 1. Theoretically this cannot happen, since the processing work flow of our printing process contains neither any contrast enhancement nor any similar processing. This problem turned out to be related only to the physic of the printer not with the concept of the method itself.

The exceeding MTF values showed a random behavior and no systematic rule could be concluded from the measurements of several printouts. This suggested that the problem was due to the presence of strong noise in the patches. The easiest way to gain better results, was to use higher amplitudes for the sinusoidally modulated patches. Thus the influence of the amplitude on the measurement has been analyzed in two steps. First the influence has been quantified and qualified (section 3.3.1) and second the test page from Jang and Allebach has been modified on the basis of this new knowledge (section 3.3.2).

3.3.1 Finding an optimal amplitude

To examine the influence of the amplitude $\Delta Y = (Y_{max} - Y_{min})/2$ on the measurement, a special test page has been designed. This test page does not contain sinusoidal patches with different bias levels and constant amplitudes as proposed in[1], but the whole test page has the same bias level Y_{bias} and the rows differ in their amplitudes. The amplitude in the n-th row is $\frac{n}{19} \cdot \min(Y_{bias} - Y_{low}, Y_{high} - Y_{bias})$, $n \in [1, 19]$. That means that the amplitudes grow linearly along the columns. The maximum amplitude is chosen such that the sinusoidal wave then touches either the upper or the lower border of the printable interval $[Y_{low}, Y_{high}]$. For example for a bias level of $Y_{bias} = 30$ and a lower border of $Y_{low} = 17.8$ the amplitudes are $n \cdot 0.64$ as indicated at the ordinate of figure 4(a).
The images in figure 4 illustrate for four different bias levels the MTF values as a gray scale image. These images show that for very low amplitudes (top rows) the measured MTF values become significantly noisy and for larger amplitudes the MTF values are more constant.
To investigate this behavior the variances in the first column of the four MTFs with different biases have been plotted (figure 5). The images in figure 4 clearly show that the biggest changes take place for the lower amplitudes. Therefore it is interesting to calculate the variance of the values from the n-th to the last row. For small n the



(a) $Y_{bias} = 30$ (b) $Y_{bias} = 40$ (c) $Y_{bias} = 50$ (d) $Y_{bias} = 60$

Figure 4. Amplitude dependency of the MTF for different mean values. The MTF value is shown as a gray level (the brighter the rectangle the higher the corresponding MTF value)

noisy values from the first rows are considered and induce a high variance; for high n there are only left over the stable MTF values and the variance is smaller. The curves are from the first column of each of the four MTFs. On the abscissa there is the row number n from which on the MTF values have been considered for the variance.

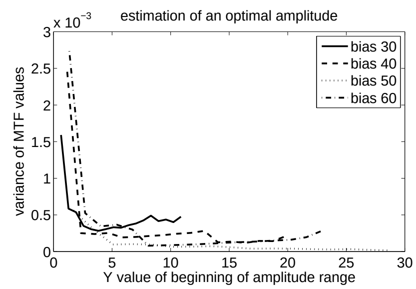


Figure 5. Variances of MTF values vs different amplitude ranges; curves from 4 MTFs with different bias levels

Figure 5 shows that, from a certain amplitude on the variances of the measured MTF values are quite low compared to the variance of all the measured MTF values. This amplitude is approximately $\Delta Y \approx 5$ and it is advisable to use sinusoidal modulations with higher amplitudes. Further on, the low variance of MTF values above the limit amplitude allows to elevate the amplitude without strongly affecting the measured MTF values.

3.3.2 Modifying Jang and Allebach's test image

In the paper from Jang and Allebach[1] the amplitude ΔY is restricted to not having to crop the sinusoidal patches at the borders of the printable interval $[0 \ldots 255]$ – in our case, this amplitude corresponds to approximatively 3 on the Y axis.

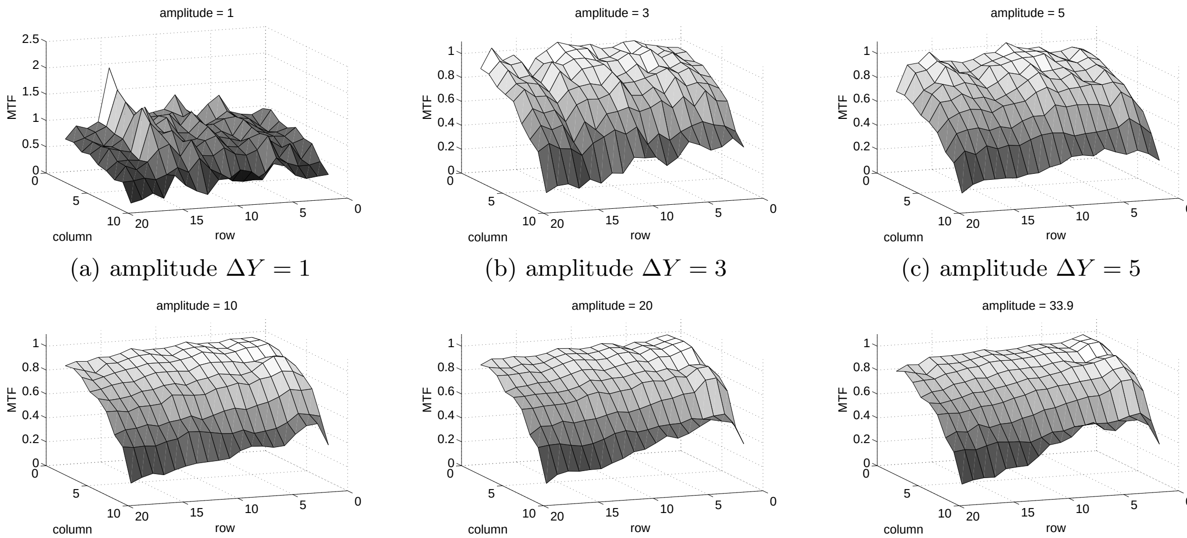
In our new test image the bias levels Y_i, $i = [l, 19]$ are chosen such that $\{Y_{low}, Y_1, \ldots, Y_{19}, Y_{high}\}$ are equidistantly distributed. The amplitude is constant where possible and greater than 5. Exceeding Y values are avoided by reducing the amplitude for those bias levels where it is necessary. The amplitudes are thus constant in the middle part of the test image and fall down at the most upper and lowest rows of the test image (the borders of the printable Y range).

Figure 6 shows measured MTF values from test pages with different amplitudes (1, 3, 5, 10, 20, 33.9). 33.9 is the highest amplitude possible for our printer which can only be realized for the bias level in the middle row. One can clearly see, that the measured MTF values become less noisy as the amplitude ΔY rises from 1 to 5 and the values for the lowest frequency become stable and are just beneath 1. The noise for the low amplitude levels is mainly due to the few quantization steps. For much higher amplitudes ($\Delta Y = 10$, 20, 33.9) the measured MTF is even less noisy and does not change significantly any more. But those MTFs fall linearly from dark to bright patches (rows from rank 1 to 19) – even for the lowest frequency.

Thus we propose to set the amplitude to $\Delta Y = 5$ which gives us a good compromise between noise reduction (see figures 6(a) to 6(c)) and reasonable stable MTF values for the lowest frequency (unlike those in figures 6(d) to 6(f)).

3.4 MTF in horizontal and vertical directions

We compare the MTF of two test images with spatial modulation in the horizontal and the vertical directions with the amplitude $\Delta Y = 5$. Figure 7(a) and 7(b) show the vertical and the horizontal MTF respectively; figure

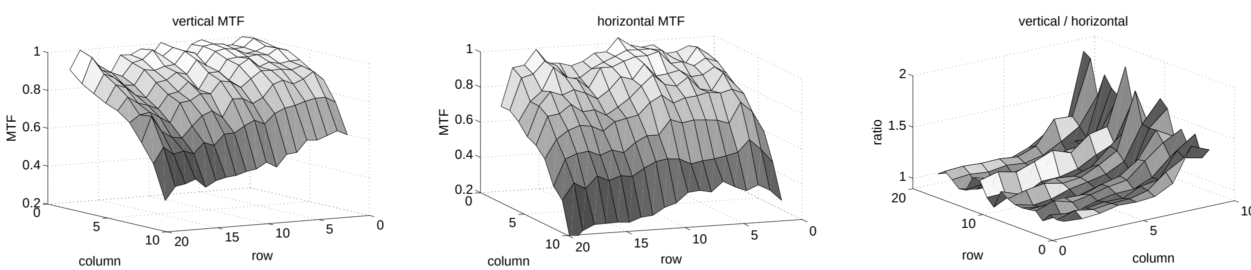


(a) amplitude $\Delta Y = 1$ (b) amplitude $\Delta Y = 3$ (c) amplitude $\Delta Y = 5$

(d) amplitude $\Delta Y = 10$ (e) amplitude $\Delta Y = 20$ (f) amplitude $\Delta Y = 33.9$

Figure 6. MTFs measured from new test images with different amplitudes; row: increasing bias level, column: increasing frequency

7(c) shows their ratio (the surface has been rotated counterclockwise by 90° to avoid occlusions).
As one can see, the MTF values in both directions are very close to 1 for the lowest frequency and they are not any more exceeding 1. Furthermore, the distinction between the two directions emerges as being important as can be seen in figure 7(c): for high frequencies (columns of high rank) the reproduction of sine waves is significantly better in the vertical than in horizontal direction.
Finally there is a falloff in the light region (high row number) which is due to the halftoning process. The brighter a patch, the lower the droplet density and the higher the mean distance between them. When this mean distance is in the order of magnitude of the sinusoidal wavelength, it becomes very difficult to reproduce the frequency. This can be observed in figure 8.

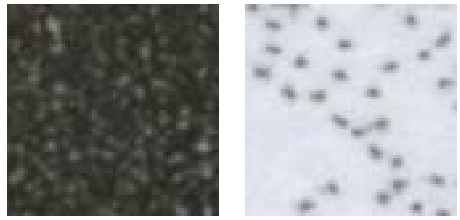


(a) vertical direction (b) horizontal direction (c) ratio of vertical to horizontal direction

Figure 7. Comparison of vertical and horizontal MTF and their ratio; row: increasing bias level, column: increasing frequency

3.5 MTF in intermediate angle directions

Since natural images contain not only horizontal and vertical modulations but a mixture of modulations in any possible direction, the MTF should be known for intermediate angles to fully describe a printer's behavior. In section 2.3 we showed that the printer's principal directions are the horizontal and the vertical. For reasons of simplicity it would be beneficial to interpolate the MTF for intermediate angles from the two MTFs in the principal directions.
To test whether this is possible we numerically rotate the test page with the amplitude $\Delta Y = 5$ by 45° (clockwise)



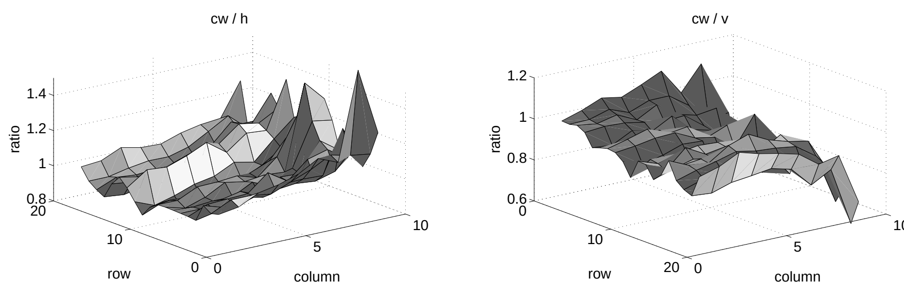
(a) darkest patch (b) brightest patch

Figure 8. Darkest (row 1) and brightest (row 19) patches of the column (rank 19) with the highest frequency (80×80 pixels at 1200 dpi)

and $-45°$ (counterclockwise) and print those two versions. Then we scan and analyze them as usual.
The first important point to notice is that the two MTFs ($45°$ and $-45°$) are very similar. We calculated their ratio patch by patch. The $(9 \times 19 =)171$ ratios have a mean of 0.99 and a standard deviation of 0.005, which means that they are (seen apart from noise) identical and the next analyze can be undertaken with only one of them.
We then calculate the ratio of the $45°$ MTF to both the horizontal and the vertical MTFs; these two ratios are illustrated in figure 9.
As one can see, the values of the clockwise MTF lie between the vertical and the horizontal MTF values. This is interesting because it could be possible to interpolate the MTF for an angle θ as the weighted sum of the horizontal and the vertical MTF:

$$\mathrm{MTF}_{|\theta|} = f(\theta) \cdot \mathrm{MTF}_{hor} + (1 - f(|\theta|)) \cdot \mathrm{MTF}_{ver}\,, \quad \text{where } f(|\theta|) : [0°, 90°] \longmapsto [0, 1]\,.$$

$f(\theta)$ is most likely a monotone function and – due to the symmetry – has to be defined only for angles in the interval $[0°, 90°]$ instead of $[-90°, 90°]$.



(a) $45°$ / horizontal (b) $45°$ / vertical

Figure 9. Ratios of the $45°$ MTF to the horizontal and the vertical MTF; row: increasing bias level, column: increasing frequency

3.6 Discussion

We have considered the noise in the method proposed by Jang and Allebach[1]. For that, we have designed a special test image where the bias level is constant and the amplitude is different in each row. The results show clearly that low amplitudes have two disadvantages: measurements are noisy and tend to be significantly smaller than 1 – even for low frequencies.
Furthermore, it has been found that the MTF values do not any more change importantly with the amplitude if the amplitude is above a certain level ($\Delta Y \approx 5$ in our case). We have then introduced a modification to Jang and Allebach's test image by setting the amplitude to an optimal value and by reducing it only for patches which would exceed the printable range otherwise.
Finally, we showed that the MTFs from test pages which had been rotated ($45°$ and $45°$) before printing have the same values and that these values lie between the horizontal and the vertical MTF values. Hence, it is most likely possible to interpolate the MTF for an intermediate angle from the horizontal and vertical MTF.

4. SURVEY OF THE METHOD PROPOSED BY HASEGAWA ET AL.

4.1 Introduction to the method

The method proposed by Hasegawa et al.[2] does not measure the MTF directly but through the calculation of the Contrast Transfer Function (CTF). This is achieved by generating a test page with six different pattern types (vertical and horizontal lines, vertical and horizontal stripes, 25 % and 50 % duty dot), where each pattern is repeated with different frequencies. An example of one of those patterns ("Vertical Stripe Pattern") is presented in figure 10 (upper part). In the next step the image is printed and scanned (middle part). Then the input and the output image are matched on top of each other; the difference between the two after the matching is shown in the bottom part of figure 10.
One considers for each position x_{ij} its absorbance value $A_{ij} = 1 - Y_{ij}$ at the output (Y_{ij} is the Y value

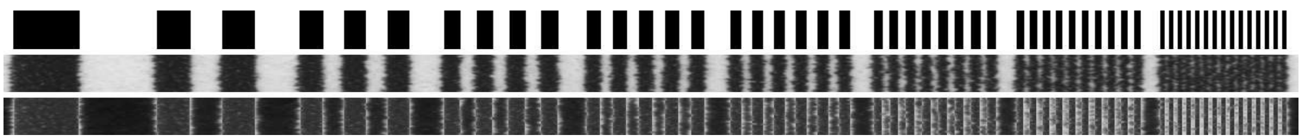


Figure 10. Vertical Line Pattern, input (top) and output (middle) image and difference (bottom), total width = 1 inch

of CIE-XYZ), an intended value at the input $I_{ij} \in \{0 = \text{white}, 1 = \text{black}\}$ and a local spatial frequency $f_{ij} \in \mathcal{F}$. The means $\bar{A}_{r,f}$ and the variances $V_{r,f}$ ($r \in 0,1$) are calculated for all pixels in the light region $\Omega_{0,f} = \{x_{ij} | I_{ij} = 0, f_{ij} = f\}$ and all pixels in the dark region $\Omega_{1,f} = \{x_{ij} | I_{ij} = 1, f_{ij} = f\}$ and for each frequency $f \in \mathcal{F}$ respectively:

$$\bar{A}_{r,f} = \frac{1}{n_{r,f}} \sum_{x_{ij} \in \Omega_{r,f}} A_{ij}, \quad V_{r,f} = \frac{1}{n_{r,f} - 1} \sum_{x_{ij} \in \Omega_{r,f}} \left(A_{ij} - \bar{A}_{r,f}\right)^2 ,$$

where $n_{r,f} = \#\{x_{ij} \in \Omega_{r,f}\}$ is the cardinality of $\Omega_{r,f}$, $r \in \{0 = \text{light}, 1 = \text{dark}\}$, $f \in \mathcal{F}$.

In addition to that a solid black (100 % ink) and a paper white (0 % ink)field are analyzed to calculate the averaged maximum value $A_{SolidBlack}$ and minimum value $A_{PaperWhite}$.
The CTF is then defined as:

$$\text{CTF}(f) = 100 \cdot \max\left(\bar{A}_{1,f} - \bar{A}_{0,f}, 0\right) \Big/ \left|A_{SolidBlack} - A_{PaperWhite}\right| ,$$

where the max-function sets the CTF to zero if the bright region is darker than the dark region.
The MTF can then be calculated with a conversion equation cited in[2]

$$\text{MTF}(f) = \frac{\pi}{4}\left[\text{CTF}(f) + \frac{\text{CTF}(3f)}{3} - \frac{\text{CTF}(5f)}{5} + \frac{\text{CTF}(7f)}{7} - \ldots\right] . \tag{1}$$

The variances are used for a significance test which estimates whether the two means are significantly different with respect to their variances and cardinalities. Then the two quantities

$$t_f = \frac{\bar{A}_{1,f} - \bar{A}_{0,f}}{\sqrt{\frac{V_{1,f}}{n_{1,f}} + \frac{V_{0,f}}{n_{0,f}}}} \quad \text{and} \quad \Phi^{\star}_f = \frac{\left(\frac{V_{0,f}}{n_0} + \frac{V_{1,f}}{n_1}\right)^2}{\left(\frac{V_{0,f}}{n_{0,f}}\right)^2 / \Phi_{0,f} + \left(\frac{V_{1,f}}{n_{1,f}}\right)^2 / \Phi_{1,f}} \tag{2}$$

are calculated where $\Phi_{r,f} = n_{r,f} - 1$ are the degrees of freedom of the light and dark region respectively. The statistic t_f is related to the Student's distribution $\mathcal{S}(\Phi^{\star}_f, \alpha)$ with the significance level α and the estimated degrees of freedom $\Phi^{\star}_f$. This leads to the inequality

$$\left|\bar{A}_{1,f} - \bar{A}_{0,f}\right| \geq z^{\Phi^{\star}_f}_{(1-\alpha)} \cdot \sqrt{\frac{V_{1,f}}{n_{1,f}} + \frac{V_{0,f}}{n_{0,f}}} ,$$

which is fulfilled if and only if the dark and the light region can be considered as being significantly distinct with a significance level α. The value z^{Φ}_{α} is the α-quantile of the Student's distribution with Φ degrees of freedom. In[2] α has been set to 0.6.

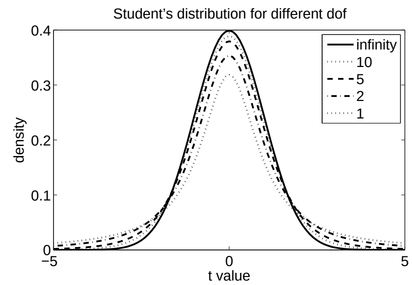


Figure 11. Student's distribution for different degrees of freedom converging against normal distribution

4.2 Analysis

It was proposed to use vector images for this method.[2] The target image sent to the printer is a postscript file (or comparable) with non pixel data inside. Many vector graphic documents such as text, technical drawings or sketches use no intermediate gray (color) levels. In this case it is sufficient to measure the Contrast Transfer Function for black and white (or full color and white) only.
Generating a vector graphic offers the possibility to generate patterns with any frequency. There is no limit to the resolution or a constraint that the wavelength has to be a multiple of the pixel size.
The use of rectangular signals instead of sinusoidal ones avoids the introduction of quantization noise observed in section 3.3 but in exchange causes a mixture of different frequencies.

The test whether the two areas are significantly different or not offers a good add-on to the CTF. This test judges the quality of the measured CTF values. However we have not been able to neither reproduce the results of the significance tests in[2], nor retrace the choice of the significance level α which has been made in[2]. We will discuss that in section 4.3.

In order to compare the results from this method from the one's from the method in section 3 we have decided to use the same frequencies, so that $\mathcal{F} = \{10, 20, 30, 40, 50, 60, 80, 100, 150\}\frac{cycles}{inch}$.

4.3 Significance test

We first propose the simplification of the Student's distribution for the present usage (section 4.3.1) and then discuss the choice of α (section 4.3.2). Further on the importance of a quantity in the test has to be highlighted (section 4.3.3).

4.3.1 Simplification of Student's distribution

For a sufficiently high degree of freedom (approx. > 100) Student's distribution converges against a normal distribution $\mathcal{N}(0, 1)$ (see [4, ch 9.7.3], figure 11). In our case the degree of freedom (number of scanned pixels) is far above this limit. For example at a scan resolution of 1200 dpi an area of $1\,mm^2$ has already more than 2000 pixels. Thus the simplification can be used. In this case $\Phi^\star$ from equation 2 does not need to be calculated and the normal distribution can be used, which simplifies the calculation with standard matlab functions.
This is why we will consider only the simplified case.

4.3.2 The choice of α

The basic idea of all hypothesis tests is to test a hypothesis $\mathcal{H}_0$ against an alternative $\mathcal{H}_1$ while limiting the risk of a wrong decision. There are two types of wrong decisions: deciding that $\mathcal{H}_1$ is true even though $\mathcal{H}_0$ is true and vice versa. The probability of the first error is named α and the second β. Normally the two hypothesis are chosen in that manner, that the first error is the worse case and hence α is the probability which should be limited to a low level.
In our case we dispose of two random variables $\mathcal{A}_0 \sim \mathcal{N}(\mu_0, \sigma_0^2)$ and $\mathcal{A}_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ which are the absorbances of the light and dark region respectively. Since the absorbance of the dark region should be higher, the question is whether μ_1 is greater than μ_0. To justify the meaningfulness of the result the hypothesis have to be chosen:

$$\mathcal{H}_0 : \mu_1 - \mu_0 \leq 0 \qquad \mathcal{H}_1 : \mu_1 - \mu_0 > 0\,.$$

The first error (probability α) is then to wrongly assume that μ_1 is greater than μ_0. In other words: if one estimates that the dark region absorbs better than the light region, there is a probability of α that this decision is wrong.
With n_0 (n_1) drawings $A_{0,0} \dots A_{0,n_1}$ ($A_{1,0} \dots A_{1,n_1}$) from the random variable $\mathcal{A}_0$ ($\mathcal{A}_1$), one can numerically estimate the mean μ_0 (μ_1) and the variance σ_0^2 (σ_1^2). The estimators (itself again random variables) are:

$$\bar{\mathcal{A}}_r = \frac{1}{n_r}\sum_{i=1}^{n_r} \mathcal{A}_{r,i}, \quad \mathcal{V}_r = \frac{1}{n_r - 1}\sum_{i=1}^{n_r}(\mathcal{A}_{r,i} - \bar{\mathcal{A}}_r)^2 \qquad r \in \{0, 1\}\,.$$

For n_0, n_1 sufficiently high, one has[4]

$$\frac{(\bar{\mathcal{A}}_1 - \bar{\mathcal{A}}_0) - (\mu_1 - \mu_0)}{\sqrt{\frac{\mathcal{V}_1}{n_1} + \frac{\mathcal{V}_0}{n_0}}} \overset{approx.}{\rightsquigarrow} \mathcal{N}(0,1)\,.$$

We can thus estimate that hypothesis $\mathcal{H}_1$ is right with an error probability smaller than α if

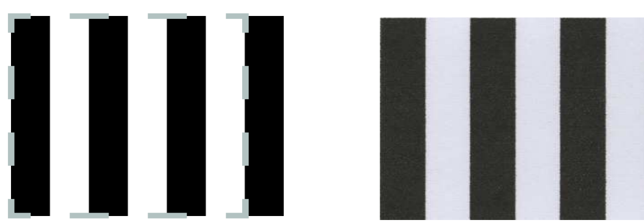
$$(t \equiv)\frac{\bar{A}_1 - \bar{A}_0}{\sqrt{\frac{V_1}{n_1} + \frac{V_0}{n_0}}} > z_{1-\alpha}\,, \tag{3}$$

where t is the test statistic and z_p is the p-quantile of the normal distribution.
Since α is an error probability it should be chosen quite small; for example $\alpha = 0.05$ is a typical value and $z_{0.95} = 1.64$.

4.3.3 Dependency on the scan resolution

The significance test depends directly on the cardinalities. If the cardinalities are very high, the variances and means can be considered as well estimated and they will not change. Then the only parameters that change in the calculation of the test variable t are the two cardinalities n_0 and n_1.

In order to proof this dependency, three test patches have been generated. One with a very low frequency (3 cycles/inch), another with a higher frequency (50 cycles/inch) and a last one with 300 cycles/inch. All of them have been printed with a resolution of 600 dpi and scanned with different resolutions (between 72 dpi and 2400 dpi). Figure 12(a) shows the low frequency test patch with a dashed rectangle, which indicates the part of the image that has been considered for the analysis. On the left hand side nothings has to be attached since there it is already white. The last stripe attached to the right hand side serves to have a continuously periodic signal within the area of analysis. Figure 12(b) shows the corresponding scan with 600 dpi.

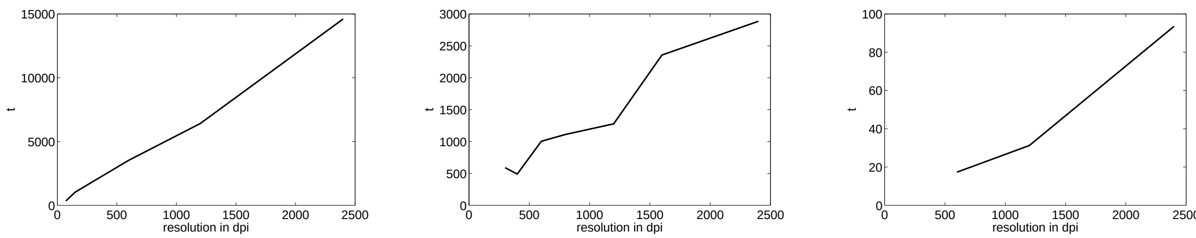


(a) generated test patch (b) scan with 600dpi

Figure 12. Low frequency test patch for the resolution dependency of the significance test

After scanning the printouts each with different resolutions (multiples of their frequencies), the images have been analyzed. The variable t versus the scan resolution is shown in figure 13. Since the square root of the number of drawings $\sqrt{n}$ is proportional to the scan resolution, the statistic should be proportional to the scan resolution too – this is clearly visible.
From inequation 3 it is also evident that there is indeed a cutoff frequency (statistic normally falls with rising frequency) from which on the inequation is not any more fulfilled. The figures 13(a) to 13(c) clearly show that the test statistic t falls with rising frequency and thus becomes smaller than the limit $z_{1-\alpha}$.

Hence a scan resolution much higher than the print resolution is not justifiable, as this will not result in more information about the printout. In almost the same manner a scan resolution much lower than the print resolution is not of interest, since this causes a loss of information especially at the borders between light and dark areas. It is therefore advisable to chose a scan resolution in the same order as the printer resolution; e.g. twice the printer resolution as in.[1]
But whatever one chooses as scan resolution, it is important to be conscious that the result of the hypothesis test would be different for a different scan resolution.



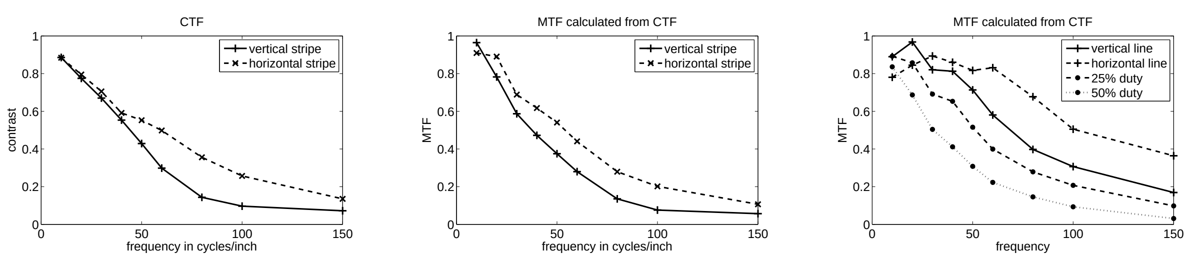
(a) patch with 3 cycles/inch (b) patch with 50 cycles/inch (c) patch with 300 cycles/inch

Figure 13. Statistic t versus the scan resolution, relation approximatively linear

4.4 Conversion equation: CTF to MTF

Seen apart from the inconvenient resolution dependency of the hypothesis test one can calculate the values of the CTF and neglect their significance. To better compare the results with the ones from Jang and Allebach's method, we have decided to use the same frequencies. Thus we have designed a test page following the design from Hasegawa et al., but the frequencies are set to $\{10, 20, 30, 40, 50, 60, 80, 100, 150\}\frac{cycles}{inch}$. For each of the six pattern types one obtains a CTF. Figure 14(a) shows the CTF for the horizontal and the vertical stripe pattern and figure 14(b) the MTF calculated with the proposed conversion equation. The two stripe patterns have been chosen because their structure is closer to Jang and Allebach's patches as the structure of the other four patterns. The MTF from the other 4 patterns is shown in figure 14(c).
One sees that the MTF obtained with the method from Hasegawa et al. are differ significantly from each other and it is not obvious which one has to choose to get a good estimation of the real MTF.
To calculate the MTF with equation 1 for the frequency f one needs the CTF at impair multiples of f. In our conversion we interpolate intermediate CTF values linearly and extrapolate values for frequencies above $150\frac{cycles}{inch}$ but set them to zero when they become negative.

Like the MTF obtained from Jang and Allebach's method, this method also estimates a better transfer for modulations in vertical direction (attention: horizontal stripes correspond to vertical modulation and vice versa). So there is a coherency between the two methods.



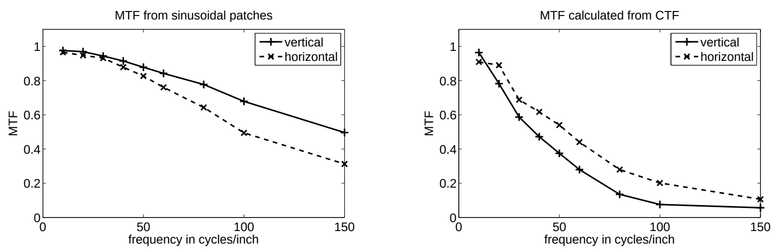
(a) CTF for vertical and horizontal stripe pattern (b) MTF for vertical and horizontal stripe pattern (c) MTF for vertical and horizontal line pattern and 25% and 50% duty pattern

Figure 14. CTF and MTF for the horizontal and the vertical stripe pattern and MTFs for the other 4 test patterns

5. CONCLUSION – COMPARISON OF THE TWO METHODS

Both examined methods attempt to measure the behavior of a printing system. Jang and Allebach measure the MTF directly with the help of sinusoidally modulated patches which are sent to the printer as a pixel image. Hasegawa et al. use black and white patterns which are sent to the printer as a vector graphic to measure the CTF which can be converted into the MTF. They

For a better comparison we show the MTF from the method from Hasegawa et al. in figure 15(b) (same as figure 14(b)) and the MTF obtained from Jang and Allebach's method in figure 15(a) (mean of all gray levels, in order to have a 2D plot too).
Seen apart from the fact that they both estimate a better transfer for modulations in vertical direction (vertical



(a) Jang and Allebach (b) Hasegawa et al.

Figure 15. Comparison of the MTFs obtained with the different methods

modulation corresponds to horizontal stripes), the differences between the MTFs from both methods are indeed significant. In our opinion the falloff of the MTF values in figure 15(b) is quite strong and does not really match with the impressions one has while looking at the printout.
But it is difficult to judge the MTFs and to decide which one describes better the printing system.

Our interest in estimating a printer MTF lies in the goal of processing photographic images. Thus we want MTF values not only for black and white images but also for intermediate gray levels. The method from Jang and Allebach offers this possibility and our experiences with it show that this is really necessary since the gray level (bias) influences the MTF (section 3.6). In addition to that in natural images rarely contain abrupt changes from black to white. Thus using the test patches from Jang and Allebach is closer to our needs than the patterns from Hasegawa et al.

At the end of the method from Hasegawa et al. there is a step for converting the CTF in the MTF. In this step there are assumptions on extrapolated CTF values which could invalidate the conversion. The method from Jang and Allebach estimates the MTF values directly with the use of the sinusoidally modulated patches.

We finally conclude that the method from Jang and Allebach is better adapted to our needs which is the processing of photographic images.

Further research could be done as follows:

- Print the patterns from Hasegawa et al. with two different gray values instead of pure black and white.
- Find a connection between the CTF/MTF obtained from the different pattern types from Hasegawa et al.
- Find a good interpolation for intermediate angles – $f(\theta)$.

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