Overcoming the limitations of traditional model-updating approaches

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ABSTRACT

Model updating is useful for improving structural performance assessments. This paper examines an important assumption of traditional model-updating approaches. This assumption requires the error independence between points where predictions and measurements are compared. Simulations performed on a full-scale bridge show that uncertainties are correlated for both static and dynamic predictions. Therefore, traditional model-updating techniques are not appropriate in such situations. Model updating limitations related to randomness and independence of uncertainties may be overcome by an interpretation strategy called Candidate Model Search for System Identification (CMS4SI). Instead of judging a model by its ability to fit measured data, the approach falsifies models using threshold values that are based upon uncertainties. Uncertainties may be correlated, systematic, independent or random.

KEYWORDS

Uncertainties, Correlation, Systematic errors, Performance assessment, Model Updating, Model Calibration, System Identification, CMS*4*SI

Structural performance assessment tasks such as reliability analyses, remaining fatigue life evaluations, and earthquake response predictions require behavior models of structures. The quality of such performance assessments is thus determined by the accuracy of model predictions. To improve accuracy, engineers take measurements and then perform model-updating (also known as model-calibration, model-tuning and curve fitting) in order to reduce uncertainties related to models.

There are two types of traditional model updating. The first type aims to minimize the discrepancy (residual) between predicted and measured values (for example (Bakir et al. 2008; Bell et al. 2007; Ka-Veng Yuen 2006; Sanayei et al. 1997; Schlune et al. 2009; Teughels and De Roeck 2004). In the presence of several *comparison points* (measurements and predictions), it is often not possible to obtain a residual of zero. Therefore, there are procedures to minimize the overall residual over comparison points. All proposals have the same goal; adjust the parameter values to get the best overall fit between predicted and measured values (residual minimization).

The second group of model-updating approaches involves maximizing the likelihood of a model (Beck and Katafygiotis 1998; Cheung and Beck 2009; Hadidi and Gucunski 2008; Tarantola 2005; Yuen et al. 2006). This approach usually accounts for uncertainties which come from both measurements and modeling. Posterior probability (sometimes called likelihood) for possible model parameter values (model instances) are computed according to uncertainty sources. The goal is to select the model instance(s) which maximizes the posterior probability. More than one model instance may be selected if they reveal an equivalent likelihood.

This paper discusses the reasons why such traditional model updating approaches may not be reliable for the identification of full-scale structures. The first section presents the limitations associated with traditional model-updating approaches. The second section then describes the candidate model search for system identification algorithm (CMS4SI) which overcomes the limitations of traditional model-updating.

LIMITATIONS OF TRADITIONAL MODEL-UPDATING

Several successful model-updating applications can be found among the methodologies reported above. However, most of these examples employed simulated data and they either implicitly or explicitly include the assumption that uncertainties at different comparison points are independent. When independence is present and with a large amount of comparison points, either minimizing the residual or maximizing the likelihood may lead to valid results since the average errors are likely to cancel. Results from such processes lead to best estimates analogously to what is done in linear regression based on the Gauss-Markov theorem (Plackett 1972). This theorem says that for a linear regression, if errors can be represented as Gaussian, uncorrelated and equally distributed random variables, the best unbiased estimate is obtained from least-square regression. If the assumption of uncertainty independence of comparison points does not hold, the premise behind least-square regression as well as the assumption of traditional model updating is no longer valid.

Is the hypothesis of independence valid for the identification full-scale structures? For measurement uncertainties such as sensor resolution and repeatability, it is reasonable to assume such independence. However, modeling uncertainties, such as model simplifications, geometry variations and some values for constant, are usually affected by bias. Model simplification errors reflect the accuracy with which the model is representative of the full-scale structure. Such uncertainty is always present in models and in most cases; one can rely only upon experience to quantify its extent. In this paper, the correlation between prediction quantities and their locations is studied for the model of a full-scale bridge. The structure studied is the Langensand Bridge (Switzerland).

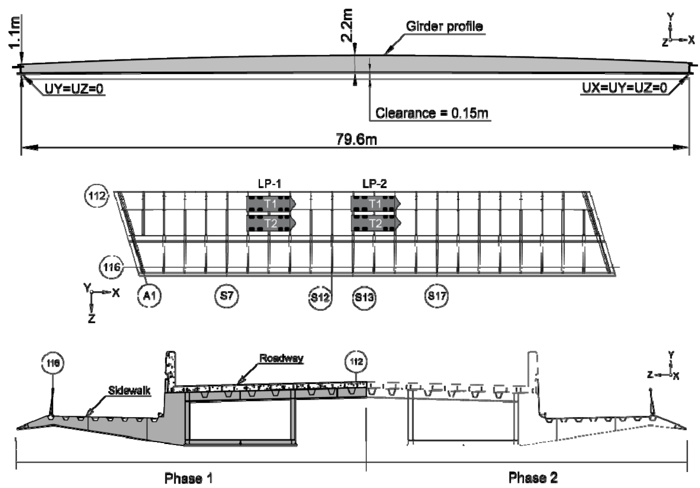
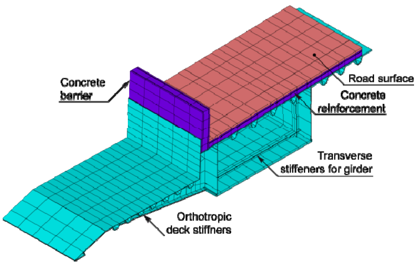


Figure 1 – Langensand Bridge elevation, top view and cross section, reproduced with permission from ASCE (Goulet et al. 2010)

This structure is an 80m long single span bridge. Only half of the structure (Phase 1) is studied. Figure 1 shows the elevation, the top view and cross section of the structure. Key reference axes, describing the sensor and load configuration layout, are also presented. In this study, six displacement sensors were used along with seven rotations and three strain measurements. Details of sensor placement and load configuration may be found in a publication by Goulet et al. (Goulet et al. 2010). Dynamic modal frequencies are simulated for 15 modes. The cross section of the finite element model used for the analysis is shown in Figure 2 along with the uncertainties associated with the FE model. Every uncertainty source presented in Figure 2 is a parameter of the model. Stochastic sampling is performed using the Monte-Carlo method in order to combine uncertainty sources.



Uncertainty source	unit	Mean	STD
$\Delta\nu$ concrete	-	0	0.025
Truck weight	Ton	35	0.125
Δt steel plates	%	0	1
Δt pavement	%	0	5
Δt concrete	%	0	2.5
Strain sensor positioning	mm	0	5

Figure 2 – Finite element model of the bridge, reproduced with permission from ASCE (Goulet et al. 2010) and uncertainty data associated with the FE model

For each instance, predictions are computed for quantities such as displacement, rotation, strain and modal frequencies. The objective is to examine the correlation between the prediction quantities and their locations.

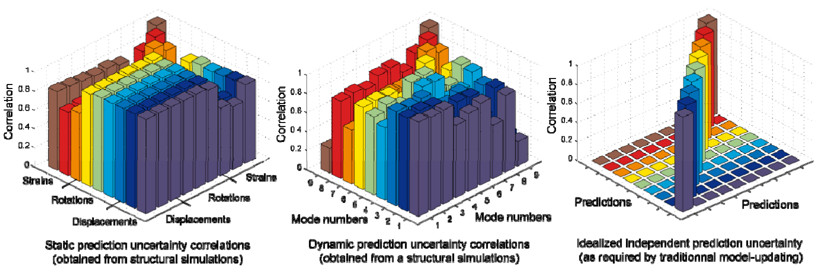


Figure 3 – Correlations between prediction quantities using a full-scale bridge simulation

Figure 3 shows a three-dimensional representation of the correlation matrices for model-dependent prediction uncertainties. In this figure, the horizontal planes are prediction quantities and locations and the height of vertical bars is the correlation between two predictions. The left graph shows results obtained from the stochastic combination of uncertainties for static prediction. Correlations between static predictions (displacement, rotations and strains) are high. Table 1 summarizes the average correlation between static prediction types. Displacement and rotations are almost perfectly correlated. Strains show a less correlated dependency due to the local character of these predictions. The correlation between load cases is also tested. On average, the uncertainty correlation between static-load cases is 0.98.

The center graph shows the correlation between prediction uncertainties for dynamic frequencies. In this case, correlations are lower than for static predictions and tend to decreases as the modes are significantly different (i.e. modes close to each other are highly correlated). However, the correlation still remains generally high. For instance the correlation between the first and second mode is of 0.98 and 0.6 between the first and the tenth.

Table 1 – Average uncertainty correlation between static prediction quantities

Prediction quantities	Displacement	Rotations	Strains
Displacement	0.99	0.98	0.74
Rotations	0.98	0.97	0.75
Strains	0.74	0.75	0.59

The right graph represents the idealized case of independent prediction uncertainty as required in traditional model-updating. Prediction uncertainties in both cases (for static and dynamic) do not satisfy the independence requirements. Therefore, traditional model-updating is not appropriate for such a structure. Either the best-fit model or the most probable model(s) are likely to be biased due to correlated modeling errors and therefore, results may not be representative of the real structure behavior. This underlines the need for a methodology which is not limited to cases where uncertainties are independent.

CANDIDATE MODEL SEARCH FOR SYSTEM IDENTIFICATION (CMS*4*SI)

In many scientific communities, it has been acknowledged for centuries that it is not possible to fully validate a hypothesis (model); it is only possible to falsify it. Tarantola (2006) explicitly acknowledged that fact and suggested that inverse tasks such as structural identification may only be solved by falsifying model instances. Therefore the challenge is to separate, in a rigorous and systematic way, candidate and rejected models. Several attempts have been made (Goulet et al. 2010; Ravindran et al. 2007; Robert-Nicoud et al. 2005) without fully succeeding in capturing the complexity of uncertainty combination, especially for multiple measurements.

Candidate model search for system identification is proposed to overcome the limitations mentioned in the previous section and to account for uncertainties and their correlations. This algorithm does not find a best match between predictions and measurements and it does not find the most likely model instance. Starting from an initial population of models, the approach filters out the models instances for which the discrepancy between predicted and measured values is sufficient to be sure that the right model will remain in the set according to a desired target reliability. The limits separating accepted and rejected models are called the thresholds (one for each comparison point). Threshold values are maximal plausible errors that occur through combining uncertainties from modeling and measuring. The algorithm defining the threshold values is summarized in the graph presented in Figure 4.

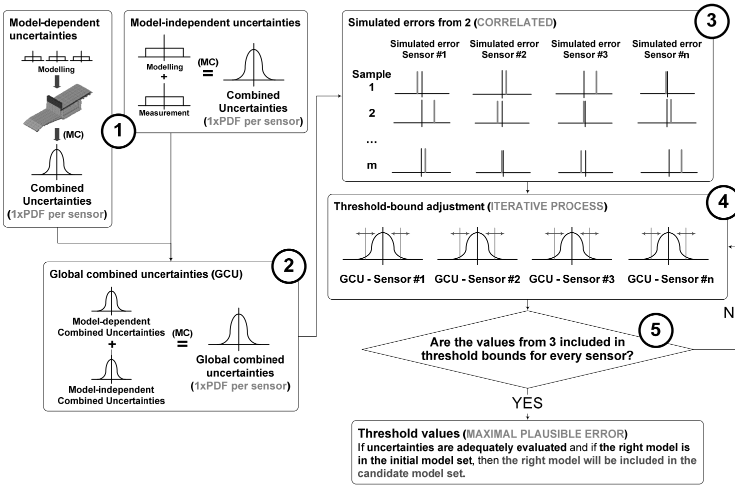


Figure 4 – Steps of Threshold Computation (PDF = Probability density function; MC= Monte-Carlo combination)

In step one, uncertainties are combined using Monte-Carlo (MC) stochastic sampling into two separate processes. Model-dependent uncertainties have to be combined through the finite element template model used for generation of model instances. The output is a matrix containing the predicted values for an instance of input parameter on each line (model-dependent errors). Each column is a comparison point used in the analysis. In Figure 4, this number is referred as a sensor for the purpose of simplification. Several thousand input parameter instances are used to obtain an uncertainty distribution for each comparison point. The mean of each distribution is subtracted from each of its samples in order to obtain the variation compared to a mean model. This result is named model-dependent combined uncertainties. Separately, model-independent uncertainties are combined in a MC process which does not involve the template model. Several million model instances are generated in order to obtain representative probability density functions (PDFs) of model independent uncertainties. For each instance, a sample is drawn in each uncertainty distribution and then summed to obtain a combined distribution.

Due to computational limitations, much fewer model-dependent uncertainties can be managed compared with model-independent uncertainties. Therefore, the data in the matrix has to be copied several times and concatenate in order to have the same size as the second. In step two, both model independent and dependent errors are summed. This results in global combined uncertainties, presented as a matrix having several million rows and as many columns as there are comparison points. Next, we define the global uncertainty coverage interval that ensures that the right model is in the candidate model set with a probability of Ψ. For common identification purposes, Ψ is set to 95%. By definition, the right model should be able to predict every observation; only one observation is sufficient to discard a model instance. The uncertainty between different comparison points is not always perfectly correlated. Therefore, the use of several comparison points requires a larger coverage interval than that defined for one measurement. Each time a comparison point is added to filter model instances, threshold values increase to account for the additional chance of wrongly rejecting the right model. The amount of increase is dependent upon the error independence between comparison points. Threshold values are defined in the third to fifth steps.

In the third step, correlated samples are drawn from each global uncertainty distribution and then added to a randomly chosen instance of model-dependent combined error. In the fourth and fifth steps, correlated simulated errors are used to define the coverage interval required in order to include the simulated errors simultaneously for each sensor. The validity of Ψ is verified and the coverage interval is adjusted for every comparison point in order to be sure that simulated errors are within the threshold bounds. Under the assumption that the uncertainties have been adequately evaluated, this procedure ensures that the right model is not wrongly discarded according to Ψ, the desired target reliability. The outcome obtained from filtering corresponds to the set of models that are able to explain the measured behavior while accounting for uncertainties and their correlations.

The system identification methodology explained here for the case of structural identification can be generalized to be applicable to any inverse problem where models and observations are compared.

CONCLUSIONS

Traditional model-updating approaches are only valid for situations where uncertainties are random and independent. Other situations are often called biased uncertainty. For most complex structures (for example, full scale bridges, buildings and dams) the uncertainties related to models do not fulfill such requirements.

Simulations performed on a full-scale bridge showed that uncertainties are correlated for both static and dynamic predictions. Therefore traditional model-updating techniques are not appropriate in such situations.

Model updating limitations related to randomness and independence of uncertainties may be overcome by an interpretation strategy called Candidate Model Search for System Identification (CMS4SI). Instead of judging a model by its ability to fit measured data, the approach is based on the principle that it is only possible to falsify a model. Therefore, a threshold which accounts for systematic and random uncertainties along with correlations is appropriate discarding models from the initial model set. This strategy is scientifically sound and further work is evaluating its universal applicability.

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REFERENCES

Bakir, P. G., Reynders, E., and Roeck, G. D. (2008). "An improved finite element model updating method by the global optimization technique [`]Coupled Local Minimizers'." *Computers & Structures*, 86(11-12), 1339-1352.

Beck, J. L., and Katafygiotis, L. S. (1998). "Updating Models and Their Uncertainties. I: Bayesian Statistical Framework." *Journal of Engineering Mechanics*, 124(4), 455-461.

Bell, E. S., Sanayei, M., Javdekar, C. N., and Slavsky, E. (2007). "Multiresponse parameter estimation for finite-element model updating using nondestructive test data." *Journal of Structural Engineering-Asce*, 133(8), 1067-1079.

Cheung, S. H., and Beck, J. L. (2009). "Bayesian Model Updating Using Hybrid Monte Carlo Simulation with Application to Structural Dynamic Models with Many Uncertain Parameters." *Journal of Engineering Mechanics*, 135(4), 243-255.

Goulet, J.-A., Kripakaran, P., and Smith, I. F. C. (2010). "Multimodel Structural Performance Monitoring." *Journal of Structural Engineering*, 136(10), 1309-1318.

Hadidi, R., and Gucunski, N. (2008). "Probabilistic Approach to the Solution of Inverse Problems in Civil Engineering." *Journal of Computing in Civil Engineering*, 22(6), 338-347.

Ka-Veng Yuen, J. L. B., Lambros S. Katafygiotis,. (2006). "Efficient model updating and health monitoring methodology using incomplete modal data without mode matching." *Structural Control and Health Monitoring*, 13(1), 91-107.

Plackett, R. L. (1972). "Studies in the History of Probability and Statistics. XXIX: The Discovery of the Method of Least Squares." *Biometrika*, 59(2), 239-251.

Ravindran, S., Kripakaran, P., and Smith, I. F. C. (2007). "Evaluation reliability of multiple-model system identification." 14th EG-ICE Workshop, Maribor, Slovenia.

Robert-Nicoud, Y., Raphael, B., Burdet, O., and Smith, I. F. C. (2005). "Model identification of bridges using measurement data." *Computer-Aided Civil and Infrastructure Engineering*, 20(2), 118-131.

Sanayei, M., Imbaro, G. R., McClain, J. A. S., and Brown, L. C. (1997). "Structural model updating using experimental static measurements." *Journal of Structural Engineering-Asce*, 123(6), 792-798.

Schlune, H., Plos, M., and Gylltoft, K. (2009). "Improved bridge evaluation through finite element model updating using static and dynamic measurements." *Engineering Structures*, 31(7), 1477-1485.

Tarantola, A. (2005). "Inverse Problem Theory and Methods for Model Parameter Estimation." SIAM, ed., Philaderphia.

Tarantola, A. (2006). "Popper, Bayes and the inverse problem." *Nat Phys*, 2(8), 492-494.

Teughels, A., and De Roeck, G. (2004). "Structural damage identification of the highway bridge Z24 by FE model updating." *Journal of Sound and Vibration*, 278(3), 589-610.

Yuen, K.-V., Beck, J. L., and Katafygiotis, L. S. (2006). "Unified Probabilistic Approach for Model Updating and Damage Detection." *Journal of Applied Mechanics*, 73(4), 555-564.