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ACOUSTIC IMPEDANCE SYNTHESIS AT THE DIAPHRAGM OF MOVING COIL LOUDSPEAKERS USING OUTPUT FEEDBACK CONTROL

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This paper discusses a time-domain technique for synthesizing acoustic impedance at the diaphragm of a loudspeaker using a proportional-plus-derivative output feedback. The dynamics of electroacoustic transducers such as moving-coil loudspeakers can be readily controlled either by direct feedback principle on acoustic quantities, or by plugging a shunt network at the electrical terminals. Any conventional loudspeaker first intended to be a sound transmitter may then become a versatile electroacoustic resonator capable of absorbing (or of reflecting as much) the incident sound energy in a frequency-dependent way by simple electronic controls. Instead of counteracting some unwanted sound by using superposition principle, as is the case for conventional active noise control, such actuator-based strategy aims at monitoring the reaction of a loudspeaker embedded into walls so as to control the proportion of reflected sound waves on this boundary. After a short description of the dynamics of moving-coil loudspeakers giving emphasis on the advantage of electromechanical coupling reversibility, a proportional-plus-derivative output feedback combined to a feed-forward action is proposed for synthesizing of desired acoustic impedance. As a conclusion, the overall performance of the proposed method is presented along with computed results and general discussions on practical implementation.

1. Introduction

The dynamics of electroacoustic transducers such as moving-coil loudspeakers can be readily controlled either direct feedback on acoustic quantities [1, 2, 3, 4], or by plugging a shunt networks at the electrical terminals [5, 6, 7]. With the help of such very basic control strategies variable acoustic impedance can be achieved at the transducer diaphragm. A conventional loudspeaker can then be transformed into a versatile electroacoustic resonator capable of absorbing sound energy within a large frequency range, typically more than a frequency decade around its natural frequency [7]. Generally speaking, feedback-based techniques are viewed as a specialized section of control engineering, whereas shunt-based methods relate more to a specialized section of electrical engineering. In the area of acoustic impedance control, both strategies aim at preventing (or at reinforcing) the reflection of incident sound waves by controlling the dynamics of the electroacoustic transducer. A common feature is that design procedures most often rely on the frequency response approach (Bode diagram) for determining the appropriate control parameters. The performance of the transient response is then specified in an indirect manner in terms of phase margin, gain margin, bandwidth, resonant peak

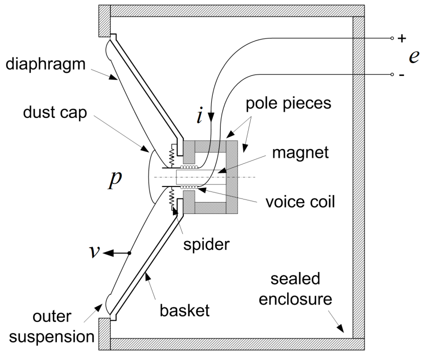

Figure 1. Schematic of a moving-coil loudspeaker in closed-box.

magnitude, etc., which depend on the ratio of the feedback gains [7]. One way of achieving a desired acoustic impedance by direct feedback is to apply to the electric terminals a command voltage which is proportional to a linear combination of both velocity and sound pressure at the vicinity of the diaphragm [3, 4, 8]. With a purely proportional feedback however, one single closed-loop pole can take on a pre-assigned value in the left hand part of the complex plane. In order to more effectively control the dynamics of a system, a proportional-plus-derivative (PD) output feedback may be introduced. [9]. The idea of synthesizing a desired acoustic impedance by designing a PD controller within state space approach is the main motivation for this paper.

The remainder of this paper is organized as follows. First, a brief description of the dynamics of a moving-coil loudspeaker will be detailed, giving emphasis on the advantage of electromechanical coupling reversibility. The problem of controlling the dynamics of the transducer will be then formulated in state space. A pole placement technique will be provided for determining the output feedback gains that satisfy a desired behavior for the closed-loop system. As a conclusion, the overall performance of the proposed method for synthesizing a desired acoustic impedance will be presented along with simulation results and general discussions on practical implementation.

2. System dynamics

2.1 Moving-coil loudspeaker model

A common description of the moving-coil loudspeaker for small displacements and below the first modal frequency of the diaphragm is given by the following set of linear differential equations [10]:

$$\begin{aligned} Sp(t) &= M_{ms}\,\dot{v}(t) + R_{ms}\,v(t) + \frac{1}{C_{mc}}\xi(t) - Bl\,i(t) \\ e(t) &= R_e\,i(t) + L_e\,\dot{i}(t) - Bl\,v(t) \end{aligned} \tag{1}$$

where $p(t)$ is the input sound pressure acting on the loudspeaker diaphragm, $v(t) = \dot{\xi}(t)$ is the diaphragm velocity, $\xi(t)$ is the diaphragm displacement, $i(t)$ is the electrical current flowing through the voice coil, and $e(t)$ is the input voltage applied at the electrical terminals. For the model parameters, S is the effective piston area, Bl is the force factor, M_{ms} and R_{ms} are the mass and mechanical resistance of the moving part, R_e and L_e are the dc resistance and the inductance of the voice coil. Here, $C_{mc} = (1/C_{ms} + \rho c^2/V_b)^{-1}$ is the equivalent mechanical compliance accounting for both the flexible edge suspension and spider of the loudspeaker and the cabinet, where ρ and c are the density and celerity of air and V_b is the volume of the cabinet. The terms $Bl\,i(t)$ and $Bl\,v(t)$ in Eq. (1) are the Laplace force induced by the current circulating through the coil and the back electromotive force (emf) induced by the motion of the voice coil within the magnetic field, respectively. They both express the electromechanical coupling on the mechanical and electrical side, respectively.

Table 1. Loudspeaker Visaton® AL-170 technical data.

Description	Notation	Value	Unit
dc Resistance	R_e	5.6	Ω
Voice coil inductance	L_e	0.9	mH
Force factor	Bl	6.9	T.m
Moving mass	M_{ms}	13	g
Mechanical resistance	R_{ms}	0.8	N.s.m^{-1}
Mechanical compliance	C_{ms}	1.35	mm.N^{-1}
Effective piston area	S	133	cm^2

2.2 Advantage of electromechanical coupling reversibility

Any conventional electroacoustic transducer can be employed either as a sound transmitter, when it converts electrical energy into acoustical energy, or as a sound receiver when operating in the opposite way [10]. When the system is driven by an auxiliary voltage source and also subjected to an exogenous sound source these conversion processes happen simultaneously. A complete description of the transducer implies thus to account for the nature of the input voltage. When operating as a sound transmitter for instance, an auxiliary voltage source $e_s(t)$ is connected at the electrical terminals and the driven voltage can be expressed as:

$$e(t) = e_s(t) - Z_s\, i(t) \tag{2}$$

where Z_s is the internal impedance of the power source. When operating in reverse as a sound receiver, $e_s(t)$ and Z_s may be tailored in view of modifying the transducer dynamics so that it more or less reflects some sound energy in a frequency-dependent way [7].

3. Problem formulation

3.1 State space representation

Looking at the loudspeaker as a dynamical system, the input voltage $e(t)$ can be viewed as a command (or controllable) input for controlling its dynamics. The output, namely the process variable we are interested in controlling, is the diaphragm velocity. Being not directly controllable, the sound pressure $p(t)$ may be regarded as a disturbance input that may affect the output. Likewise, the physical quantities $v(t)$, $\xi(t)$ and $i(t)$ in Eq. (1) are internal state variables since they are involved in the circulation of energy within the dynamical system. Introducing the state vector $\boldsymbol{x}(t) = (\xi(t), v(t), i(t))^T$ and the scalar command variable $u(t) = e(t)$ the set of equations of Eq. (1) can be written in a state form [11]. The general state space representation of this linear single-input-single-output (SISO) system can be expressed as:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\,\boldsymbol{x}(t) + \boldsymbol{b}\,u(t) + \boldsymbol{b}_p\,p(t) \\ y(t) &= \boldsymbol{c}^T\,\boldsymbol{x}(t) \end{aligned} \tag{3}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\dfrac{1}{C_{mc}M_{ms}} & -\dfrac{R_{ms}}{M_{ms}} & \dfrac{Bl}{M_{ms}} \\ 0 & \dfrac{Bl}{L_e} & -\dfrac{R_e}{L_e} \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{L_e} \end{bmatrix} \quad \boldsymbol{b}_p = \begin{bmatrix} 0 \\ \dfrac{S}{M_{ms}} \\ 0 \end{bmatrix} \quad \boldsymbol{c}^T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \tag{4}$$

Note that matrices $\boldsymbol{A}$, $\boldsymbol{b}$ and $\boldsymbol{b}_p$ are composed of the model parameters of the loudspeaker whereas the output matrix $\boldsymbol{c}^T$ depends on which state variable of $\boldsymbol{x}(t)$ is measured. Note also that a command variable $u(t)$ can be constructed to transfer the system from any initial output $y(t_0)$ to any final output

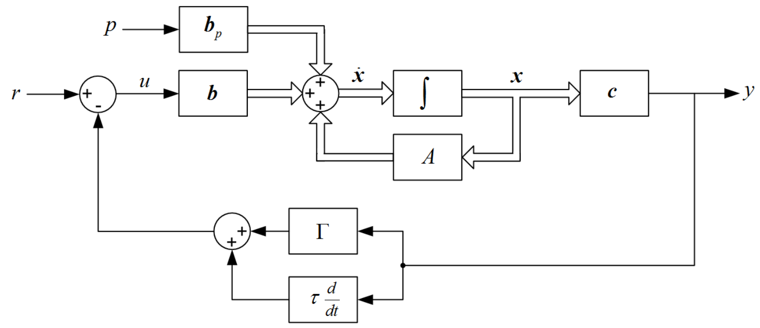

Figure 2. Block diagram of the system with proportional-derivative controller in feedback.

$y(t_1)$ in a finite time interval $t_0 \leq t \leq t_1$ since the observability matrix $\begin{bmatrix} \boldsymbol{c}^T & \boldsymbol{c}^T\boldsymbol{A} & \boldsymbol{c}^T\boldsymbol{A}^2 \end{bmatrix}^T$ is of rank 3, and the controllability matrix $\begin{bmatrix} \boldsymbol{c}^T\boldsymbol{b} & \boldsymbol{c}^T\boldsymbol{A}\boldsymbol{b} & \boldsymbol{c}^T\boldsymbol{A}^2\boldsymbol{b} \end{bmatrix}$ is of rank 1, meaning that the system is both observable and controllable [12].

3.2 Output feedback law

Generally speaking, designing a state (or output) feedback means developing an appropriate control law so that the closed-loop system behaves with acceptable performance in terms of static accuracy, disturbance rejection and transient response [11]. Output feedback control is simply more practical since it covers situations where all states are not accessible to measurement. A common strategy for implementing output feedback control relies on the pole placement techniques. The basic idea is to specify the desired location of the poles of the closed-loop system, and then determine the feedback gains to achieve these poles. Applying a proportional-plus-derivative output feedback law, as shown in Fig. 2, can be written as [9]:

$$u(t) = -\Gamma y(t) - \tau \dot{y}(t) + r(t) \tag{5}$$

where Γ and τ are constant proportional and derivative feedback gains and $r(t)$ is the reference value. The negative sign simply indicates a negative feedback on the output and its derivative.

3.3 Pole placement problem

The problem of pole placement of Eq. (3) is of determining the output feedback gains such that the closed-loop system poles take on pre-assigned values $\Lambda = \{\lambda_i,\ i = 1, 2, 3\}$. Substituting the command variable given by Eq. (5) in the system described by Eq. (3) yields the closed-loop system as:

$$\dot{\boldsymbol{x}}(t) = \left(\boldsymbol{I} + \boldsymbol{b}\tau\boldsymbol{c}^T\right)^{-1}\left(\boldsymbol{A} - \boldsymbol{b}\Gamma\boldsymbol{c}^T\right)\boldsymbol{x}(t) + \left(\boldsymbol{I} + \boldsymbol{b}\tau\boldsymbol{c}^T\right)^{-1}\left(\boldsymbol{b}_p p(t) + \boldsymbol{b} r(t)\right) \tag{6}$$

provided that $\boldsymbol{I}+\boldsymbol{b}\tau\boldsymbol{c}^T$ is invertible. The transient response of the closed-loop system is determined by the eigenvalues of the system matrix $\boldsymbol{A}_c = \left(\boldsymbol{I} + \boldsymbol{b}\tau\boldsymbol{c}^T\right)^{-1}\left(\boldsymbol{A} - \boldsymbol{b}\Gamma\boldsymbol{c}^T\right)$. Noting that the characteristic polynomial $H(s) = |s\boldsymbol{I} - \boldsymbol{A}_c|$ may be written in terms of the feedback gains Γ and τ as:

$$H(s) = \frac{1}{|\boldsymbol{I} + \boldsymbol{b}\tau\boldsymbol{c}^T|}\left|s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{b}\left(\Gamma + s\tau\right)\boldsymbol{c}^T\right| \tag{7}$$

and that the desired characteristic polynomial for a third-order system can be written as:

$$H(s) = \prod_{i=1}^{3}\left(s - \lambda_i\right) = s^3 + \beta_2 s^2 + \beta_1 s + \beta_0 \tag{8}$$

the output feedback gains are readily determined by equating Eq. (7) and Eq. (8). Substituting Eq. (4) into Eq. (7) the feedback gains can be expressed as a function of the loudspeaker's parameters and of coefficients of the desired characteristic polynomial as:

$$\begin{aligned} \Gamma &= \frac{R_{ms}R_e}{Bl} + \frac{L_e}{C_{mc}Bl} - Bl - \frac{L_e M_{ms}}{Bl}\beta_1 \\ \tau &= \frac{R_e M_{ms}}{Bl} + \frac{L_e R_{ms}}{Bl} - \frac{L_e M_{ms}}{Bl}\beta_2 \end{aligned} \tag{9}$$

Given that the closed-loop system order exceeds twice the number of inputs or outputs only two poles of the closed-loop system can be specified with the control law given by Eq. (5) [9]. In view of improving the dynamic compensation of the system by tempting to assign a third pole one additional output must be envisaged so that the system order does not exceed twice the number of inputs or outputs.

3.4 Improvements using feed-forward control

Generally speaking the control system performance can be improved by combining the feedback (or closed-loop) action of the proportional-integral-derivative (PID) controller with a feed-forward (or open-loop) element. In the case of controlling the dynamics of a loudspeaker, the feed-forward controller will anticipate the influence of all sound pressure disturbance on the diaphragm velocity and will deploy control actions so that the loudspeaker behaves as expected in a timely fashion.

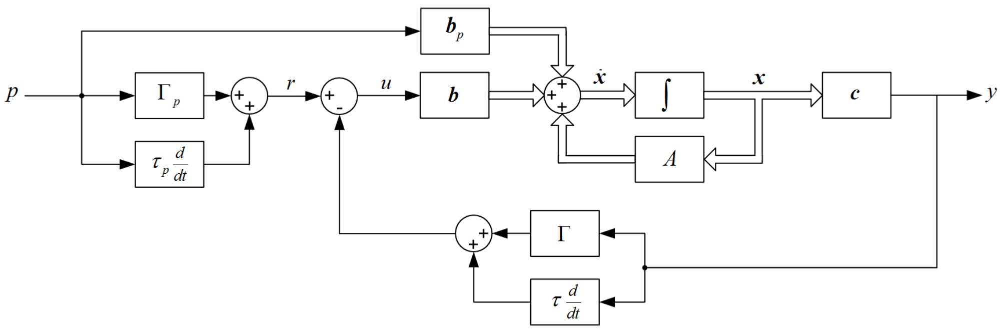

Figure 3. Block diagram of the system with proportional-derivative controllers in feedback and feed-forward.

In order to obtain a broadband absorption at the loudspeaker diaphragm, it has been shown that the ratio of the velocity feedback gain over the sound pressure gain should equal the target acoustic impedance value [7]. The feed-forward gains to match the impedance of the diaphragm to the characteristic impedance of air are simply given by:

$$\Gamma_p = \frac{\Gamma}{\rho c} \qquad \tau_p = \frac{\tau}{\rho c} \tag{10}$$

Now combining the feed-forward action to the control law given in Eq. (5) yields a new expression for the command voltage as:

$$u(t) = -\Gamma y(t) - \tau \dot{y}(t) + \Gamma_p\, p(t) + \tau_p\, p(t) = e(t) \tag{11}$$

Substituting Eq. (11) into Eq. (1) leads, after Laplace transform and some rearrangements, to the acoustic admittance $Y(s) = V(s)/P(s)$ of the electroacoustic absorber, where $V(s)$ and $P(s)$ are the Laplace transforms of velocity $v(t)$ and sound pressure $p(t)$, as:

$$Y(s) = \frac{s^2\left(SL_e + Bl\tau_p\right) + s\left(SR_e + Bl\Gamma_p\right)}{s^3 M_{ms}Le + s^2\left(M_{ms}R_e + R_{ms}L_e - Bl\tau\right) + s\left(R_{ms}R_e + \dfrac{L_e}{C_{mc}} - (Bl)^2 - Bl\Gamma\right) + \dfrac{R_e}{C_{mc}}} \tag{12}$$

Note that the feed-forward element may alter the zeros of the acoustic admittance, thus offering additional degrees of freedom for attaining any desired specifications. Regarding stability, the feed-forward action should never cause oscillation into the system since it is outside the feedback loop.

4. Results and discussion

To gain an understanding of the achievable performance from the PD feedback controller, the acoustic admittance of the loudspeaker diaphragm is computed for various control settings. The specifications of the Visaton® AL 170 low-midrange loudspeaker mounted in a sealed cabinet, the volume of which is $V_b = 10$ l, are given in Tab. 1. The different control settings considered in this section are listed in Tab. 2.

Table 2. Control settings of the different strategies investigated.

Case	Γ V.(m/s)$^{-1}$	τ V.(m/s^2)$^{-1}$	Γ_p V.Pa^{-1}	τ_p V.(Pa/s)$^{-1}$
Closed circuit	0	0	0	0
P controller	-40	0	0.1	0
PD controller	-40	-0.5	0.1	0.0012

As shown in Fig. 4, the proportional-derivative controller combined to a feed-forward action makes possible to assign two poles and two zeros such that the dynamics of the loudspeaker is better controlled than with a purely proportional controller. In the frequency domain this should result in an extension of the bandwidth where the diaphragm fully absorbs the sound energy. Working together, the combined open-loop feed-forward controller and closed-loop PD controller should provide a more responsive, stable and reliable control system. Implementing an electroacoustic absorber appears quite straightforward and consists of first adjusting the ratio of gains for achieving a desired acoustic resistance, and then increasing simultaneously the gains while their ratio remains constant with respect to stability margins.

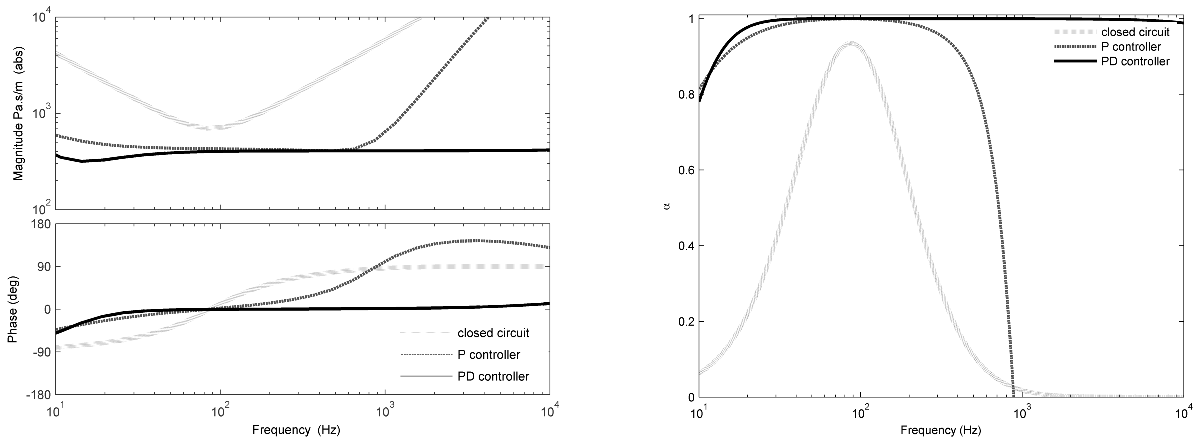

Figure 4. Bode diagram of the computed acoustic impedance and computed absorption coefficient.

5. Conclusions

In this paper a time-domain method has been described for synthesizing acoustic impedance at the diaphragm of an electroacoustic transducer by means of a proportional-plus-derivative (PD) output feedback controller combined to a feed-forward action. It as been shown that the problem of synthesizing an acoustic impedance can be solved by the techniques developed for pole assignment throughout control theory. In comparison with a purely proportional output feedback that is usually used for acoustic impedance matching, the PD controller introduces one additional degree of freedom for achieving a desired acoustic impedance without increasing the order of the system. Expected results are a better control of the dynamics of the electroacoustic absorber in terms of rise time, maximum overshoot, and settling time, and hence some improvement on both bandwidth and stability.

6. Acknowledgements

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REFERENCES

[1] H.F. Olson and E.G. May, Electronic sound absorber, *Journal of the Acoustical Society of America*, **25**(6), pp. 1130-1136, (1953).

[2] E. De Boer, Theory of motional feedback, *IRE Transaction on Audio*, **9**(1), pp 15-21, (1961).

[3] P. Darlington, Loudspeaker circuit with means for monitoring the pressure at the speaker diaphragm, means for monitoring the velocity of the speaker diaphragm and a feedback circuit, World Patent No WO 1997/003536, (1997).

[4] X. Meynial, Active acoustic impedance control device, World Patent No 1999/059377, (1999).

[5] R.J. Bobber, An active transducer as a characteristic impedance of an acoustic transmission line, *Journal of the Acoustical Society of America*, **48**(1B), pp. 317-324, (1970).

[6] A.J. Fleming,D. Niderberger, S.O.R Moheimani, Control of resonant acoustic sound fields by electrical shunting of a loudspeaker, *IEEE Transaction on Control System Technology*, **15**(4), pp. 689-703, (2007).

[7] H. Lissek, R. Boulandet, R. Fleury, Electroacoustic absorbers: bridging the gap between shunt loudspeakers and active sound absorption, *Journal of the Acoustical Society of America*, **129**(5), (2011).

[8] M. Furstoss and D. Thenail and M-A. Galland, Surface impedance control for sound absorption: direct and hybrid passive/active strategies, *Journal of Sound and Vibration*, **203**(2), pp. 219-236, (1997).

[9] H. Seraji, M. Tarokh, Design of proportional-plus-derivative output feedback for pole assignment, *Proc. IEEE*, **124**(8), pp. 729-732, (1977).

[10] M. Rossi, *Audio*, Presses Polytechniques et Universitaires Romandes, (2007).

[11] R.C. Dorf, R.H. Bishop, *Modern control systems* (11th Ed.), Pearson Prentice Hall, (2008).

[12] K. Ogata, *Modern control engineering* (3rd Ed.), Prentice Hall International, (1997).