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Estimation of Modelling Errors in Structural System Identification

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ABSTRACT: The study presented in this paper builds upon previous research into system identification using multiple models at EPFL. This identification methodology is based on the selection of sets of candidate models such that differences between measurements and their predictions are below a certain threshold value. The threshold value is a function of modeling and measurement errors. This paper presents results from a preliminary study on the estimation of modeling errors in structural system identification. Modeling errors have many sources, such as uncertainty in parameter values and finite element discretization. In addition to defining the threshold value, evaluating errors may also help minimizing errors and choosing better measurement locations. Two types of modelling errors are studied. The first is the error introduced by neglecting the effect of secondary structural elements such as roadway barriers and the second is due to element discretization. A composite truss bridge is used as a case study. Results from the study show that the errors due to exclusion of a secondary structural element may be significant and in such cases, they must be taken into account for system identification. Therefore displacement or rotation measurements are better for structural identification than strain measurements near beam-slab interfaces. The approach used and the results described are expected to contribute to increasing the accuracy of probabilistic studies on full scale bridges.

1 INTRODUCTION

Most existing bridges have been prudently designed using modeling assumptions that lead to safe and economical structures. However, when taking management decisions such as repair and augmenting the load-carrying capacity, design models may not be appropriate. On-site measurement often reveals important differences between measurement data and predictions of design models. A systematic approach to interpretation of measurement data employs methodologies developed in the field of system identification (Ljung 1999). Since system identification is an inverse engineering approach, many models may predict the same measured behaviour. Raphael and Smith (1998) observed that errors coming from model and measurements may compensate each other, leading to the identification of wrong candidate behavioural models. To overcome this difficulty, the authors have proposed a multi-model approach which involves the generation of thousands of possible behavioural scenarios. Candidate models are selected by comparing the difference between values from measurement and from predictions with a threshold value that is determined using estimates of modeling and measurement errors (Kripakaran et al. 2007; Raphael and Smith 1998; Raphael and Smith 2003; Robert-Nicoud et al. 2005).

This paper focuses on the estimation of modeling errors. Accurate simulations require features such as appropriate mesh refinement, adequate element types and correct representation of the structure and physical parameters such as material proprieties. The effects of modeling parameters have to be ascertained in order to estimate and minimize modeling errors. The few studies that consider modeling errors examine only aleatory variations which include statistical uncertainties in modeling parameters (Cheung and Beck 2009; Frangopol et al. 2008; Lee et al. 2005; Mahadevan and Rebba 2006; McFarland and Mahadevan 2008; Sanayei et al. 2001). Epistemic errors that are caused by inadequate modeling assumptions are seldom considered. Current probabilistic approaches are often not reliable because epistemic errors are not known. A multiple model probabilistic approach to structural identification (Ravindran et al. 2007) must incorporate both epistemic and aleatory uncertainties in order to assess the behavior of complex coupled systems such as bridges.

Epistemic uncertainties arise due to modeling assumptions made during the identification process. One assumption could be that secondary structural elements have a negligible contribution to the structural stiffness. Design models usually do not include secondary structural elements since their purpose is to lead to a safe structure designed for limit states. However, the goal of structural identification is to identify the model that represents the true state of the structure. Hence the influence of secondary elements has to be evaluated. Several authors have already assessed the influence of secondary structural elements on the strength and on transverse load distribution (Akinci et al. 2008; Chung et al. 2006; Conner and Huo 2006; Eamon and Nowak 2002; Eamon and Nowak 2004; Mabsout et al. 1997a). However, none of these authors have studied the influence of secondary elements in order to improve structural system identification. Furthermore, the elements studied by these authors are principally barriers and sidewalks. While performing system identification, other parameters (ex: a sloped profile of the deck, haunch, pavement and concrete reinforcement), may also influence the response of the structure. Excluding the effects of such parameters may lead to the identification of wrong candidate models.

System identification using the multi-model approach requires evaluations of thousands of candidate models. Consequently, the solution time for an evaluation has to be kept to a minimum. Element types and mesh size affect the solution time. Practitioners seldom use models with solid elements due to computing constraints. A combination of beam and shell elements is commonly used to simulate bridges. Users must employ these elements with care since all element types have limitations. Each element type is based on specific fundamental hypotheses and is not suitable for all situations (ANSYS 2007; Chapelle and Bathe 1998; Cook et al. 2007; Macneal 1994). For example, shells are bi-dimensional elements that may adequately simulate the behavior of components that have a small thickness compared to its other dimensions. Model discretization may also be an important source of modeling errors. When using a combination of shells or beams and shell elements to form a composite section, results may be inaccurate if a sufficiently refined mesh is not used (Crisfield 1991; Gupta and Ma 1977; Miller 1980).

The objective of this paper is to estimate modelling errors so that reliable threshold values can be found for candidate models. Two sources of modelling errors are studied. The first is the exclusion of secondary structural elements and the second is model discretization. A composite truss bridge is used to illustrate these two types of errors.

2 SYSTEM IDENTIFICATION

The framework of multiple-model system identification research at EPFL is shown in Figure 1. The identification strategy is an iterative process that employs measurements for identification of candidate behavioral models and then information from candidate models to improve the measurement system. At the beginning, modeling assumptions and measurements from the initial measurement system are provided by engineers. Modeling assumptions define the parameters to be employed within the identification task. The set of model parameters consists of quantities such as elastic constant, connection stiffness and moment of inertia. Each set of values for the model parameters corresponds to a model of the structure. The model generation module compares measurement values with model predictions in a stochastic search to generate sets of candidate models. Candidate models are those for which the difference between the measurement at each location and the corresponding prediction lies below a threshold.

Setting an appropriate threshold value is important for generating the set of candidate models. A simple way to compute the threshold is to add the absolute value of the measurement and modeling errors. However, this may lead to a large threshold value and therefore a large set of candidate models. On the other hand, if the threshold value is too low (i.e. assuming errors are smaller than they actually are), the right model may not be included in the candidate set.

Complex structures have large numbers of candidate models. These models are input for the data mining and feature extraction module. Data mining techniques are used to extract relationships between models and cluster similar models. The measurement system design module uses the entropy among predictions of representative models from model clusters as a criterion to determine locations for subsequent measurements. This module also has algorithms for designing the initial measurement system for a structure. The model generation module, data mining module and measurement system design module involve human interaction and this interaction is accommodated within the engineer-computer interaction module. For example, this module has visualization tools for displaying results from the data mining module.

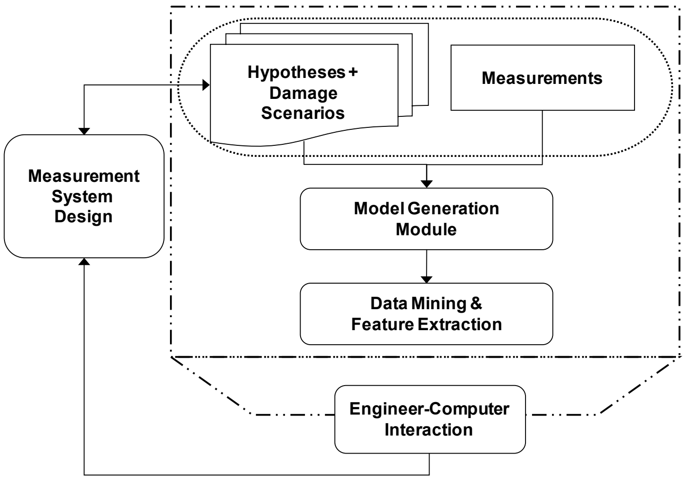

Figure 1 – System identification methodology using multiple models

This paper focuses on the incorporation of modeling errors into the system identification process. Modeling errors are classified into two classes - epistemic and aleatory. Epistemic errors arise from the lack of knowledge about the behavior of a structure, the use of simplified hypothesis, or modeling assumptions. Aleatory error describes uncertainties that are related to the parameter values. For example this type of error is associated with statistical variations in physical parameters such as Young's modulus, plate thicknesses, and loading values. Modeling errors are evaluated in three steps.

1. Identify parameters that introduce aleatory and epistemic uncertainties.
2. Evaluate the sensitivity of model predictions such as strains and displacements to the parameters.
3. Combine uncertainties associated with modeling parameters to find a threshold value for selection of candidate models.

This paper evaluates the epistemic uncertainties that originate from modeling assumptions such as the exclusion of secondary structural elements and the level of model discretization. The objective is to investigate the effects of modeling assumptions on the model predictions. The following section describes the evaluation of modeling errors through a case study of a composite truss bridge.

3 CASE STUDY

The example presented here is a preliminary study carried out on a multi-span composite truss bridge. The model is composed of six continuous spans covering a total length of 300 meters. A finite element model of the bridge is given in Figure 2. Because of the size of the structure, it is not feasible to use solid elements. Therefore, a combination of beam and shell elements is used.

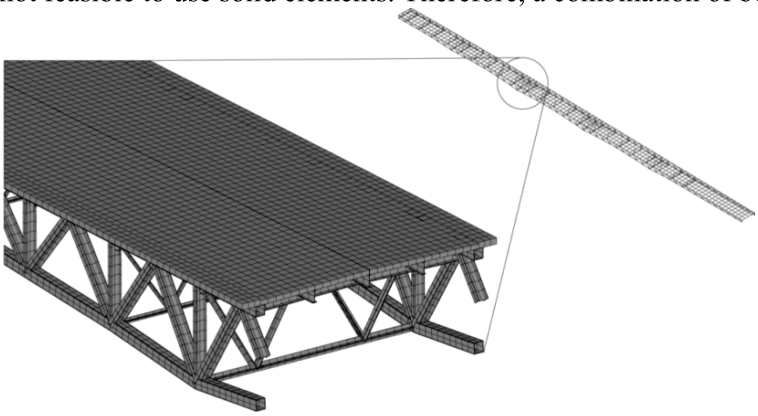

Figure 2 – Finite element model of the multi-span composite truss bridge

As the system identification process requires thousands of evaluations of the model, a mesh with 150,000 elements is chosen so that the solution time is reasonable (< 10 min per evaluation).

Only a part of the full bridge is used to assess the impact of modeling errors. This substructure represents a 20 m single span of the bridge. In addition to the concrete slab and main truss, the model includes secondary elements that are often neglected during design such as secondary beams, sloped deck, pavement, haunch, sidewalk barrier and concrete reinforcement. The model as analyzed in the ANSYS software is presented in Figure 3. This modeling approach using beams and shells has already been extensively tested by previous researchers and has been shown to adequately represent the behavior of composite members (Chiewanichakorn et al. 2004; Chung and Sotelino 2006; Dall'Asta and Zona 2004; Mabsout et al. 1997b). To show the influence of secondary elements, models that do not include a given type of secondary element are analyzed and the results are then compared with the results from the analysis of a model that includes all secondary elements. This approach brings out the influence of each parameter.

The other aspect of the study is the relation between predictions at a given location and mesh refinement. Sampled locations are presented in Figure 4. Strain predictions at the intersections of axis S3-T3, S5-T3 and S6-T1 are recorded at four different locations, namely, the upper side of the truss top chord, the haunch bottom chord, the haunch top chord and the bottom chord of the slab. In order to ensure that a modeling error is not introduced by the use of beam and shell elements to create a composite structure, the strains at the upper side of the truss top chord and the bottom part of the haunch are compared. These strain values should be the same since those two points coincide.

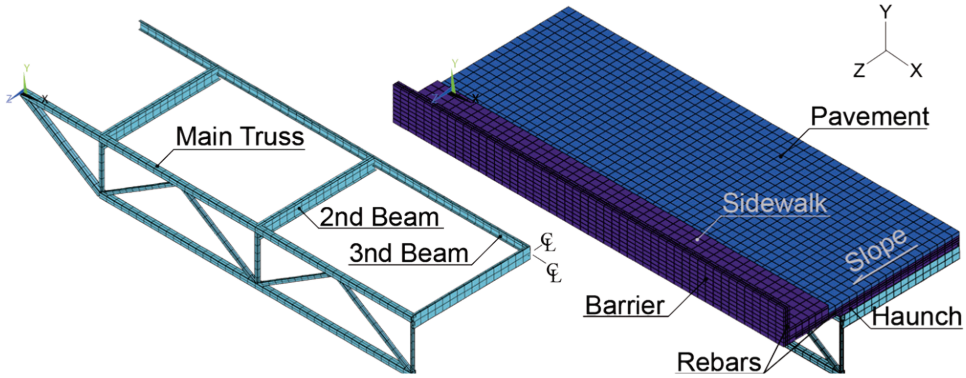

Figure 3 – Finite element model used in the case study

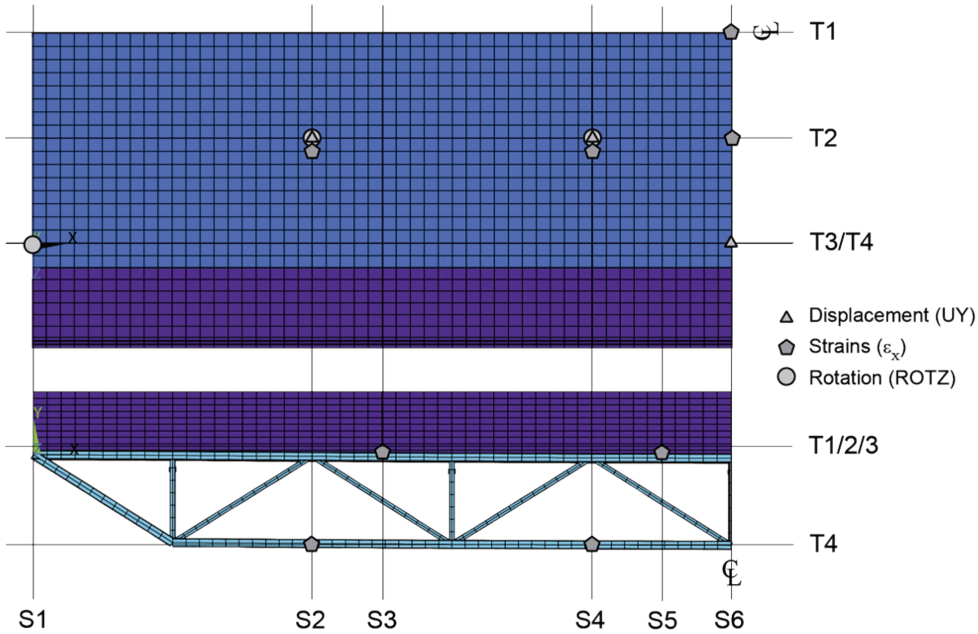

Figure 4 – Sampled locations used in the case study

4 RESULTS

4.1 Influence of secondary elements

The first part of the study on the influence of secondary elements indicates that the inclusion of these features is important. If these elements are not included (mass and stiffness) in the model, the engineer should increase the value of the threshold in order to account for the error caused by neglecting a given feature. According to the sensor locations studied, the averaged relative errors taken over the minimum and maximum prediction values for each type of prediction (rotation, displacement and strains) are presented in Table 1. A model containing all secondary elements is taken to be the reference (error = 0). Strain predictions located in the concrete deck are more sensitive to the omission of secondary structural elements. Relative errors from these strain predictions also show more variability (ranging from less than 4% to more than 160%), compared to other prediction types or locations. The behavior of a secondary element is often not understood. For example in the case of a partial interaction between the barrier and the concrete slab, strain predictions from the slab will have larger errors than other types of predictions.

Table 1 – Estimated error (%) caused by neglecting secondary elements

	Averaged relative error (%)			
Model feature excluded	Displacement	Rotation	Strains ε_x-Steel Truss (Bottom chord)	Strains ε_x-Slab
No element excluded		≡ 0		
Slope	3.4	3.5	3.8	14.1
Haunch	6.5	7.8	2.6	13.2
Sidewalk	16.3	17.9	13.6	53.7
Barrier	22.2	19.1	22.3	33.2
Pavement	13.6	13.3	10.9	39.6
Concrete Reinforcement	1.3	1.2	1.2	3.2
All elements above excluded	**57.8**	**54.3**	**43.1**	**167.6**

4.2 Influence of mesh discretization

The second part of this paper describes a study on the error due to mesh refinement. The objective is to evaluate the relationship between mesh refinement and the errors in prediction types and locations. Figure 5 presents the relative errors in strain predictions with respect to the level of mesh refinement defined in terms of the number of elements used in the model. Figure 6 similarly presents the errors in displacement and rotation predictions. In each graph it is indicated the maximum number of elements to use in order to keep solving time reasonable (< 10 min per simulation for the full bridge). Since the full bridge require 30 times more elements than the substructure studied, the number of elements used must remain in this case below 5,000.

Strain predictions located at the point of application of the load (eX-S4T2) and along the longitudinal axis of beam-shell interface (eX-S6T1) have a slow rate of convergence. Strain results taken in the concrete deck and at locations far from discontinuities (eX-S2T2) or at locations perpendicular to them (eX-S6T2) have an acceptable rate of convergence (error ≈ 1%). Finally, predictions for the bottom chord of the truss (eX-S4T4) show a fast rate of convergence (error <<1%).

The stress discontinuity at the beam-shell interface results in a relative error that reaches up to 8% when using 5,000 elements. A model composed of 250 elements shows a relative error that exceeds 700% at the discontinuities. Results presented in Figure 6 show that displacement and rotation predictions at all locations have a relatively low error (<<1%).

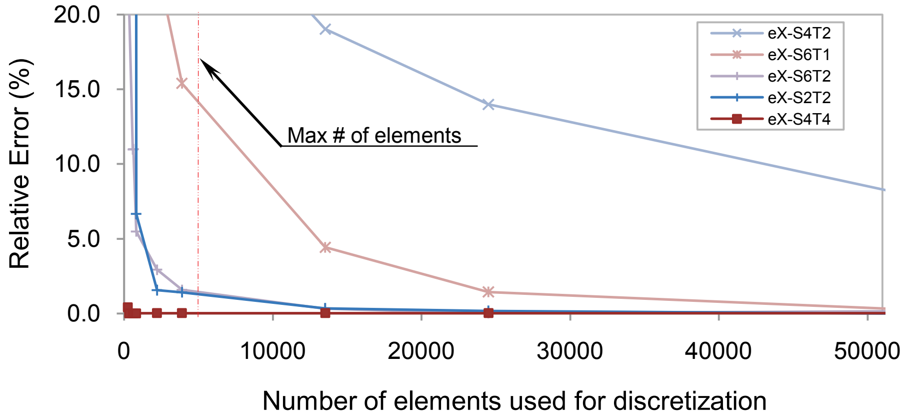

Figure 5 – Relative error in strain predictions in relation to the number of elements used for discretization

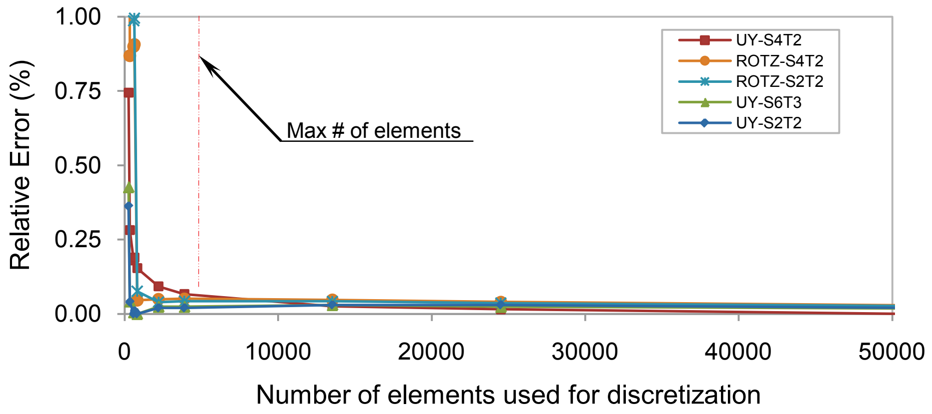

Figure 6 – Relative errors in displacement and rotation predictions in relation to the number of elements used for discretization

4.3 Discussion

The results presented here give only a qualitative description of the locations where errors are too high. The relation between the influence zone and the size of the geometric or element type discontinuity will be assessed in future work. While results are applicable for all truss bridges, the influence of discontinuities and secondary elements may vary with the geometry and material properties. Similar behaviour is also observed on others types of structures. Future work will identify the extent of the phenomenon. Another result from this study is that regions where truss members are joined together should not be selected for measurement. These regions are usually associated with high stress gradients and therefore model predictions are unlikely to be robust.

5 CONCLUSIONS

The conclusions of this research are:

- Secondary structural features such as haunch, pavement, sloped deck profile, concrete reinforcement, barrier and sidewalks have a significant influence on the response. Therefore system identification tasks must take into account modelling errors due to the omission of secondary elements when they are not included in the analysis. Moreover, strain predictions located in the deck are more sensitive to the omission of secondary elements.

- For the composite truss bridge studied in this paper, the omission of secondary elements results in strain predictions in the concrete deck that are more erroneous than displacement or rotation predictions when using the same mesh density. Consequently, strain measurements may necessitate the use of a highly-refined finite element model that is computationally expensive for multiple-model structural identification.
- Evaluating modelling errors with respect to different response types can support decisions related to sensor types to be chosen for measurement. For the composite truss bridge, displacement and rotation predictions support structural identification using finite element models of lower mesh densities compared with strain predictions from within the interface zone in the concrete deck.
- Strain predictions near connections between beams and slabs are inaccurate unless the numerical model for the structure uses a highly refined mesh. This conclusion is also valid for predictions at the position of application of a large concentrated load such as near truck wheels.
- The research improves the knowledge of errors involved when performing numerical simulations on composite bridges. Results are expected to improve the quality of methods for probabilistic system identification by having explicitly showed potential inaccuracy caused by epistemic modelling errors.

Future work will determine quantitatively the influence zone of the discontinuities as mentioned above. This study will also be applied to other types of structures.

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