Tensegrity modules for pedestrian bridges

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Abstract

Tensegrity systems are spatial structures composed of tensile and compression components in a self-equilibrated state of prestress. The tensegrity concept offers good structural efficiency and results in modular and lightweight structures. Research into tensegrity systems has resulted in reliable techniques for form finding and structural analysis. However, designing tensegrity systems for structural engineering applications is not yet part of mainstream structural design. Indeed, few examples of tensegrity structures have been used for structural engineering purposes.

This paper presents a tensegrity-ring pedestrian bridge designed according to Swiss structural engineering practice. The tensegrity pedestrian bridge is designed by assembling elementary self-stressed modules. Bridge configurations designed with different elementary modules are evaluated for the same span, internal clear space and load cases. A parametric study is performed to identify parameters that most affect the behavior of the structure. A comparison of configurations and descriptions of feasible configurations are included. Static analysis is conducted with a dynamic-relaxation algorithm. Design requirements for pedestrian bridges and results of parametric studies are used to propose a design procedure for this type of structure. A structural efficiency indicator is also presented and used to compare proposals for feasible bridge configurations. Design results illustrate that the tensegrity-ring topology can efficiently meet bridge-design criteria.

Keywords: tensegrity system, pedestrian bridge, structural design, structural efficiency.

1. Introduction

Tensegrity systems are a special class of spatial reticulated structures that are composed of struts and tendons. Tensioned and compressed components are assembled in a self-equilibrated system providing stability and stiffness to the structure. Over the last decades, the tensegrity concept has received significant interest among scientists and engineers throughout disciplines such as architecture, civil engineering, biology, robotics and aerospace. However, few examples of tensegrity structures have been used for civil engineering purposes. Paronesso and Passera proposed a tensegrity platform for the 2002 Swiss National Exhibition in Yverdon [1] and designed the tensegrity roof for the velodrome in Aigle (Switzerland). Schlaich, Bergermann and Partners built the Rostock tower (Germany) in 2003 assembling six "simplex" modules [2]. Schlaich [3] concluded that despite their inherent flexibility the potential of tensegrities for tower and roof structures is substantial. Studies on double layer tensegrity grids were initiated by Motro and Hanaor [4]. Hanaor [5] presented design aspects of double layer tensegrity grids. Quirant et al [6] studied the different stages of tensegrity grid design. A design procedure was developed and applied to a double layer tensegrity grid covering a surface of 81 m^2.

Research into tensegrity structures revealed that only small amounts of energy are needed to change the shape of such systems. Tensegrities are thus regarded as an attractive solution for controllable and deployable systems. Fest et al. [7] experimentally explored potential of active tensegrities on a five-module tensegrity structure. A quasi-static control strategy based on stochastic search is first proposed to satisfy serviceability criterion [8]. The control strategy is then extended to take into account additional robustness objectives [9]. Smaili and Motro [10] presented a design study of curved tensegrity systems and investigated the deployment process requirements of such structures. Motro et al [11] proposed a new family of tensegrity cells called "tensegrity rings" that can be assembled in a "hollow rope" and provided a general method for creating these foldable tensegrity cells starting from any n-sided prism. The concept of "hollow rope" shows promise for architecture and civil engineering applications such as pedestrian bridges. The increasing amount of research into tensegrity systems has resulted in reliable techniques for form finding and structural analysis. However, there has been relatively little research on design strategies for structural purposes. Thus, there is a need to develop systematic methods for tensegrity design.

This paper presents a tensegrity "hollow rope" pedestrian bridge that is designed according to Swiss civil-engineering practice. The tensegrity pedestrian bridge is designed by assembling elementary self-stressed modules. Three bridge configurations designed with different elementary modules are studied having the same span, internal space and load cases. A parametric study is performed to identify parameters that significantly affect the behavior of the structure. A comparison of different configurations and a description of the feasible configuration are included. Static analysis is conducted with a dynamic-relaxation algorithm. Finally, an extension of the design method is proposed for tensegrity bridges.

2. A tensegrity pedestrian bridge

The proposed structure is a pedestrian bridge with a span of 20 m. The structural system of the bridge is composed of four identical tensegrity modules (Figure 1). Symmetry about midspan is obtained by mirroring two modules.

Figure 1: The tensegrity pedestrian bridge

The structure of the bridge is designed with a rectangular internal free space with a height of 2.5 m and a width of 1.3 m. The bridge deck is supported on the bottom nodes at both ends of each module. All nodes at both ends of the bridge are blocked.

Pugh [12] proposed elementary tensegrity modules of polygonal form which have important open spaces at their centers. In these modules, each strut end is jointed to another to create circuits of struts within outer cable circuits. Previous work [11] has classified them as "*ring modules*" because of their hollow tube shape. A generic construction method for ring modules based on the geometry of straight prisms was proposed. This method provides regularity for elementary ring module topology. Three self-stressed ring modules are considered in this study. All three modules were designed and compared considering the bridge geometry requirements for length and minimum pedestrian space.
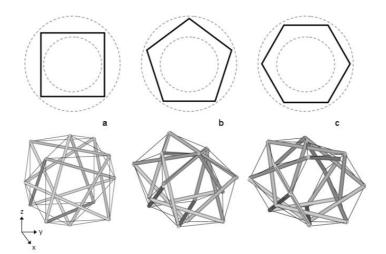
Figure 2: Cable polygons inside the ring: a) square b) pentagon and c) hexagon

Tensegrity ring modules that are studied in this paper are called square, pentagon and hexagon modules. Modules are named after the polygon defined by the cables on a module face (x-y plane). Figure 2 shows the cable polygons inside their rings for each module.

A main step in the characterization of a tensegrity system is the computation of the number of infinitesimal mechanisms and the number of independent states of self-stress [13]. Analysis of equilibrium matrix for pre-stressed ring module configurations showed that continuous ring modules (pentagon and hexagon) have no infinitesimal mechanisms and six independent states of self-stress while the square module has one infinitesimal mechanism with a base of seven elementary self-stress states.

3. Parametric analysis

3.1. Analysis

Although, the concept of tensegrity has activated interest in various fields, designing tensegrity systems for civil-engineering applications is not yet part of mainstream structural engineering. Additionally, available design codes and guidelines do not accommodate these structural systems.

Tensegrity structures behave nonlinearly in response to external loading. Since tensegrities are self-stressed and flexible, displacements can be large even for small deformations of critical elements. The behavior of this kind of structures is thus complex, and furthermore a great number of design parameters have to be taken into consideration. A complete analysis of a tensegrity system comprises three steps: form finding, implementation of self-stress and finally, the study of behavior under external loads. Form finding of a tensegrity system consists of determining a stable self-stressed state. A review of form finding methods can be found in [14]. Another important aspect of the behavior of tensegrity structures is geometric nonlinearity. Nonlinear modeling and iterative computational schemes are needed to design tensegrities. In this study, static analyses of tensegrity bridge configurations are performed using dynamic relaxation with kinetic damping. The maximum kinetic energy peak, determined through the calculation of consecutive nodal velocity vectors at each time increment, is chosen as new reference state. All nodal velocities are then reset to zero and the calculations are continued until the unbalanced forces converge to zero. The dynamic relaxation method does not require an assembled stiffness matrix. Moreover, its equations are always expressed in terms of current coordinates of the structure. Therefore, it is particularly suitable for structures having geometrical nonlinearities.

3.2 Parametric study

Parametric studies help to estimate the influence of different design parameters on the structural behavior of tensegrity bridges. The study is carried out for the three elementary configurations of ring modules. These configurations have identical element cross-sectional areas and self-stress level. Additionally, they sustain identical service loads. Design loading includes the dead load of elements except joints, pedestrian loads, snow and a horizontal

load. Each configuration is composed of four identical modules and has the same boundary conditions.
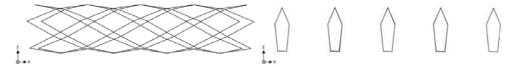

Figure 3: Illustration of the x-cables (left) and layer cables (right)

Five parameters are examined: the cross-sectional area of x-cables, the cross-sectional area of layer cables, the cross-sectional area of struts, the rigidity ratio strut-cable and the self-stress. X-cables are cables that envelop the struts, while layer cables are found at the ends of every module (Figure 3). Following standard practice for parametric analysis, a range of values for each parameter is studied while other parameters are kept unchanged revealing the influence of each parameter assuming mutual independence. The default configuration of the structure has x-cables and layer cables with a cross-section area of 1.5 cm^2 as well as struts with a cross-section area of 6 cm^2. The elastic modulus of cables is 115 GPa while the elastic modulus of struts is 210 GPa. The average vertical displacement at midspan is chosen to compare the structural performance of the configurations.

3.2.1 Cross-sectional area of x-cables and layer cables

The evolutions of the average vertical displacement at mid-span with respect to the cross-sectional area of x-cables and layer cables are illustrated in Figure 4 for all three configurations.
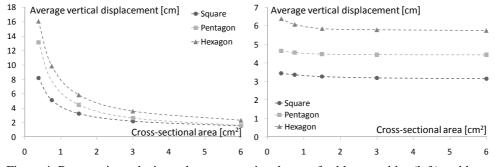

Figure 4: Parametric analysis on the cross-sectional area of cables: x-cables (left) and layer cables (right)

Results illustrated in Figure 4 show a common trend for the three modules: increasing the cross-sectional area of cables decreases the average vertical displacement at mid-span. However, the reduction on the displacement magnitude due to an increment of the cross-sectional area of layer cables is smaller than the one observed due to x-cables. An

increment of the cross-sectional area of the x-cables results in an increased stiffness of the structure without significantly increasing dead loads. An observation emerging from the parametric analysis of the cross-sectional area of x-cables is that there is a difference in the magnitude of displacements between the three modules. This difference can be attributed to the additional weight that is introduced by the presence of additional elements.

3.2.2 Cross-sectional area of struts

Ring modules have continuous struts which increase their bending stiffness compared with modules having discontinuous struts. Struts are placed respecting a rotational regularity around the longitudinal axis of the module. However, there is no symmetry in any y-z plan (Figure 5).
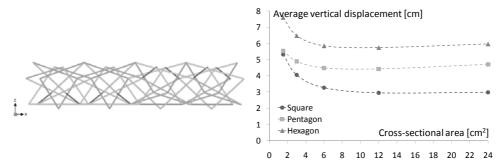

Figure 5: Illustration of struts for the pentagonal bridge and results of the parametric analysis on the cross-sectional area of struts

Displacement results in relation with the cross-sectional area of struts are shown in Figure 5. Differences in the magnitude of displacements between the three modules are again observed. Increasing the cross-sectional area of the struts decreases the vertical displacement at mid-span until reaching a cross-sectional area of approximately 12 cm^2. For all configurations the reduction in displacement becomes insignificant beyond this value of the cross-sectional area. In fact, increasing the cross-section of the struts higher than this value induces a small increment in the vertical displacement at midspan. This suggests that the increase in the stiffness of the structure is counter-balanced by an equivalent increase in dead loads.

3.2.3 Rigidity ratio cables – struts

The structural elements of a tensegrity system experience only axial loading. Tensioned and compressed elements are assembled in a self-equilibrated system providing stability and stiffness to the structure. Cable and strut characteristics were studied separately in previous sections. Since the stiffness of a tensegrity is made by the contribution of both cable and strut stiffness, the influence of the rigidity ratio between tensioned and compressed members on the behavior of the tensegrity structure was studied. The rigidity ratio includes the elastic axial rigidity expressed in Eq. (1).

$$k = \frac{EA}{L} \qquad (1)$$

where E is the modulus of elasticity, A is the cross-sectional area and L is the length of the element. As cross-sectional areas are identical for all three configurations, the ratio depends on the modulus of elasticity and the length of the elements. In order to study the influence of the rigidity ratio on the average vertical displacement, five modulus of elasticity are taken into account for cables while the modulus of elasticity is kept constant for struts. The values considered for cable modulus of elasticity are 10 GPa, 50 GPa, 115 GPa, 150 GPa and 210 GPa.
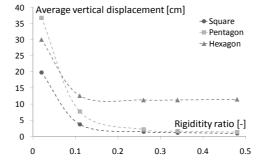

Figure 6: Parametric analysis on the rigidity ratio of steel struts and cables

For the three configurations of the bridge, the average displacement at midspan is calculated for a range of rigidity ratios (Figure 6). The evolution of midspan displacement with respect to the ratio of rigidity between cables and struts reveals a common tendency for the three modules: increasing the rigidity ratio results in a decrease in the average vertical displacement at midspan. However, the reduction in displacement becomes small beyond a certain value for the rigidity ratio. Consequently, an optimal rigidity ratio between tensile and compressive elements can be identified for each configuration. This optimal value can be used to guide the design of a tensegrity structure. For example, if design values for cross-sectional areas of struts and cables lead to a smaller rigidity ratio compared with the reference ratio, this leads to a flexible structure having large deflections. Similarly, a rigidity ratio that is higher than the reference ratio provides extra stiffness without a significant reduction in deflection.

3.2.4 Self-stress

Another important parameter of tensegrity structures is self-stress. The self-stress state is responsible for the stability and the high resistance of tensegrity structures. Self-stress can be created in many ways. In this study, cables are assumed to be pre-tensioned. Consequently, tensile stresses are induced in cables while compression occurs in the struts. Pre-stress is specified in terms of tensile ratio: a ratio between the pre-stressing internal force and the tensile strength of the cable. From a global point of view, element self-

stresses contribute to the overall rigidity of the structure. However, self-stress acting on structural elements reduces their local capacity.
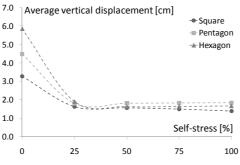

Figure 7: Parametric analysis on the self-stress

Average displacement at the mid-span section of the bridge is illustrated with respect to the self-stress level in relation with the tensile strength in Figure 7. In this case, differences in the magnitude of displacements between the three modules are observed only for values of self-stress smaller than 25 %. The evolution of midspan displacement with respect to the level of self-stress reveals also a common tendency: increasing self-stress has a positive influence on the displacement until reaching a plateau. The plateau starts at a self-stress level of approximately 25 % of cable tensile strength for the three modules.

4. Design procedure

The parametric study showed that a number of design parameters influence the behavior of the pedestrian bridge. Designing a tensegrity bridge could require repeated analyses followed by modifications. Material properties, self-stress level and rigidity ratio have to be chosen to ensure safety and serviceability of the structure.

Most modern structural codes, such as the SIA Swiss codes, define safety and serviceability criteria for footbridge design. Safety criteria ensure overall stability and include verifications of element resistance and stability. The serviceability limit state requirements for a pedestrian bridge involve limiting vertical displacement at mid-span to satisfy functionality and user comfort as well as appearance criteria. In a study of tensegrity-structure design, Quirant [6] proposed a design procedure that was applied in the case of a double layer tensegrity grid. In a similar approach and based on the conclusions of the parametric study presented in previous sections, a design procedure is proposed and tested for the tensegrity bridge (Figure 8).

The design procedure starts with a pre-defined topology for a tensegrity ring module. Once the topology and nodal positions are known, constitutive materials need to be chosen. The material choice influences the self-weight and the rigidity of the structure. Thus, it is also important for the rigidity ratio between tensile elements and compressive elements. The design rigidity ratio has to be defined for a peculiar topology based on parametric study. The next step in the design procedure is to choose a value for the cross-sectional area of

struts. Strut area is governed by slenderness limitation. The reference rigidity ratio permits the calculation of the corresponding cross-sectional area for cables. Self-stress requires a particular attention during the design process due to its dual action as a load for the elements and a rigidity amplifier for the entire structure. First, an analysis under self-weight is conducted in order to verify if both sections and self-stress state need to be modified. The self-stress level and element cross-section areas are increased progressively until the deflection limitation is satisfied. High deflection may require the definition of higher sections for the elements. The structure is then analyzed according to serviceability and ultimate limit states criteria. If one of the criteria is violated, then new cross-sectional areas have to be defined.

The design procedure provides feasible configurations using the three tensegrity ring modules. For the three configurations, steel cables with a Young modulus of 115 GPa are used. Hollow tubes having a Young modulus of 210 GPa are used for compressive components. Required levels of self-stress for the different configurations are similar. Feasible configurations obtained using the proposed design procedure are presented in Table 1.

Table 1: Feasible configurations of the tensegrity bridge

	Strut diameter × strut thickness [cm]	Cross-sectional area of struts [cm^2]	Cross-sectional area of x-cables [cm^2]	Cross-sectional area of layer cables [cm^2]
Square	9.5 × 3.2	9.23	2.83	1.00
Pentagon	9.5 × 3.6	10.34	3.00	3.00
Hexagon	10.1 × 3.6	11.10	3.00	3.00

5. Structural efficiency

The structural efficiency of tensegrity ring modules is compared in order to identify the best design of the tensegrity pedestrian bridge. Although all three ring modules studied in this paper have some similarities, they also have differences, such as the number of elements and the length of elements. Therefore, in order to have a fair comparison, the bridge configurations are designed under the same procedure and under similar conditions (self-stress, loads, boundary conditions, etc.).

A Structural Efficiency Index (SEI) is used for the comparison of the proposed tensegrity ring modules. This index is inspired from work conducted by Vu et al. [15]. The index is composed of three parts (Eq. 2) and takes into account design loads, self-weight, span and maximum deflection:

$$SEI = \alpha \times \beta \times \gamma \qquad (2)$$

where
$$\alpha = \frac{L+W}{W}, \quad \beta = \frac{S}{D} \quad \text{and} \quad \gamma = \frac{L}{D} \qquad (3)$$

L is the sum of design loads without their corresponding partial factors, W the self-weight, S the span of the bridge and D the maximum deflection at midspan. A high SEI index corresponds to a high structural efficiency. For the studied case, design loads are calculated according to the span and the internal space of the bridge that are identical for every configuration. Consequently, design loads are also the same.
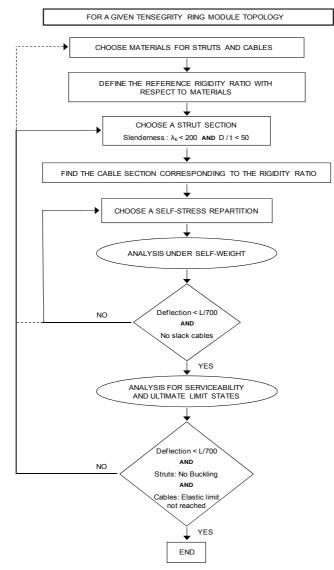

Figure 8: Design procedure for tensegrity ring structures

The first part of SEI reflects the efficiency according to the self-weight of the structure. It includes only the self-weight and loads applied to the structure. These parameters are significant especially for short span structures. In this case, design is usually guided by strength requirements (ULS) rather than serviceability constrains (SLS). The second part of SEI is a common criterion for serviceability including span over deflection. The serviceability limit for this bridge is set to S/D = 700. The last part of the SEI involves the overall rigidity of the structure through relating "pay load" and deflection. The values for the pentagon module are used to normalize values of SEI. Values of the SEI are given in Table 2.

Table 2: Structural Efficiency Index, normalized by values for the pentagon module

	$\dfrac{\alpha}{\alpha_{pentagon}}$	$\dfrac{\beta}{\beta_{pentagon}}$	$\dfrac{\gamma}{\gamma_{pentagon}}$	$\dfrac{SEI}{SEI_{pentagon}}$
Square	0.98	0.75	0.75	0.55
Pentagon	1.00	1.00	1.00	1.00
Hexagon	0.88	0.60	0.60	0.32

Results presented in Table 2 show that the pentagon bridge presents the highest SEI. The pentagon module offers a high structural efficiency regarding the three SEI components. Compared to square and hexagon configurations, the pentagon has a higher self-weight efficiency and its serviceability performance is also better due to a higher overall rigidity. The square configuration presents a self-weight efficiency that is almost as high as the pentagon. However, its serviceability performance and overall rigidity are lower. Finally, the hexagon bridge has the overall lowest structural efficiency.

6. Conclusions

Tensegrity-structure design is a challenging task due to geometrical complexity and closely coupled behavior. The most efficient module is the pentagon ring and the least efficient is the hexagon. This conclusion employed a newly developed structural efficiency index.

A useful extension of the tensegrity grid design guideline includes the ratio of rigidity between cables and struts as well as material properties. Increasing this ratio decreases vertical displacements until reaching a limit that varies with material properties. Tensegrity structures are efficient when properly designed. Further work involves construction and testing a prototype of the pentagon tensegrity bridge.

Acknowledgements

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