Model-Free Precompensator Tuning Based on the Correlation Approach

Alireza Karimi, Mark Butcher and Roland Longchamp

***Abstract*— High performance output tracking can be achieved by precompensator or feedforward controllers based on the inverse of either the closed-loop system or the plant model, respectively. However, it has been shown that these inverse controllers can adversely affect the tracking performance in the presence of model uncertainty. In this paper, a model-free approach based on only one set of acquired data from a simple closed-loop experiment is used to tune the controller parameters. The approach is based on the decorrelation of the tracking error and the desired output and is asymptotically not sensitive to noise and disturbances. From a system identification point of view the stable inverse of the closed-loop system is identified by an extended instrumental variable algorithm in the framework of errors-in-variables identification methods. By a frequency-domain analysis of the criterion, it is shown that the weighted two-norm of the difference between the controller and the inverse of the closed-loop transfer function can be minimized. The method is successfully applied to a high precision position control system.**

***Index Terms*— Feedforward control, correlation, linear motor, data-driven approach**

I. INTRODUCTION

TWO-DEGREE of freedom controllers are largely used when disturbance rejection and reference signal tracking are both considered as closed loop performance criteria. In many cases, the feedback controller is first designed to ensure robust stability and satisfy the disturbance rejection specification. Then, in the second step, a precompensator (Fig. 1) is designed to improve the tracking performance. If the plant model is perfectly known, this problem can be converted to a standard model matching problem and can be solved either analytically or by using convex optimization algorithms. However, a perfect model of the plant is never available and a nominal model with some uncertainty bounds should be considered for the design (see eg. [1], [2], [3]). When the uncertainty is above a certain level the tracking performance of the system can, in fact, be adversely affected [4].

Another approach, when a mathematical model of the plant is not available, is to tune directly the controller parameters using the data acquired from some simple experiments. This approach is called model-free because a model of the plant or closed-loop system is not used for controller tuning and measured data are directly used to tune the controller parameters and to minimize a control criterion. In [5] a method for designing feedforward controllers using data in the frequency domain is proposed. Whilst this approach is well suited to the model matching problem, it requires that time-domain specifications for the closed-loop system be approximated in the frequency domain. Iterative Feedback Tuning (IFT) is a model-free approach for tuning the parameters of two-degree of freedom controllers based on some specific closed-loop experiments to compute an unbiased estimate of the gradient of the control criterion [6]. Separate tuning of the feedforward and the feedback controllers is proposed to improve the tracking performance using the IFT approach in [7]. A similar iterative method based on the correlation approach has been proposed and successfully applied to a magnetic suspension system [8] as well as a benchmark problem [9]. An overview of this approach together with the theoretical results can be found in [10]. The main idea is that instead of minimizing the performance error, it is made uncorrelated with the reference signal. It can be shown that in this case the noise has asymptotically no effect on the controller parameter estimates. A characteristic feature of these iterative approaches is that they require many experiments on the real system for criterion evaluation. The main interest of a new controller tuning method called Virtual Reference Feedback Tuning (VRFT) is that only one set of data is required to tune a controller for the model reference problem [11]. In this approach the controller tuning problem is transformed to an identification problem by defining a virtual reference signal. It can be shown that if the ideal controller (which makes the closed-loop system equal to the reference model) is in the set of parameterized controllers, it is also the minimizer of the identification criterion. However, if it is not the case, an approximation of the control criterion is minimized. An extension of this method to two-degree of freedom controllers is given in [12].

In this paper, the correlation approach will be used to tune the parameters of the precompensator controller such that the tracking error becomes uncorrelated with the desired output. The key point is to introduce a new tuning scheme in which the position of precompensator and the closed-loop system is swapped. This way, the evaluation of control criterion does not require a new experiment on the system and only one set of data is sufficient for tuning of precompensator parameters. Therefore, similar to the VRFT approach, parameter estimation algorithms can be used for "identifying" the controller parameters. However, in the tuning scheme the measurement noise is added to the input of the precompensator, which makes the problem more difficult than classical identification problems. This problem is recognized as Errors In Variables (EIV) in literature [13]. Here, it is

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shown that the use of extended instrumental variables with a specific choice of instruments leads to unbiased estimates of the parameters. Moreover, frequency-domain analysis of the criterion shows that even when the ideal controller does not belong to the parameterized set of controllers the two-norm of the difference between the reference model and the closed-loop system can be minimized. The proposed approach can be applied straightforwardly to tuning of the feedforward controllers. The details have been omitted in this paper but can be found in [14].

The technique proposed is tested experimentally on a linear motor typically used in the semiconductor manufacturing industry where rapid, high precision motions are required thus making a two-degree of freedom controller architecture a near necessity (see eg.[15]).

The paper is organized as follows. Notation and preliminaries about the correlation approach are given in Section II. The precompensator tuning scheme together with the one shot tuning algorithm and frequency-domain analysis are presented in Section III. Simulation and experimental results are presented in Section IV. Finally, the concluding remarks are given in Section V.

II. PRELIMINARIES

Let the output $y(t)$ of a SISO linear time-invariant plant model $G(q^{-1})$ be described by:

$$y(t) = G(q^{-1})u(t) + v(t) \tag{1}$$

where $u(t)$ is the plant input, $v(t)$ a weakly stationary random process and q^{-1} the backward-shift operator. Assume that the controller $K(q^{-1})$ stabilizes the unknown plant model $G(q^{-1})$ in closed-loop with unit feedback. It is also assumed that the disturbance effect at the closed-loop output is zero-mean, i.e. either $v(t)$ is zero-mean or the controller contains an integrator. The objective is to improve the closed-loop tracking error $e(t) = y_d(t) - y(t)$ using only one set of data acquired in closed-loop operation. This can be done by filtering the desired output with a precompensator before applying it as a reference signal to the closed-loop system (see Fig. 1).

In principle, the precompensator F should be a stable (not necessarily causal, if $y_d(t)$ is a priori known) approximation of the inverse of the closed loop transfer function.

A. Controller Parameterization

Let F be parameterized as:

$$F(\rho, q^{-1}) = \beta^T(q^{-1})\rho \tag{2}$$

where $\rho^T = [\rho_0, \rho_1, \ldots, \rho_{n_\rho}]$ is the vector of controller parameters and $\beta(q^{-1})$ the vector of linear discrete-time transfer operators:

$$\beta^T(q^{-1}) = [\beta_0(q^{-1}), \beta_1(q^{-1}), \ldots, \beta_{n_\rho}(q^{-1})]. \tag{3}$$

The elements of vector $\beta^T(q^{-1})$ can be any orthogonal basis functions. In the sequel, for simplicity of presentation, we suppose that $\beta^T(q^{-1}) = [q^{\delta}, q^{\delta-1}, \ldots, q^{\delta-n_\rho}]$ which leads to the following FIR model for F:

$$F(\rho, q^{-1}) = \rho_0 q^{\delta} + \rho_1 q^{\delta-1} + \cdots + \rho_{n_\rho} q^{\delta-n_\rho} \tag{4}$$

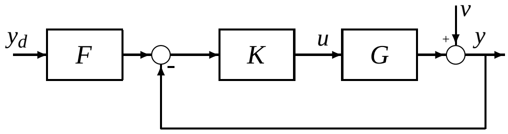

Fig. 1. Closed-loop system with precompensator

where δ is a positive scalar. In fact, the desired output is applied δ sampling periods in advance to the real system to improve the tracking error [16]. For the sake of simplicity, q^{-1} will be omitted when appropriate in the rest of the paper.

B. Correlation Approach

It is evident that if the exact inverse of the closed-loop system exists the tracking error $e(t)$ will contain only the contribution of the noise. Hence, it is reasonable to adjust the controller F in such a way that the tracking error $e(t)$ be uncorrelated with the desired output. For many systems, the exact inverse does not exist because the system is non minimum phase or of infinite order. As a result, $e(t)$ is always correlated with the desired output. However, it can be considered that a good controller F minimizes the correlation between the tracking error $e(t)$ and the desired output $y_d(t)$. In order to formulate this idea as an optimization problem, let the correlation function $f(\rho)$ be defined as:

$$f(\rho) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E\{\zeta(t)e(t)\} \tag{5}$$

where N is the number of data, $E\{\cdot\}$ denotes the mathematical expectation and

$$\zeta^T(t) = [y_d(t+n), \ldots, y_d(t), y_d(t-1), \ldots, y_d(t-n)] \tag{6}$$

with $l = 2n+1$ the dimension of $\zeta(t)$. In fact $\zeta(t)$ is a vector of instrumental variables correlated with $y_d(t)$ and uncorrelated with $v(t)$. Now, a new control criterion based on the correlation approach is defined as:

$$J(\rho) = \| f(\rho) \|_2^2 = f^T(\rho) f(\rho) \tag{7}$$

and the optimal controller parameters are:

$$\rho^* = \arg\min_{\rho} J(\rho). \tag{8}$$

Since the control criterion involves the mathematical expectation, an exact solution when only one finite set of data is available, is not attainable. However, with an ergodicity assumption on the input signal, a good estimate of the correlation function can be given by:

$$\hat{f}(\rho) = \frac{1}{N} \sum_{t=1}^{N} \zeta(t)e(t) \tag{9}$$

where N should be large enough with respect to l. The estimate of the correlation function leads to the following criterion:

$$J_N(\rho) = \| \hat{f}(\rho) \|_2^2 = \hat{f}^T(\rho)\hat{f}(\rho). \tag{10}$$

The criterion $J_N(\rho)$ goes to $J(\rho)$ when N tends to infinity. A global minimizer of $J_N(\rho)$ can be derived using the least squares algorithm. This solution together with an asymptotic frequency-domain analysis is presented in the next section.

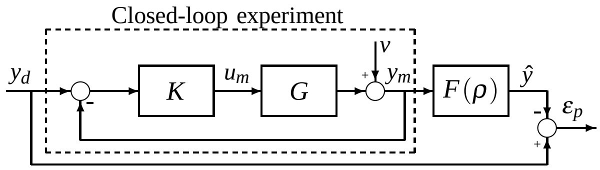

Fig. 2. Precompensator tuning scheme

III. PRECOMPENSATOR TUNING

In this section, we propose a tuning scheme to find the parameters of the precompensator F based on the correlation approach. The problems of control input weighting and controller order selection are also investigated.

A. Tuning scheme

The tracking error can be computed as (see Fig. 1):

$$e(t) = y_d(t) - y(t) = y_d(t) - F(\rho)Ty_d(t) - Sv(t) \quad (11)$$

where $T = KG(1+KG)^{-1}$ and $S = (1+KG)^{-1}$ are the closed-loop sensitivity functions. Computing $e(t)$ for different values of ρ requires many experiments on the system that can be avoided by a new tuning scheme in which the position of the closed-loop system and precompensator is interchanged so that F acts as a post-compensator (see Fig. 2). It should be mentioned that this can be done only for SISO LTI systems. In this scheme $u_m(t)$ and $y_m(t)$ are the measured input and output of the plant from a closed-loop experiment with the desired output $y_d(t)$ as the reference signal. An estimate of the tracking error now can be computed with only one set of data as follows:

$$\begin{aligned} \varepsilon_p(t) &= y_d(t) - \hat{y}(t) = y_d(t) - F(\rho)y_m(t) && (12) \\ &= y_d(t) - F(\rho)Ty_d(t) - F(\rho)Sv(t) && (13) \end{aligned}$$

It is clear that in the absence of noise ($v(t) \equiv 0$) $e(t)$ and $\varepsilon_p(t)$ are equal. However, even in the presence of noise we have:

$$f(\rho) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E\{\zeta(t)e(t)\} = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E\{\zeta(t)\varepsilon_p(t)\} \quad (14)$$

if the disturbance signal $v(t)$ is independent of $y_d(t)$.

It should be mentioned that the variance of the tracking error $E\{e^2(t)\}$ is not equal to the variance of the tracking error estimate $E\{\varepsilon_p^2(t)\}$. Therefore, the minimization of the variance of the tracking error cannot be carried out with only one experiment and should be done iteratively with several experiments on the real system.

B. Algorithm

The estimate of the tracking error $\varepsilon_p(t)$ can be presented in the linear regression form:

$$\varepsilon_p(t) = y_d(t) - F(\rho)y_m(t) = y_d(t) - \phi^T(t)\rho \quad (15)$$

where

$$\phi^T(t) = [y_m(t+\delta), y_m(t+\delta-1), \ldots, y_m(t+\delta-n_\rho)]. \quad (16)$$

This leads to the following expression for the correlation function estimate:

$$\hat{f}(\rho) = \frac{1}{N} \sum_{t=1}^{N} \zeta(t)[y_d(t) - \phi^T(t)\rho] = Z - Q\rho \quad (17)$$

where

$$Z = \frac{1}{N} \sum_{t=1}^{N} \zeta(t)y_d(t) \quad , \quad Q = \frac{1}{N} \sum_{t=1}^{N} \zeta(t)\phi^T(t) \quad (18)$$

Finally, if Q^TQ is nonsingular (i.e. $y_d(t)$ is sufficiently rich), straightforward calculation gives:

$$\hat{\rho} = (Q^TQ)^{-1}Q^TZ \quad (19)$$

where $\hat{\rho}$ is the global minimizer of the correlation criterion in (10).

C. Frequency-domain analysis

The correlation criterion in (7) can be reformulated as:

$$J(\rho) = f^T(\rho)f(\rho) = \sum_{\tau=-n}^{n} R_{ey_d}^2(\tau) \quad (20)$$

where $R_{ey_d}(\tau)$ is the cross-correlation function between the desired output $y_d(t)$ and the tracking error $e(t)$ defined by:

$$\begin{aligned} R_{ey_d}(\tau) &= \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E\{e(t)y_d(t-\tau)\} \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E\left\{[y_d(t) - F(\rho)Ty_d(t)]y_d(t-\tau)\right\} && (21) \end{aligned}$$

The correlation criterion can be represented in the frequency domain by applying Parseval's theorem when n tends to infinity:

$$\begin{aligned} \lim_{n\to\infty} J(\rho) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |\Phi_{ey_d}(\omega)|^2 d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} |1 - F(\rho, e^{-j\omega})T(e^{-j\omega})|^2 \Phi_{y_d}^2(\omega) d\omega && (22) \end{aligned}$$

where $\Phi_{ey_d}(\omega)$ is the cross-spectral density between $e(t)$ and $y_d(t)$ and $\Phi_{y_d}(\omega)$ is the spectral density of $y_d(t)$. This expression shows that:

- The criterion is asymptotically unaffected by noise.
- ρ^* the minimum of the correlation criterion in (7) gives $F(\rho^*) = T^{-1}$ in the ideal case (i.e. T is minimum phase and F is properly parameterized).
- If $y_d(t)$ is white noise the correlation criterion becomes

  $$J(\rho) = \|1 - F(\rho)T]\|_2^2$$

  so the difference between FT and 1 is minimized in the two-norm sense using the correlation approach.
- If $y_d(t)$ is a deterministic signal $|1 - FT|$ is minimized in the frequencies where the spectrum of y_d is large.

Remarks:

1) The model following problem in two-norm also can be treated with this model-free approach. Consider that we aim to compute the precompensator F such that

$$\|M - F(\rho)T\|_2$$

is minimized. To proceed, let us define

$$\varepsilon_M(t) = My_d(t) - \phi^T(t)\rho$$

and compute ρ such that $\varepsilon_M(t)$ is not correlated with $y_d(t)$ which is chosen to be a white noise signal independent of $v(t)$. If for practical reasons, $y_d(t)$ cannot be chosen as a white noise but can be expressed as $y_d(t) = H(q^{-1})w(t)$ where $w(t)$ is a white noise, the use of filtered error $H^{-1}\varepsilon_M(t)$ and filtered instrumental variable $H^{-1}\zeta(t)$ leads to minimization of $\|M - F(\rho)T\|_2$.

2) If instead of the correlation criterion in (10) the variance of $\varepsilon_p(t)$ is minimized, unacceptable results may be obtained even if the noise to signal ratio is not very high. This can be shown by the frequency expression of the variance of $\varepsilon_p(t)$:

$$E\{\varepsilon_p^2(\rho)\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left\{|1 - F(\rho, e^{-j\omega})T(e^{-j\omega})|^2\Phi_{y_d}(\omega) + |F(\rho, e^{-j\omega})S(e^{-j\omega})|^2\Phi_v(\omega)\right\} d\omega \quad (23)$$

It is clear that in order to minimize the variance of the estimate of the tracking error the two positive terms in the integral should be minimized. However, minimizing the first term requires that $F(\rho)$ be close to a high-pass filter (as T is usually a low-pass filter) which consequently increases the effect of high-frequency noise in the second term of the integral.

3) Although, theoretically n in (22) should go to infinity in order to obtain the frequency interpretation of the criterion, in practice a large value for n is sufficient. The reason is that $R_{ey_d}(\tau)$ is close to zero for τ greater than the settling time of the impulse response of the transfer function between $y_d(t)$ and $e(t)$ when $y_d(t)$ is white noise. This gives a guideline to choose the value of n. Additionally the number of data N should be chosen much greater than n (e.g. $N > 10n$).

D. Control input weighting

When a precompensator is added to the feedback controller to improve the tracking performance, it is possible that the control input becomes too large for certain desired outputs. Therefore, it is reasonable to take into account the control input in the design of the precompensator. The measured control input $u_m(t)$ that corresponds to the control input when the feedback controller alone is used can be represented by $u_m(t) = KS[y_d(t) - v(t)]$ (see Fig. 2). In the presence of the precompensator, an estimate of the control input can be obtained by $\hat{u}(t) = F(\rho)u_m(t) = \varphi^T(t)\rho$ where

$$\varphi^T(t) = [u_m(t+\delta), u_m(t+\delta-1), \ldots, u_m(t+\delta-n_\rho)] \quad (24)$$

It is clear that in the absence of noise, $\hat{u}(t)$ is equal to $u(t)$ and in the presence of noise we have:

$$g(\rho) = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E\{\zeta(t)u(t)\} = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E\{\zeta(t)\hat{u}(t)\}. \quad (25)$$

Now in order to consider the spectrum of the control input in the control design, let the following correlation criterion be defined:

$$\begin{aligned} J(\rho) &= f^T(\rho)f(\rho) + \lambda g^T(\rho)g(\rho) \\ &= \sum_{\tau=-n}^{n} R_{ey_d}^2(\tau) + \lambda R_{uy_d}^2(\tau) \end{aligned} \quad (26)$$

where $R_{uy_d}(\tau)$ is the cross-correlation function between the control input and the desired output, and λ a positive scalar weighting factor. This new criterion can be interpreted in the frequency-domain as:

$$\begin{aligned} \lim_{n\to\infty} J(\rho) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \left[|\Phi_{ey_d}(\omega)|^2 + \lambda|\Phi_{uy_d}(\omega)|^2\right] d\omega \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \left[|1 - F(\rho, e^{-j\omega})T(e^{-j\omega})|^2 \right. \\ &\left. + \lambda|F(\rho, e^{-j\omega})K(e^{-j\omega})S(e^{-j\omega})|^2\right] \Phi_{y_d}^2(\omega)d\omega. \end{aligned} \quad (27)$$

Therefore, using the criterion in (26) and an appropriate choice of λ the magnitude of the frequency response of the control input can be reduced in the frequency range where the spectrum of the desired output is large.

For a finite number of data, an approximation of the criterion can be obtained by:

$$J_N(\rho) = \hat{f}^T(\rho)\hat{f}(\rho) + \lambda\hat{g}^T(\rho)\hat{g}(\rho) \quad (28)$$

where

$$\hat{g}(\rho) = \frac{1}{N}\sum_{t=1}^{N} \zeta(t)\varphi^T(t)\rho = Q_u\rho \quad (29)$$

and

$$Q_u = \frac{1}{N}\sum_{t=1}^{N} \zeta(t)\varphi^T(t). \quad (30)$$

The global minimum of this criterion is given by:

$$\hat{\rho} = (Q^TQ + \lambda Q_u^TQ_u)^{-1}Q^TZ. \quad (31)$$

E. Controller structure selection

The controller structure given in (4) has only two parameters δ and n_ρ to be chosen. Here, a simple algorithm to select these parameters is presented. It is clear that for a given value of δ, increasing n_ρ will reduce the correlation criterion in (7). However, it is not reasonable to continue increasing the controller order when the design objective (decorrelation of the tracking error and the desired output) is already achieved. It can be shown that if $J_N(\rho)$ is within a confidence interval the tracking error and the desired output can be considered uncorrelated. This confidence interval can be computed using the fact that, when the tracking error and the desired output are uncorrelated, the random variable $\sqrt{N}\hat{R}_{\varepsilon_p y_d}(\tau)$ converges in distribution to a normal distribution when N goes to infinity [17]:

$$\sqrt{N}\hat{R}_{\varepsilon_p y_d}(\tau) = \frac{1}{\sqrt{N}}\sum_{t=1}^{N} \varepsilon_p(t)y_d(t-\tau) \to \mathcal{N}(0, P) \quad (32)$$

where

$$P = \sum_{\tau=-\infty}^{\infty} R_{\varepsilon_p}(\tau)R_{y_d}(\tau) \quad (33)$$

with $R_{\varepsilon_p}(\tau)$ and $R_{y_d}(\tau)$ being the autocorrelation functions of $\varepsilon_p(t)$ and $y_d(t)$, respectively. Thus from the criterion (10) it follows that:

$$\frac{N}{P} J_N(\rho^*) \rightarrow \chi^2(l) \tag{34}$$

where ρ^* is the parameter vector that achieves decorrelation. Denoting the α-level of the $\chi^2(l)$ distribution as $\chi^2_\alpha(l)$ the condition to be satisfied in selecting the controller order is:

$$J_N(\rho) \leq \frac{\hat{P}}{N}\chi^2_\alpha(l) \tag{35}$$

where $\hat{P}$ is an estimate of P based on the calculated parameter vector ρ. This condition allows an algorithm to be proposed for the selection of the values of the parameters δ and n_ρ (the order selection can be done using a new data set as well):

Algorithm: $n_\rho = 1$

I : $\delta^* = \arg\min_{\delta} J_N(\rho, n_\rho, \delta)$ for $\delta = 0 : \delta_{max}$

if $J_N(\rho, n_\rho, \delta^*) \leq \frac{\hat{P}}{N}\chi^2_\alpha(l)$

stop; $n^*_\rho = n_\rho$ and $\rho^* = \rho$

else $n_\rho = n_\rho + 1$ and Goto I

IV. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Results

Example 1: The correlation approach for precompensator tuning is applied to a closed-loop system presented by the following transfer function:

$$T(q^{-1}) = \frac{0.2q^{-4}}{1 - 1.5q^{-1} + 0.7q^{-2}}. \tag{36}$$

This simple model is used in this example as its exact inverse exists in FIR form and therefore the precompensator $F = T^{-1}$ should be found by the proposed method under ideal conditions. The desired output is the response of the discrete-time second-order system:

$$\frac{0.0941q^{-1} + 0.0708q^{-2}}{1 - 1.262q^{-1} + 0.4274q^{-2}} \tag{37}$$

to a square-wave signal (between -1 and 1) of six periods (number of data is $N = 1200$). The desired output $y_d(t)$ is applied to the closed-loop system without precompensator to obtain the simulated measured output as:

$$y_m(t) = T(q^{-1})y_d(t) + S(q^{-1})v(t) \tag{38}$$

where $S(q^{-1}) = 1 - T(q^{-1})$ and $v(t)$ is a uniformly distributed zero-mean white noise with a variance of 0.00746. Fig. 3 shows one period of the desired output $y_d(t)$ and the measured noisy output without precompensator $y_m(t)$. The precompensator tuning algorithm in Eqs.(17)-(19), together with the controller structure selection algorithm, are used to calculate the precompensator. A value of $n = 25$ is chosen approximately based on the estimated settling time of $y_m(t)$. Fig. 4 shows the value of $J_N(\rho, n_\rho, \delta^*)$ for different values of n_ρ. Additionally the corresponding values of $\frac{\hat{P}}{N}\chi^2_\alpha(l)$ for $\alpha = 0.05$ and $l = 2n + 1 = 51$ are shown. It is clearly seen that the condition (35) is satisfied for $n_\rho \geq 2$, thus $n_\rho = 2$ was chosen. Additionally 4 sampling periods of preview ($\delta = 4$)

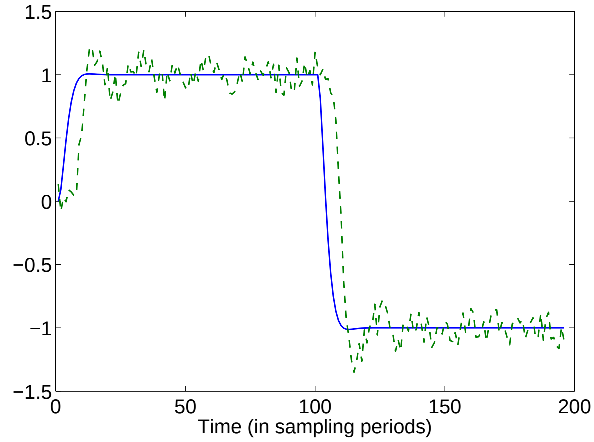

Fig. 3. Desired output (solid) and measured, noisy output without precompensator (dashed) for Example 1

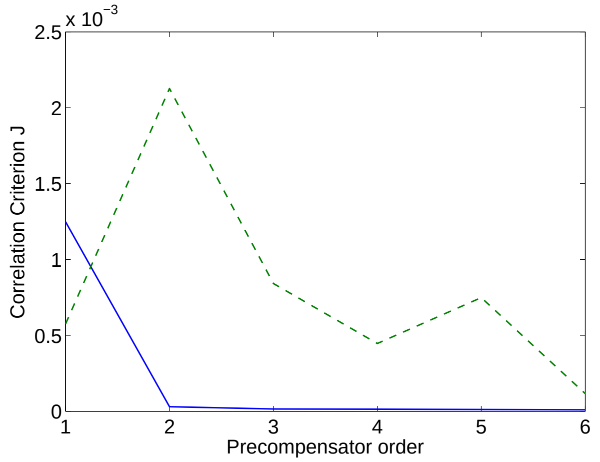

Fig. 4. Correlation criterion (solid) and controller order selection criterion (dashed) versus n_ρ for Example 1

gives the minimum value of the correlation criterion for this order. The calculated non-causal precompensator is, thus :

$$F(q^{-1}) = 4.950q^4 - 7.380q^3 + 3.430q^2$$

which can be seen to be very close to $T^{-1}(q^{-1})$, the difference being due to the use of a finite number of data. Fig. 5 compares the output of the original closed-loop system without a precompensator with the output of the system using the precompensator in the absence of noise. Noise is not present in this validation-type simulation so that the true tracking obtained using a precompensator, tuned in the presence of noise, is clearly visible. The output of the system with a precompensator tuned to minimize the variance of $\varepsilon_p(t)$ is also shown, the precompensator found using this method being

$$F(q^{-1}) = 0.867q^4 + 0.150q^3 - 0.050q^2$$

which is far from the inverse of $T(q^{-1})$. Table I compares the 2-norm of the tracking error obtained in the validation simulation i.e. when noise was not present. It can be observed that near perfect tracking performance can be obtained with only one set of data using the proposed method. However, the

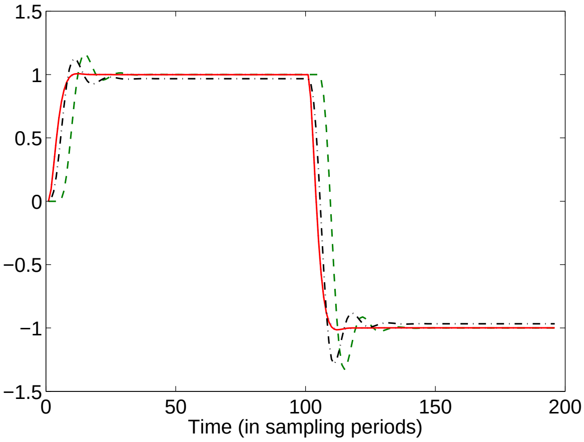

Fig. 5. Desired output (dotted), tracking obtained in the validation simulation for Example 1, in the absence of noise, for system: without precompensator (dashed), with precompensator tuned using the correlation approach (solid/superimposed on desired output) and with precompensator tuned to minimize the variance of $\varepsilon_p(t)$ (dash-dot).

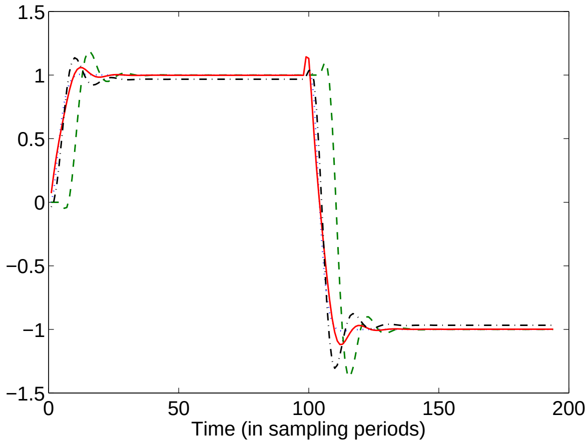

Fig. 6. Desired output (dotted), tracking obtained in the validation simulation for Example 2, in the absence of noise, for system: without precompensator (dashed), with precompensator tuned using the correlation approach (solid/partially superimposed on desired output) and with precompensator tuned to minimize the variance of $\varepsilon_p(t)$ (dash-dot).

TABLE I

2-NORM OF TRACKING ERROR OBTAINED IN THE NOISE-FREE VALIDATION SIMULATION FOR EXAMPLE 1

System	$\|e(t)\|_2$
Without Precompensator	0.3420
With Precompensator tuned with the correlation approach	0.0023
With Precompensator tuned by minimizing the variance of $\varepsilon_p(t)$	0.1142

TABLE II

2-NORM OF TRACKING ERROR OBTAINED IN THE NOISE-FREE VALIDATION SIMULATION FOR EXAMPLE 2

System	$\|e(t)\|_2$
Without Precompensator	0.4056
With Precompensator tuned with the correlation approach	0.0521
With Precompensator tuned by minimizing the variance of $\varepsilon_p(t)$	0.0899

presence of noise on $y_m(t)$ causes a bias on the parameters of the precompensator identified by minimising the variance of $\varepsilon_p(t)$, leading to reduced tracking improvement.

Example 2: A second simulation example is carried out with a different closed-loop system:

$$T(q^{-1}) = \frac{-0.2q^{-3} + 0.4q^{-4}}{1 - 1.5q^{-1} + 0.7q^{-2}}. \tag{39}$$

This $T(q^{-1})$ is chosen to represent a more complicated system whose exact inverse is unstable and cannot be represented as an FIR. The same $y_d(t)$ is used as in Example 1 and the precompensator is tuned in the same way, with $n = 25$ again. The controller structure selection algorithm gives $n_\rho = 2$ and $\delta = 6$ and the precompensator calculated is

$$F(q^{-1}) = 3.851q^6 - 6.157q^5 + 3.305q^4.$$

The precompensator tuned to minimize the variance of $\varepsilon_p(t)$ is found as

$$F(q^{-1}) = 0.643q^6 + 0.166q^5 + 0.159q^4.$$

Again a noise-free validation simulation is carried out to test the precompensator tuned in the presence of noise. The noiseless output of the closed-loop system without a precompensator and with precompensators tuned with the two methods is shown in Fig. 6. Table II shows the corresponding $\|e(t)\|_2$ values. It can be seen that due to the non-minimum phase zero in $T(q^{-1})$ and the fact that a stable exact inverse in FIR form does not exist the near perfect tracking achieved in Example 1 is not possible in this example. Nonetheless the proposed method improves the tracking greatly and gives a $\|e(t)\|_2$ value which is just over half that obtained using the precompensator which minimizes the variance of $\varepsilon_p(t)$.

B. Experimental Results

The proposed precompensator tuning method is applied to a linear, permanent magnet, synchronous motor (LPMSM) (see Fig. 7). LPMSM's are very rigid and have no mechanical transmission components. This means they are not afflicted by backlash and thus very high precision positioning is achievable. Additionally they are capable of high velocities and accelerations. These properties make them a very appealing, and thus common, choice for use in the inspection process in the semi-conductor industry, where rapid, high precision movements are required. However, their use in an industrial situation means they are liable to wear and so parameter change. A two-degree-of-freedom controller is thus well suited to this application as the feedback controller can be tuned to give robust stability and the precompensator tuned separately for high tracking accuracy. The traditional route for precompensator tuning is very labour intensive as requires extremely precise identification in order to obtain the precision necessary, if in fact possible. Additionally, it requires that this be repeated each time the system parameters change. The proposed method is thus highly suited to this application as it provides a fast

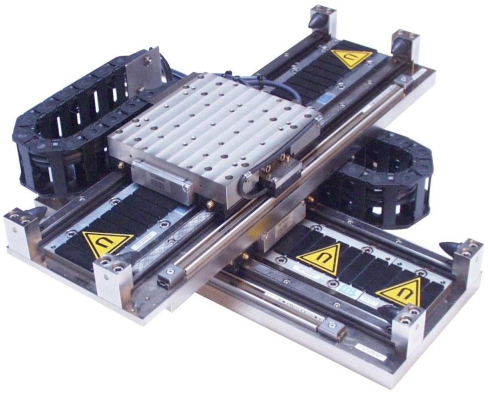

Fig. 7. Linear, permanent magnet, synchronous motor (courtesy of ETEL)

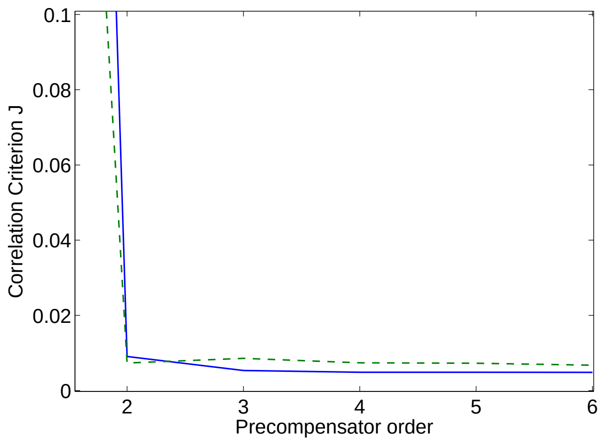

Fig. 8. Correlation criterion (solid) and controller order selection criterion (dashed) versus n_p for the experimental results

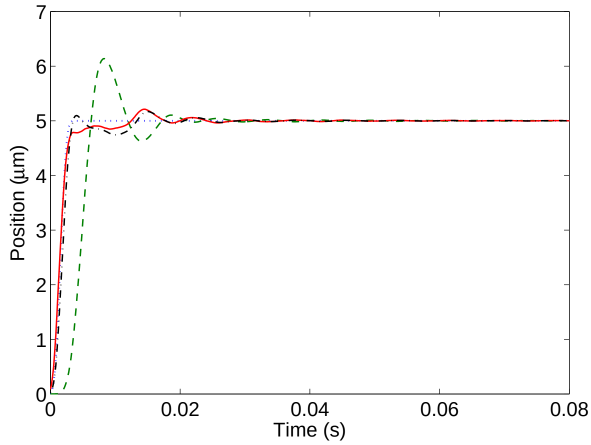

Fig. 9. Desired output (dotted), output without precompensator (dashed), output with precompensator tuned using the correlation approach (solid) and output with precompensator tuned to minimize the variance of $\varepsilon_p(t)$ (dash-dot) for the experimental results

and efficient way to tune the precompensator parameters. This method, being based directly on measured data, does not suffer from modelling uncertainty, and thus is capable of achieving the required tracking accuracy.

The motor used in the experiment is controlled by a standard two-degree-of-freedom position controller, which contains an integrator and operates at a sampling frequency of 6kHz. It was tuned previously in order to achieve robust stability. An analog position encoder using sinusoidal signals with periods of 2μm, which are then interpolated with 8192 intervals/period to obtain a resolution of 0.24nm, is used to measure the motor's position. However, the accuracy of this type of encoders is limited to 20nm.

The proposed algorithm was used to tune a precompensator for a desired output, $y_d(t)$. The desired output was taken as a so-called "S-Curve", a low-pass filtered version of a step which is less abrupt than a true step, and is a typical movement made during the wafer inspection process. The S-curve is defined in terms of the desired displacement, the maximum velocity, the maximum acceleration and the jerk time. The particular S-curve desired had a maximum displacement of 5μm, a velocity of 0.5m/s, an acceleration of 3 m/s^2 and a jerk time of 0s. The duration of $y_d(t)$ was such that $N = 1200$.

$y_d(t)$ was applied to the closed-loop system, without a precompensator, to obtain $y_m(t)$. The precompensator order and preview value were selected using the controller structure selection algorithm. A value of $n = 110$ was used, this value being estimated by measuring the settling time of the error signal of the system, without a precompensator, when an impulse was applied as a reference signal. The controller structure selection algorithm leads to a precompensator with an order of $n_p = 3$ and a preview of $\delta = 4$ being used. It can be seen from Fig. 8 that this order is the first to satisfy condition (35). The calculated precompensator was applied to the system and the resulting system output can be seen in Fig. 9. In a similar way to the simulation a precompensator, with the same structure, was also calculated in order to minimize the variance of $\varepsilon_p(t)$. The system response using this precompensator is also shown in Fig. 9. As measures of performance the 2-norm, $\|e(t)\|_2$, of the error signal, the maximum overshoot and the settling time to within 2% of the final value were taken. Table III shows the results obtained without and with the precompensator. It is clearly seen that the proposed technique greatly improves the system's tracking performance compared to the original performance. Compared to the performance obtained using the precompensator tuned to minimize the variance of $\varepsilon_p(t)$, the proposed technique can be seen, from $\|e(t)\|_2$, to give better general tracking. The benefit of the proposed technique, however, is not as obvious as in the simulation because the noise-to-signal ratio in this application is much less.

V. CONCLUSIONS

A model-free approach to precompensator tuning based on the correlation approach has been proposed. It was shown that using only one set of data and some specific tuning schemes the controller parameters can be tuned for desired output tracking or the model following problem. The approach is based on a correlation criterion which is not asymptotically sensitive to noise and can be minimized using the least squares algorithm. The frequency-domain analysis of the criterion showed that

TABLE III

SYSTEM TRACKING PERFORMANCE

System	$\|e(t)\|_2$ (μm)	Overshoot (μm)	Settling Time (s)
Without Precomp.	0.5128	1.1367	0.0188
With Precomp. tuned with the correlation approach	0.0490	0.0212	0.0162
With Precomp. tuned by minimizing the variance of $\varepsilon_p(t)$	0.0763	0.0171	0.0163

the resulting controller is a weighted approximation of the inverse of the closed-loop system in the two-norm sense. The effectiveness of the method has been illustrated via simulation and experimental results.

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